

ML & Opt Lecture 18

Convolutional Neural Networks (CNNs)

- convolutional filters
- CNN architecture
- LeNet 5

Why CNNs

- appropriate for computer vision tasks:

- bounding boxes for objects
- background subtraction
- image classification

(also used for problems from other domains)

- key to good performance: choice of feature map

$$\phi: \mathbb{R}^{k \times k} \rightarrow \mathbb{R}^D \quad (\text{implemented as a NN})$$

these feature maps can be learned on one dataset and used on others

- so far in this class, to do image classification:

- ϕ from flattening image into a vector

- ϕ from autoencoder (use encoder as feature map)

- ϕ from MLP

then use logistic regression

CNNs: another, biologically inspired, approach to learning good feature maps for vision problems

use convolutional filters

1	2	1	0	0
2	1	1	2	0
0	0	1	2	1
2	1	1	1	0
0	1	2	0	2

I

*

-1	0	-1
0	4	0
-1	0	-1

K

=

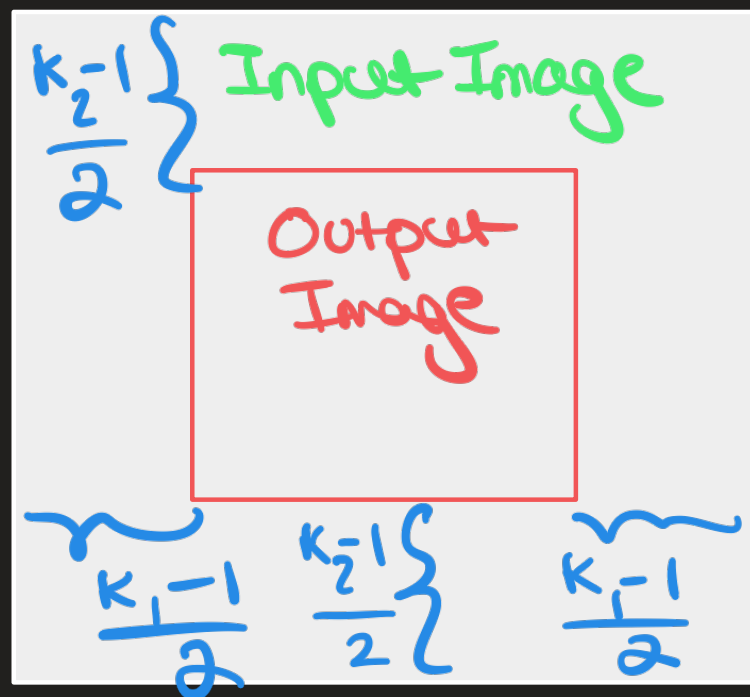
1	0	5
-6	-1	6
1	1	-2

$I * K$

- range may be different
- $I * K$ is smaller than I

our basic CNN layer takes as input an image I, convolves it with filter K, and returns $\sigma(I * K)$

Note that the filters always have size $k_1 \times k_2$ where k_1 & k_2 are odd



If the input image is size $d_1 \times d_2$, the output image is size

$$d_1 - (k_1 - 1) \times d_2 - (k_2 - 1)$$

-1,1	0,1	1,1
-1,0	0,0	1,0
-1,-1	0,-1	1,-1

indices into K

Formula for Convolutions

$$(I * K)_{i,j} = \sum_{m = -\frac{(k_1-1)}{2}}^{\frac{(k_1-1)}{2}}$$

$$\sum_{n = -\frac{(k_2-1)}{2}}^{\frac{(k_2-1)}{2}} I_{i+m, j+n} K_{mn}$$

Idea of CNNs: use convolutional filters to build our nonlinear feature map

Q: Why? A: it has very desirable inductive biases

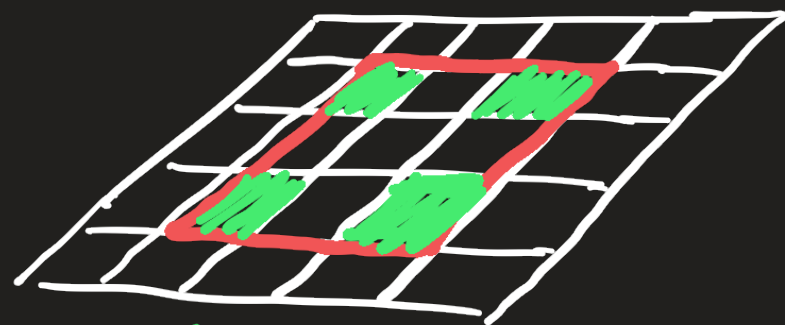
- we expect that vision is translational invariant
- intuition that ^{image} features should be local low-level!

- cheap compared to MLP layer

($k_1 \cdot k_2$ parameters vs $(d_1 - k_1 + 1)(d_2 - k_2 + 1) \times d_1 d_2$)

- easier to optimize b/c parameter sharing

Convolutional layers



Input: $d_1 \times d_2$
Filter: $k_1 \times k_2$
Output: $(d_1 - k_1 + 1) \times (d_2 - k_2 + 1)$
(Stride: 1)

(assuming $k_1 = k_2 = 3$)



$\sigma(I * K)$ learn K with backprop



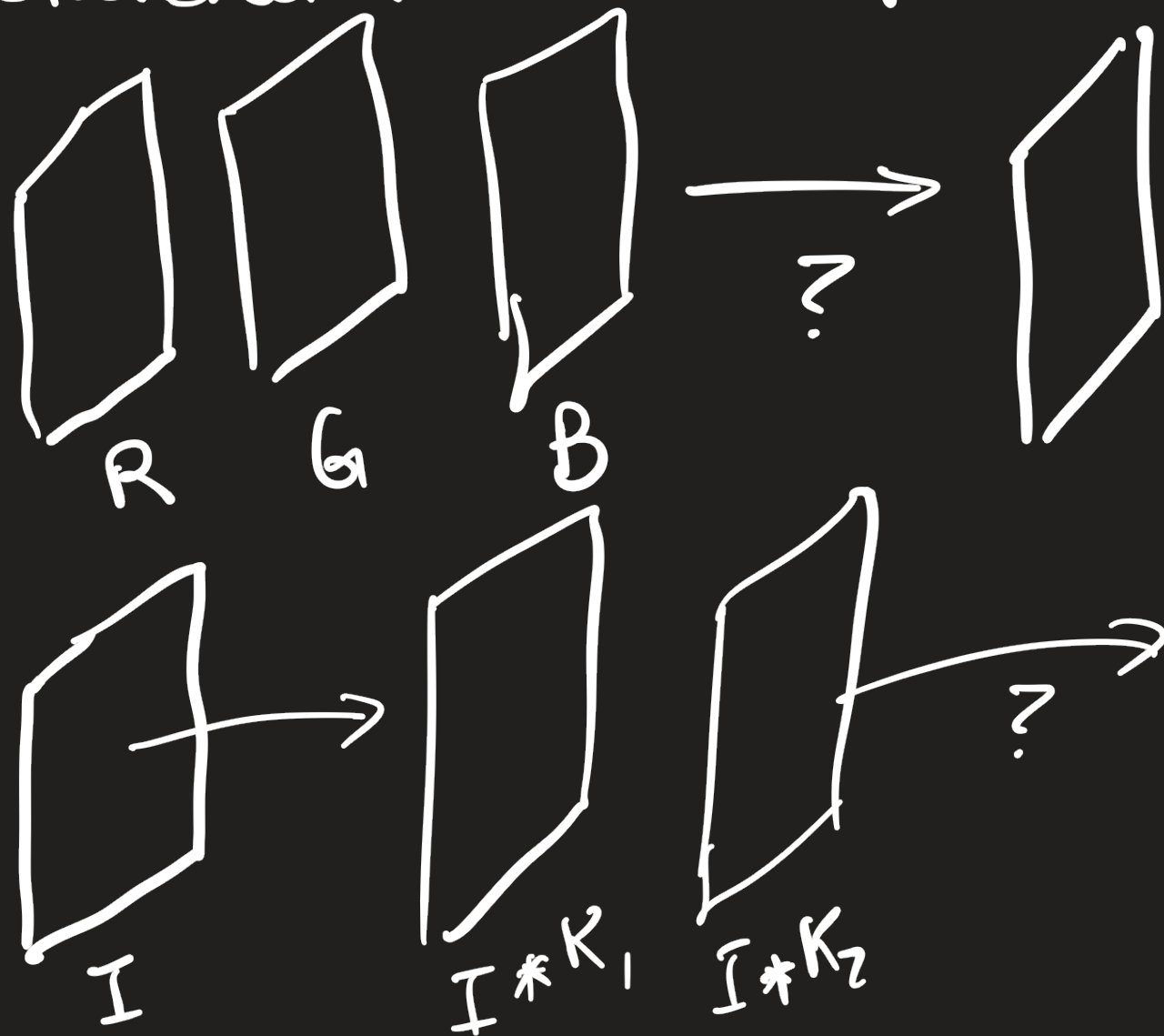
Stride: 2 in each direction.

In general, with a stride of $s_1 \times s_2$ we evaluate the filter at x -intervals of s_1 and y -intervals of s_2

- Equivalent to computing $I * K$ and then throwing away the unused outputs

Multichannel Images

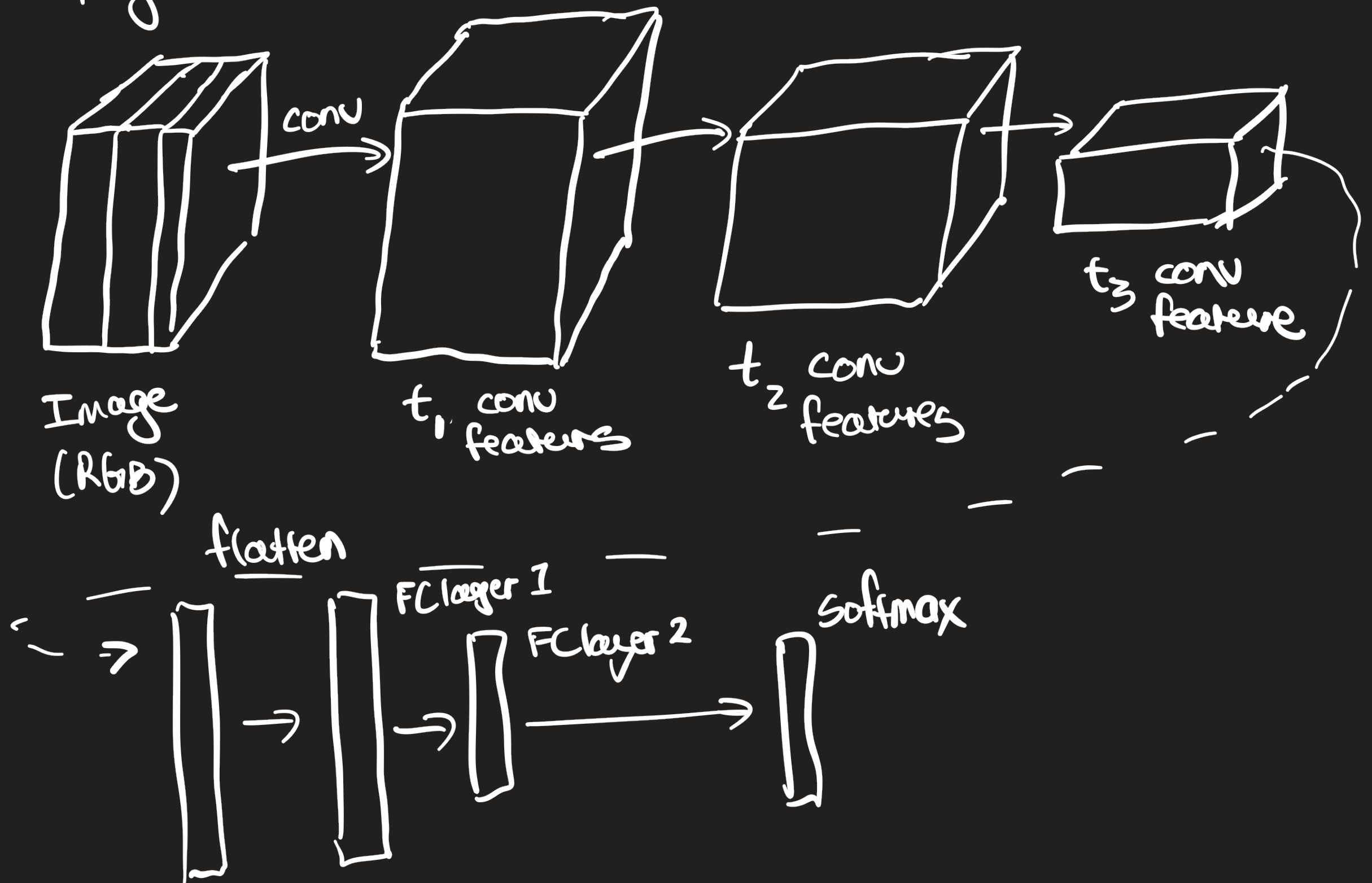
- arise as RGB images or if we use multiple convolutional filters in the previous layer



Given an n channel image $\{I_1, \dots, I_m\}$, one convolutional layer takes the form $\sigma(I_1 * K_1 + I_2 * K_2 + \dots + I_m * K_m)$

CNNs:

- combine convolutional layers with fully-connected layers



Pooling

Pooling reduces the dimensionality of a convolutional layer by aggregating locally

Typical pooling types:

- max pooling
- average pooling

29	15	28	184
0	100	70	39
12	12	7	2
12	12	45	6

pool over
2x2 blocks
→
max

100	184
12	45

Pooling's advantages:

- decreases computation downstream
- makes model more robust to shifts in location of features

Example of a CNN LeNet5 (1997)

5-layer CNN for 10 class classification

Layer	Features	Filter Size	Stride	Activation
Input	1	—	—	—
Conv2D	6	5x5	1	tanh
Avg Pooling	6	2x2	2	—
Conv2D	16	5x5	1	tanh
Avg Pooling	16	2x2	2	—
Conv2D	120	5x5	1	tanh
FC	84	—	—	tanh
FC	10	—	—	softmax