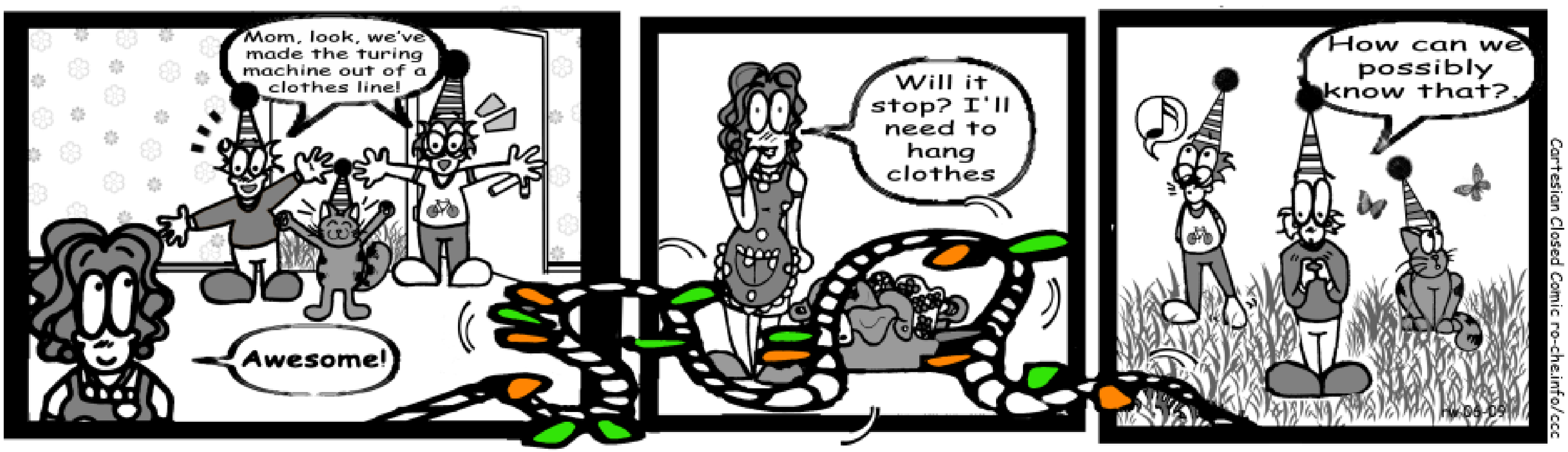
Foundations of Computer Science
Lecture 27

Unsolvable Problems

No Automatic Program Verifier for Hello-World
No Ultimate Debugger or Algorithm for PCP
The Complexity Zoo

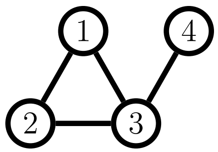


Last Time: Turing Machines

$$\begin{array}{rcl} \text{Intuitive notion of algorithm} & \equiv & \text{Turing Machine} \\ \text{Solvable problem} & \equiv & \text{Turing-}\textit{decidable} \end{array}$$

$$\mathcal{L} = \{\langle G\rangle \mid G \text{ is connected}\}$$

$$\langle G\rangle = \texttt{2; 1; 3; 4 \# 1,2; 2,3; 1,3; 3,4}$$



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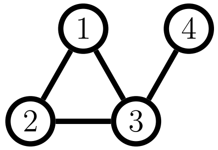

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M = Turing Machine that solves graph connectivity

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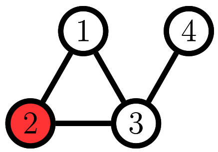

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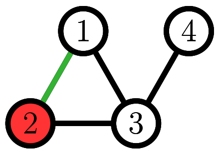

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Mark the unmarked node or GOTO step 3 if there is no such edge.

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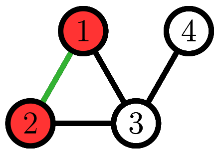

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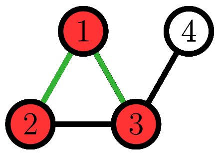

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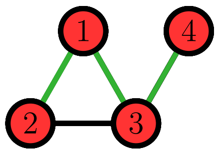

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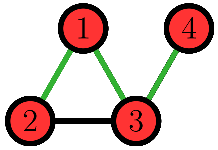

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Mark the unmarked node or GOTO step 3 if there is no such edge.
3: REJECT if there is an unmarked vertex remaining in G; otherwise ACCEPT.

To tell your friend on the other coast about this fancy Turing Machine M, encode its description into the bit-string $\langle M \rangle$ and send over the telegraph.

You want to solve a different problem? Build another Turing Machine!

Today: Unsolvable Problems

1. Programmable Turing Machines.
2. Examples of unsolvable problems.
   - Post's Correspondence Problem (PCP)?
   - HalfSum?
   - Auto-Grade?
   - Ultimate-Debugger?
3. $\mathcal{L}_{\text{TM}}$: The language recognized by a Universal Turing Machine.
   - $\mathcal{L}_{\text{TM}}$ is undecidable – cannot be solved!
4. Auto-Grade and Ultimate-Debugger do not exist.
5. What about HalfSum?

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computer program program input

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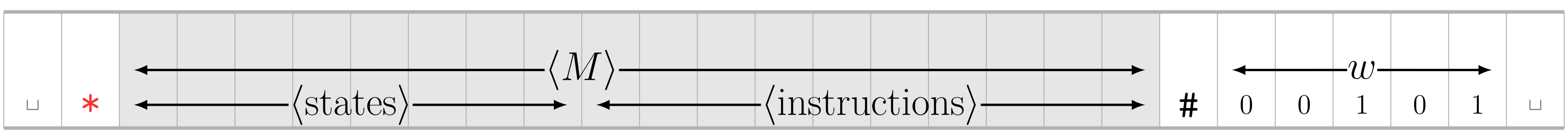

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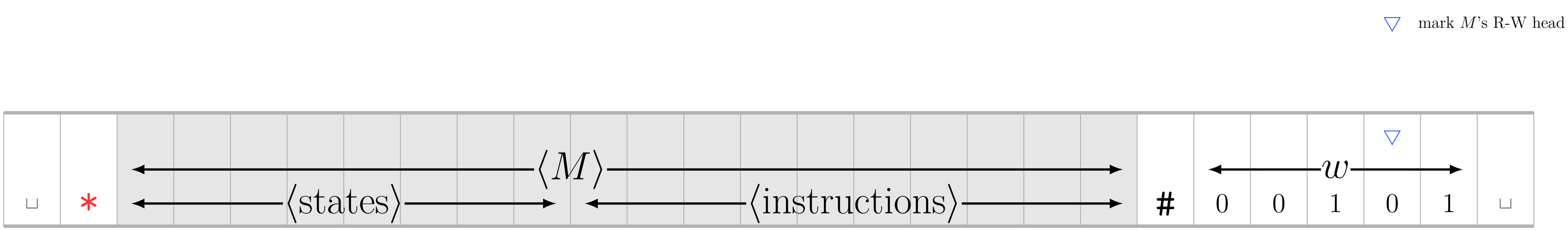

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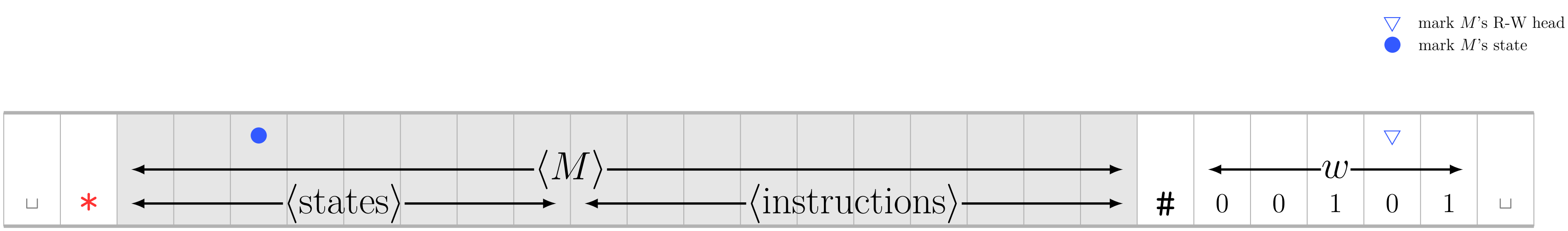

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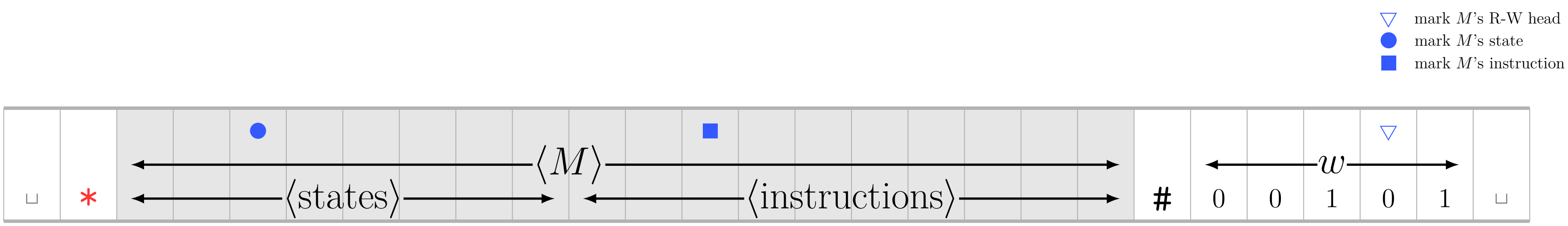

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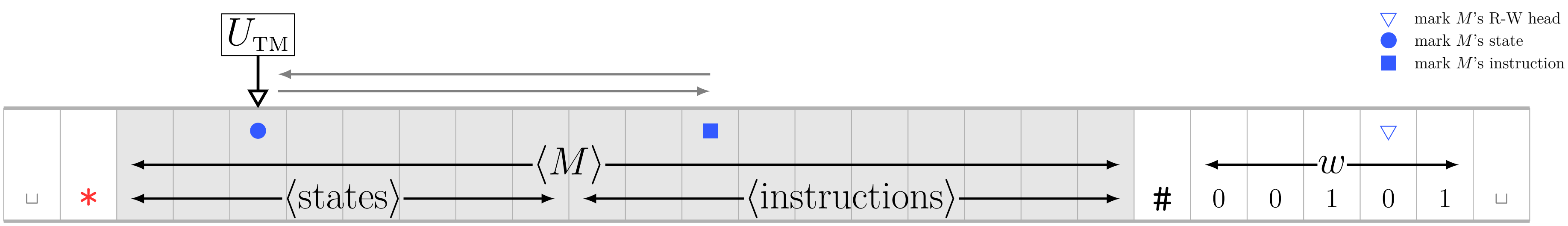

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Post's Correspondence Problem (PCP) and HalfSum

PCP: Consider 3 dominos:

d_1	d_2	d_3
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$$\begin{array}{ccc} d_1 & d_2 & d_3 \\ \boxed{\begin{array}{c} 0 \\ \hline 100 \end{array}} & \boxed{\begin{array}{c} 01 \\ \hline 00 \end{array}} & \boxed{\begin{array}{c} 110 \\ \hline 11 \end{array}} \end{array}$$

$$d_3 d_2 d_3 d_1 \;=\; \boxed{\begin{array}{c} 110 \\ \hline 11 \end{array}}\boxed{\begin{array}{c} 01 \\ \hline 00 \end{array}}\boxed{\begin{array}{c} 110 \\ \hline 11 \end{array}}\boxed{\begin{array}{c} 0 \\ \hline 100 \end{array}} \;=\; \boxed{\begin{array}{c} 110011100 \\ \hline 110011100 \end{array}}$$

$\leftarrow$ Top and bottom strings match. That's the goal.

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INPUT: Dominos $\{d_1, d_2, \ldots, d_n\}$. For example $\left\{\left[\frac{10}{101}\right], \left[\frac{011}{11}\right], \left[\frac{101}{011}\right]\right\}$.

TASK: Can one line up finitely many dominos so that the top and bottom strings match?

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$$d_3d_2d_3d_1 = \begin{array}{|c|c|c|c|}\hline 110 & 01 & 110 & 0\\ \hline 11 & 00 & 11 & 100\\ \hline\end{array} = \begin{array}{|c|}\hline 110011100\\ \hline 110011100\\ \hline\end{array}$$

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HalfSum: Consider the multiset $S = \{1, 1, 1, 3, 4, 4, 5, 6, 9\}$, and subset $A = \{1, 3, 4, 9\}$.

$$\text{sum}(A) = 17 = \tfrac{1}{2} \times \text{sum}(S).$$

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INPUT: Multiset $S = \{x_1, x_2, \ldots, x_n\}$. For example, $S = \{1, 1, 1, 3, 4, 4, 5, 6, 9\}$.

TASK: Is there a subset whose sum is $\frac{1}{2} \times \text{sum}(S) = \frac{1}{2} \times (x_1 + x_2 + \cdots + x_n)$?

Auto-Grade

Your first CS assignment: Write a program to print "Hello World!" and halt.

CS1: 700+ submissions!

Naturally, we do not grade these by hand.

Auto-Grade: runs each submission and determines if its correct. ←**program verification**

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What does Auto-Grade say for this program:

```
n = 4;
while(n > 0){
  if(n is not a sum of two primes){
    print("Hello World!") and exit;
  }
n ← n + 2;
}
```

ULTIMATE-DEBUGGER

Wouldn't it be nice to have the ULTIMATE-DEBUGGER. ← solves the *Halting Problem*

HALTS(
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) $= \begin{cases} \boxed{\text{YES}} & \text{if program halts} \\ \boxed{\text{NO}} & \text{if program infinitely loops} \end{cases}$

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$$\text{HALTS}\left(\boxed{\begin{array}{l}\texttt{n = 4;}\\ \texttt{while(n > 0)\{}\\ \quad\texttt{if(n is not a sum of two primes)\{}\\ \qquad\texttt{print("Hello World!") and exit;}\\ \quad\texttt{\}}\\ \texttt{n} \leftarrow \texttt{n + 2;}\\ \texttt{\}}\end{array}}\right) = \begin{cases}\boxed{\text{YES}} & \text{if program halts}\\ \boxed{\text{NO}} & \text{if program infinitely loops}\end{cases}$$

- We can grade the students program correctly.
- We can solve Goldbach's conjecture.
- Just think what you could do with ULTIMATE-DEBUGGER.
  - No more infinite looping programs.

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- A decider must *always* halt with an answer.
- U_{TM} may loop forever if M loops forever on w.
- Question: What do these mean: $M(\langle M\rangle)$ and $A_{\text{TM}}(\langle M\rangle\texttt{\#}\langle M\rangle)$?

Verification: Does A Program Successfully Terminate?

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U_{TM} is a *recognizer* for $\mathcal{L}_{\text{TM}}$.

Is there a Turing Machine A_{TM} which **decides** $\mathcal{L}_{\text{TM}}$?

- A decider must *always* halt with an answer.
- U_{TM} may loop forever if M loops forever on w.
- Question: What do these mean: $M(\langle M\rangle)$ and $A_{\text{TM}}(\langle M\rangle\#\langle M\rangle)$?

A diabolical Turing Machine D built from A_{TM}:

D = "Diagonal" Turing Machine derived from A_{TM} (the decider for $\mathcal{L}_{\text{TM}}$)

input: $\langle M\rangle$ where M is a Turing Machine.

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D does the *opposite* of A_{TM}. Is D a decider?

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A diabolical Turing Machine D built from A_{TM}:

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input: $\langle M\rangle$ where M is a Turing Machine.
1: Run A_{TM} with input $\langle M\rangle\#\langle M\rangle$.
2: If A_{TM} accepts then REJECT; otherwise (A_{TM} rejects) ACCEPT

D does the *opposite* of A_{TM}. Is D a decider?

Theorem. A_{TM} does not exist ($\mathcal{L}_{\text{TM}}$ Cannot be Solved)

A_{TM} exists $\rightarrow$ D exists.

D exists means it will appear on the list of all Turing Machines,

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Exercise. Show that AUTO-GRADE does not exist.

Exercise. Show that HALFSUM is solvable by giving a decider.

Ultimate-Debugger and Auto-Grade Don't Exist

No *general* program/algorithm to analyze *any* other program M and tell if M will accept or not a particular input.

- No Ultimate-Debugger to analyze other programs and tell if they halt.
- No Auto-Grade for CS-1 programs.
- No solver for PCP.

Suppose Ultimate-Debugger H_{TM} exists and *decides* if any other program halts.

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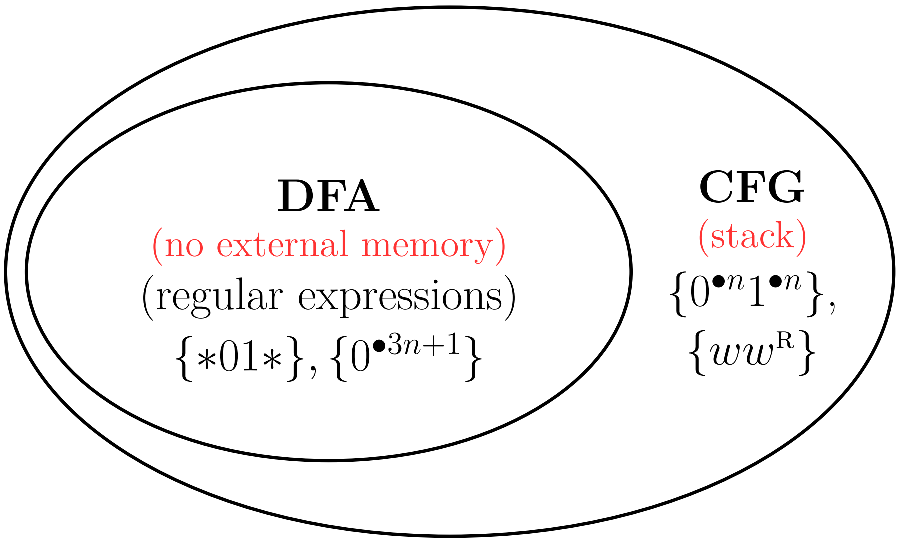
DFA
(no external memory)
(regular expressions)
$\{*01*\}, \{0^{\bullet 3n+1}\}$

The Landscape



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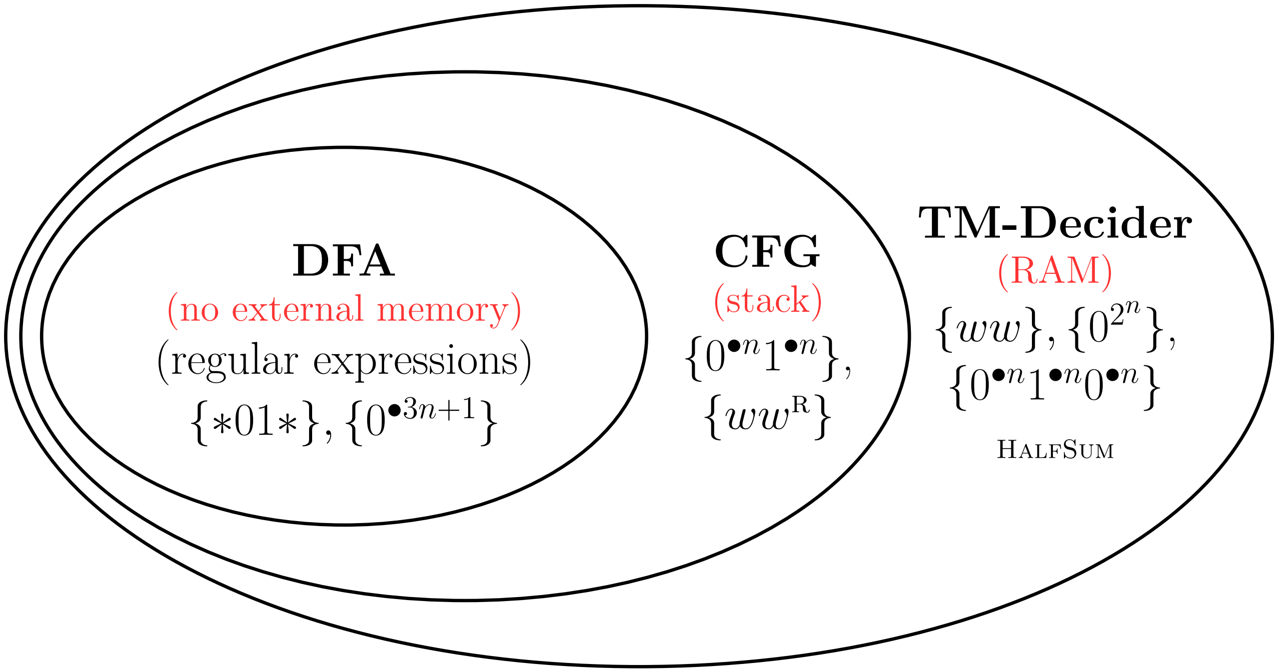

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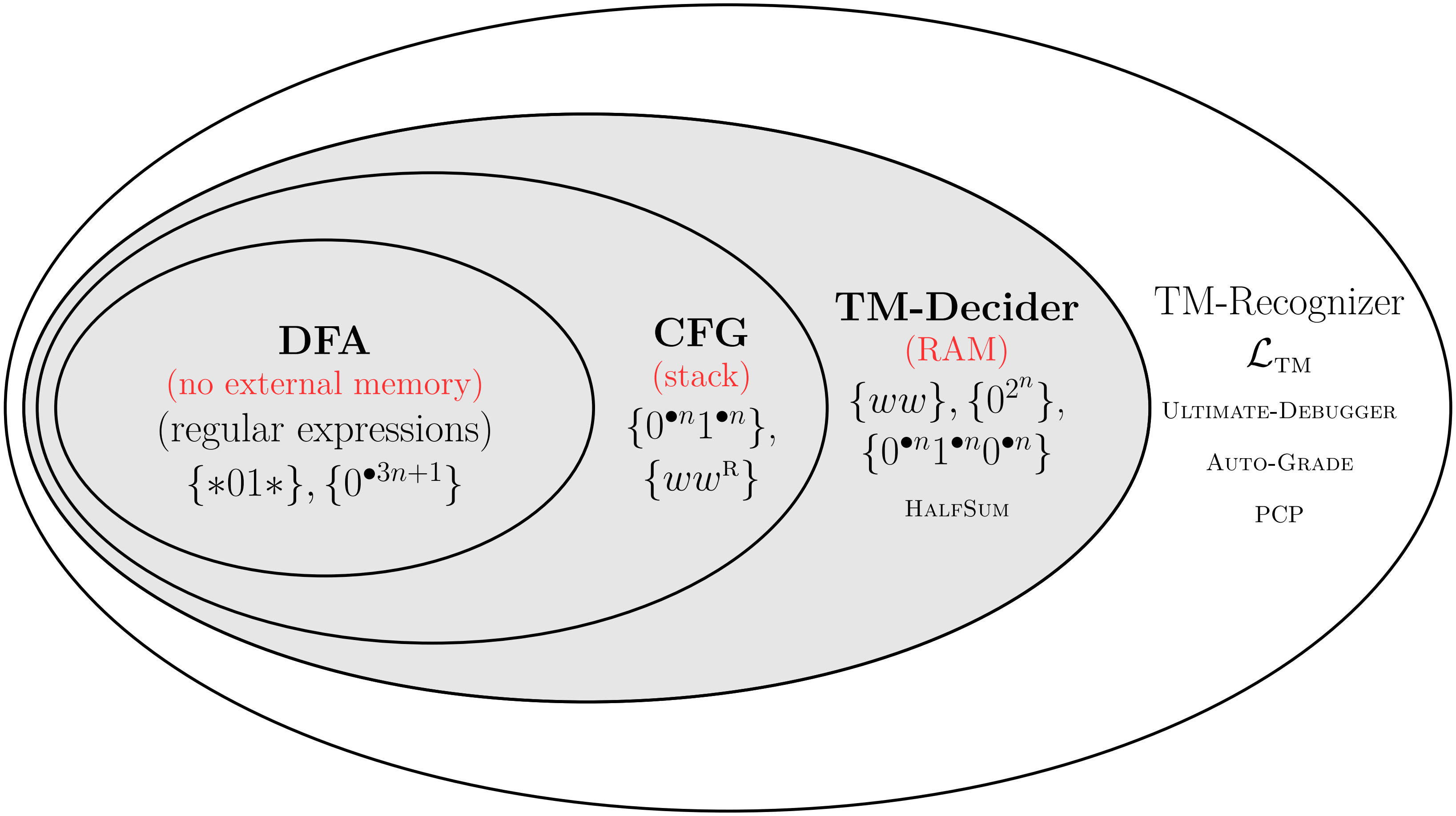

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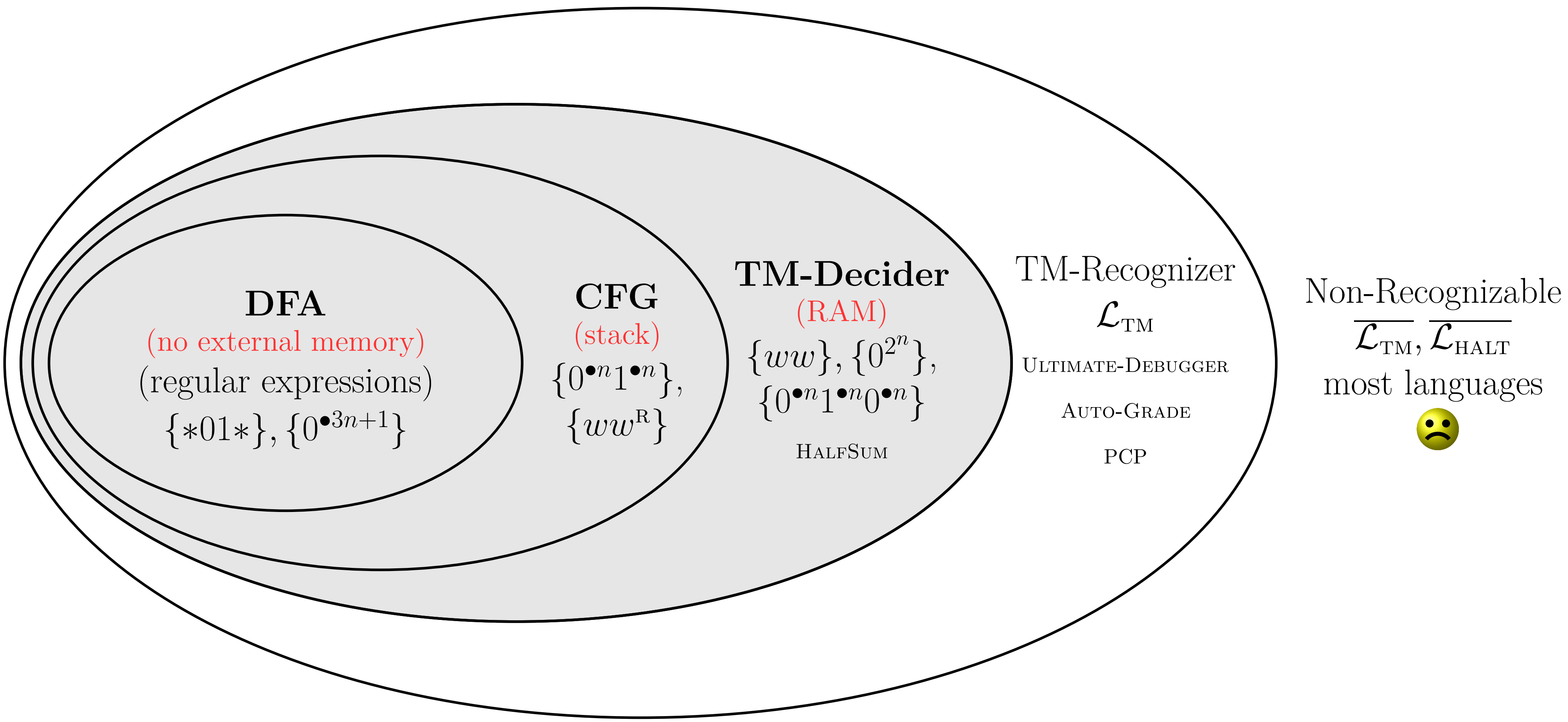

The Path Forward: Focus on Decidable Problems

FOCS

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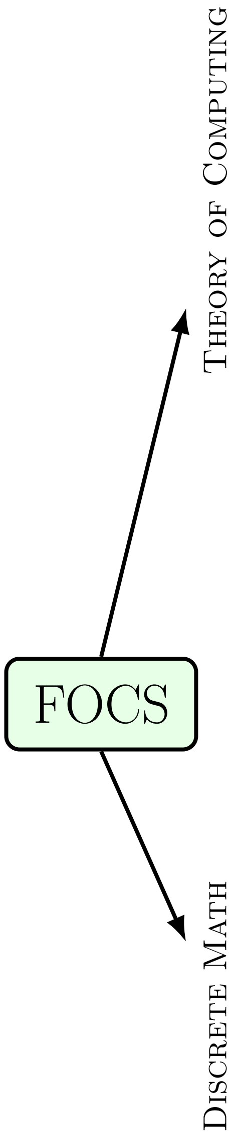

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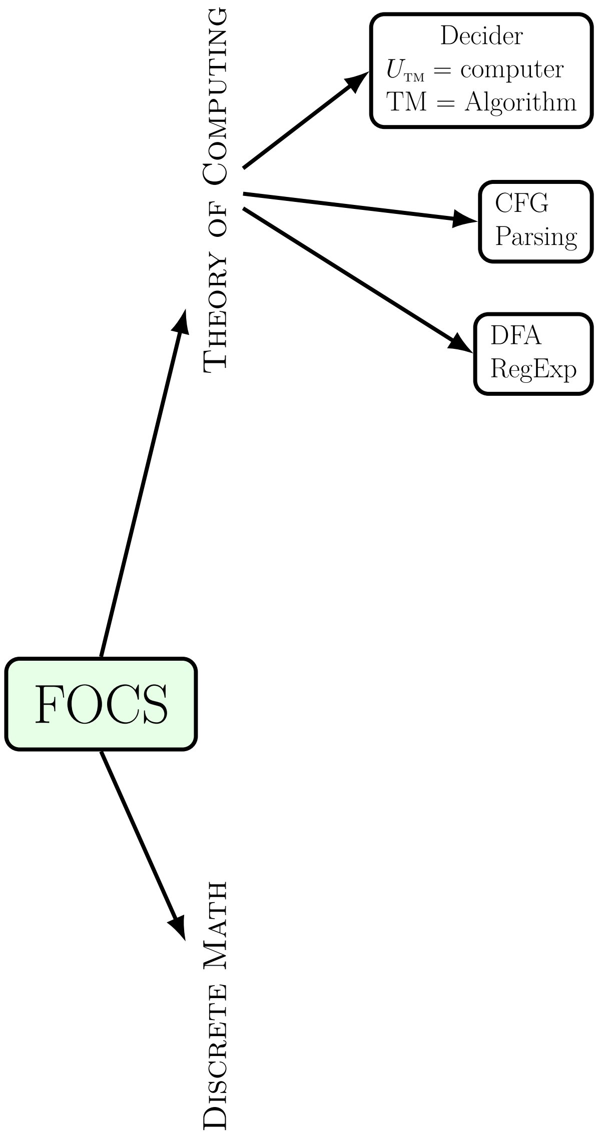

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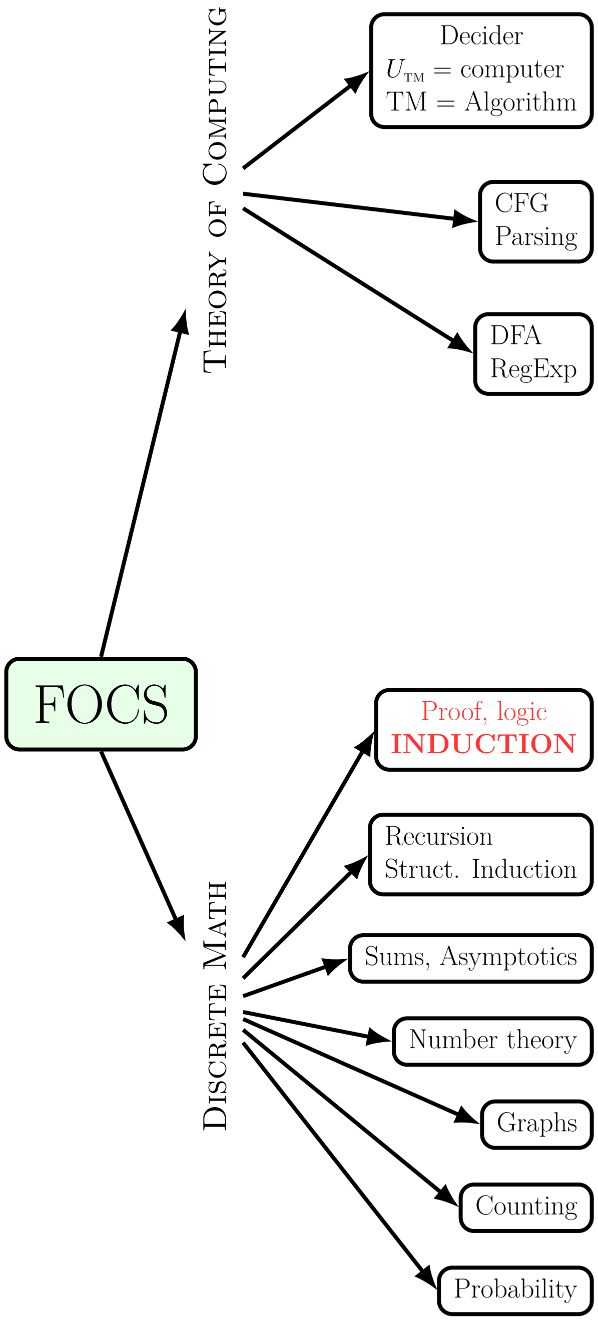

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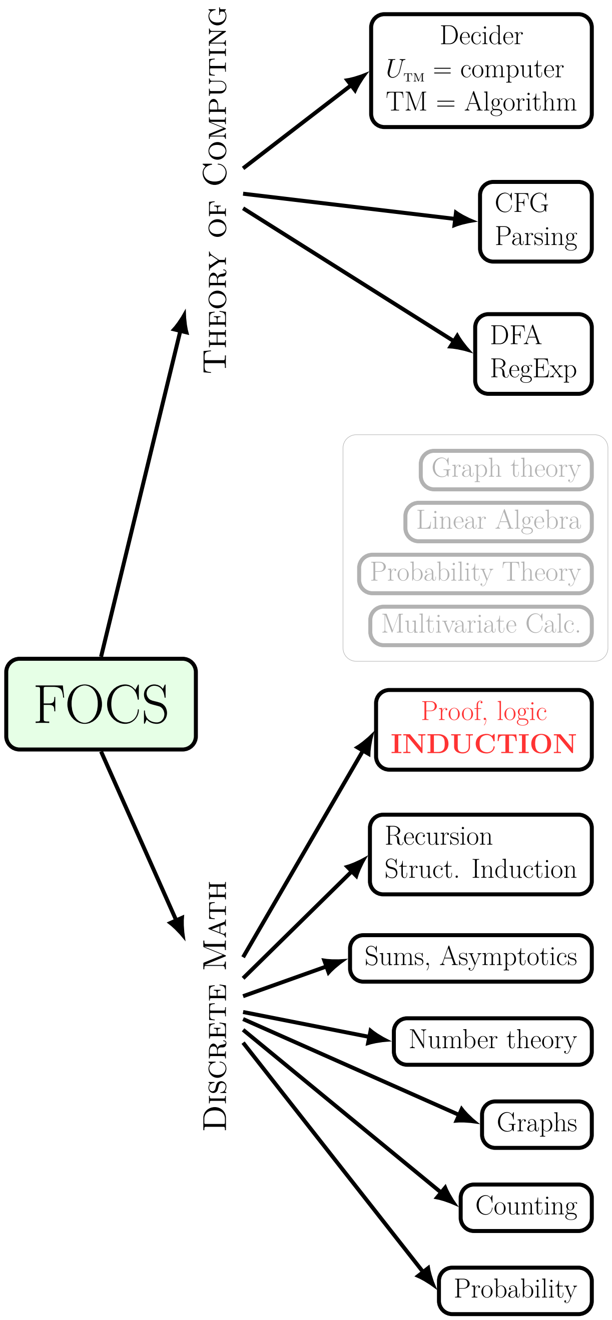

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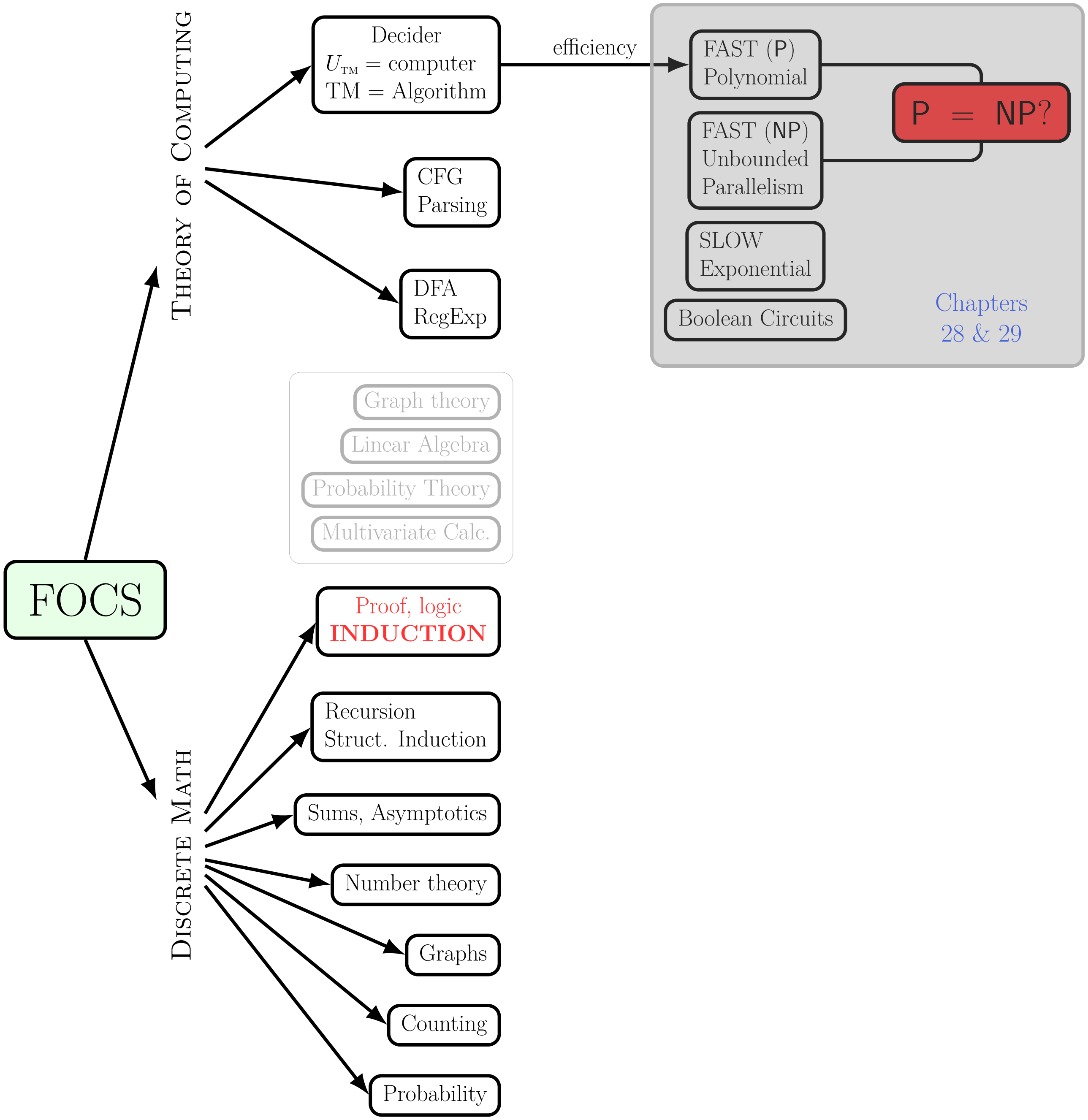

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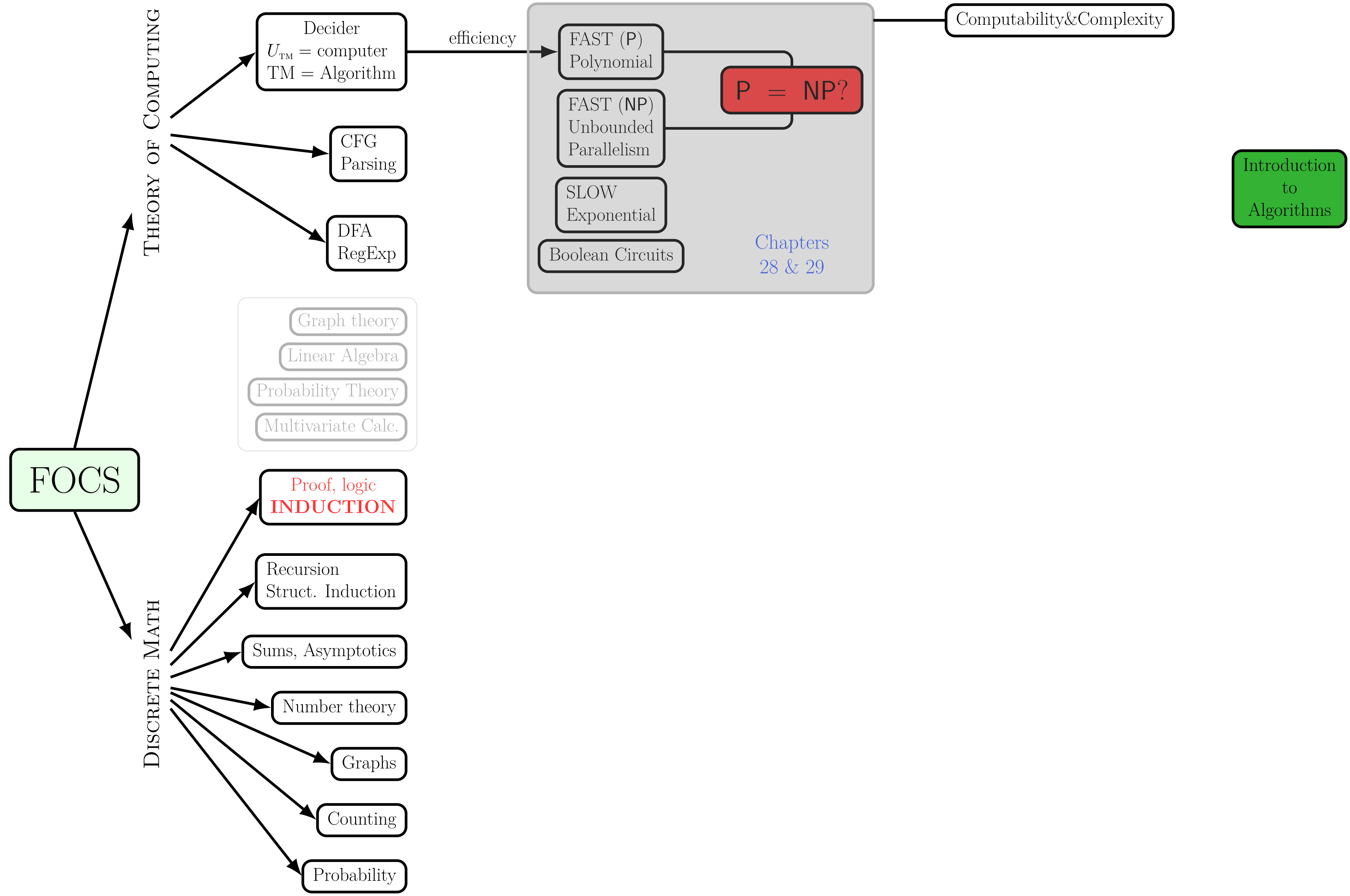

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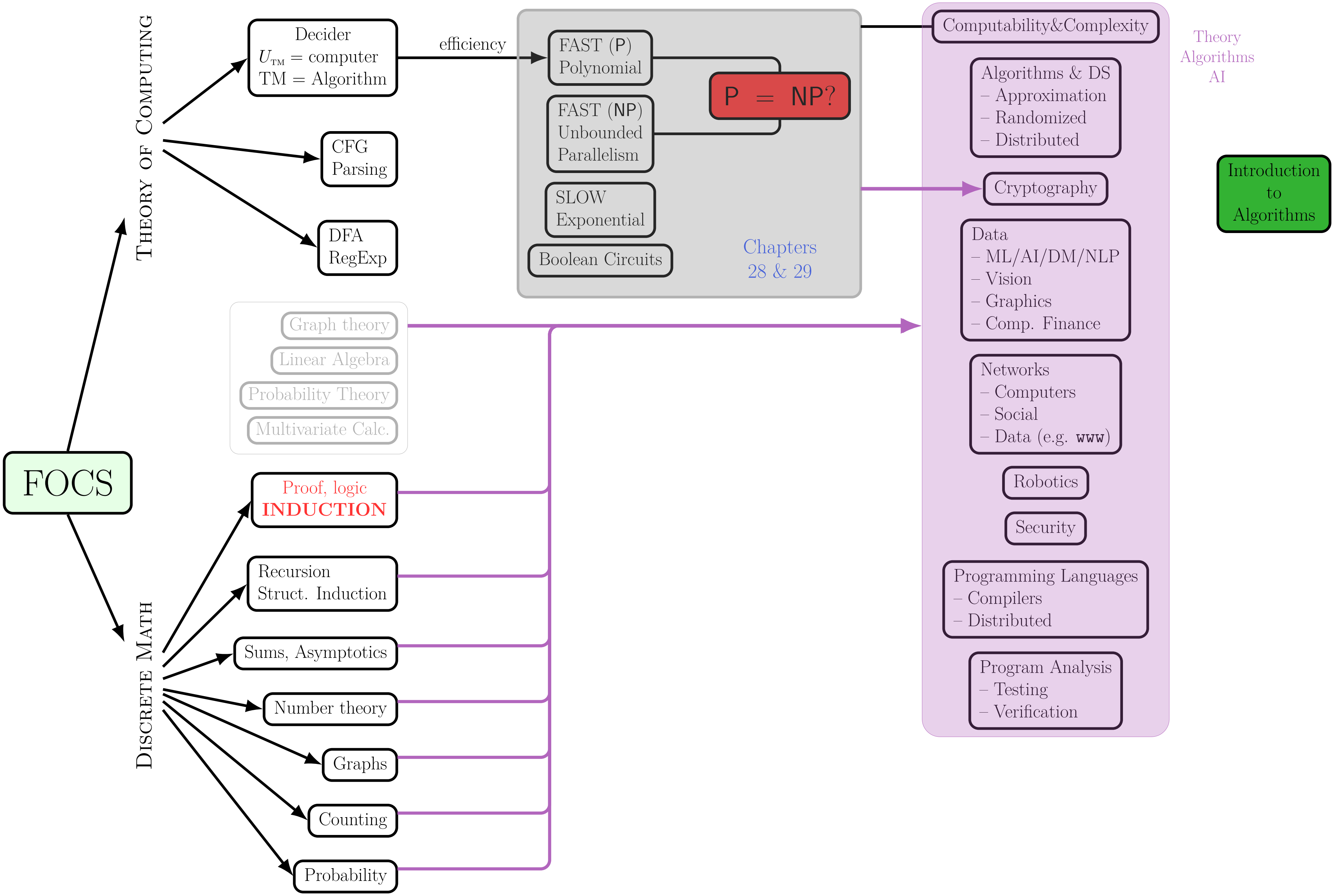

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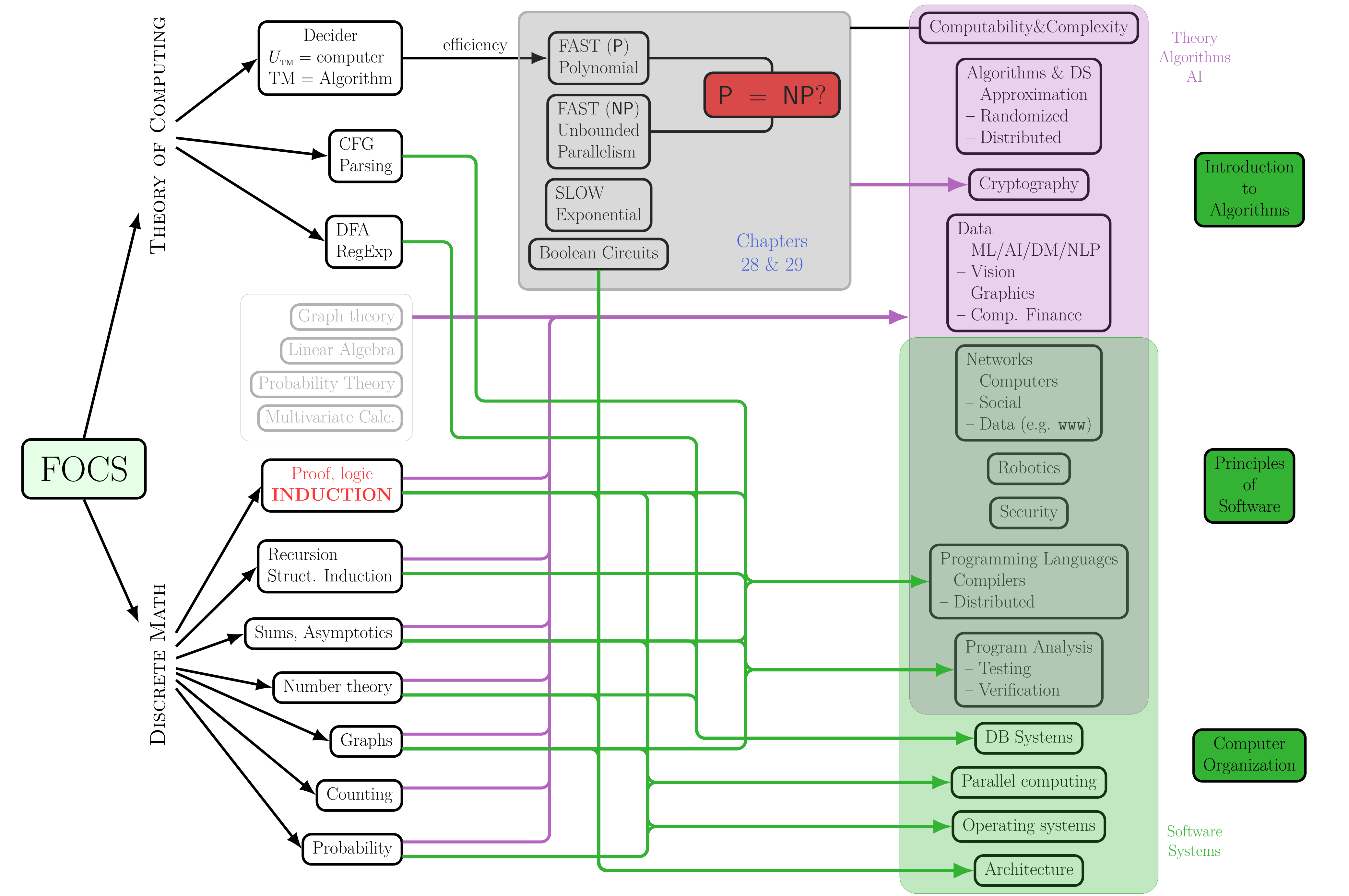

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- **Therac 25 (1985).** Concurrent programming bug killed patients through massive 100$\times$ radiation overdose.
- **AT&T Lines Go Dead (1990).** 75 million calls dropped (one line of buggy code in software upgrade).
- **Patriot missile defense fails (1991). 28 soldiers dead, 100 injured** (rounding error in scud-detection).
- **Pentium floating point long-division bug (1993).** Cost: $475 million – flawed division table.
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- **Y2K (1999).** Cost: $500 billion spent because year was stored as 2 digits to save space.
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- **Tesla Self-Driving Car (2016). 1 dead**. Auto-pilot didn't "see" tractor-trailer.
- **Financial Disasters:** London Stock Exchange down due to single server bug (**2009**; billions of pounds of trading); Knight Capital computer glitch trigers stock sale (**2012**; 500 million lost and Knight's value drops by 75%).
- **Airline Disasters:**
  - AirFrance 447 2009, **228 dead**: pitot-tube failure feeds inconsistent data to programs which then panic pilot.
  - Spanair 5022, 2008, **154 dead**: malware virus.
  - AdamAir 574, 2007, **102 dead**: navigation system errors (and pilot errors).
  - KoreanAir 801, 1997, **228 dead**: ground proximity warning system bug.
  - AeroPerú 603, 1996, **70 dead**: altimeter failures.
  - Scottish RAF Chinook, 1994, **29 dead**: faulty test program
  - AirFrance 296, 1988, **3 dead**: altimeter bug.
  - IranAir 655, 1988, **290 dead**: shot down by US Aegis combat system (misidentified as attacking military plane).
  - KoreanAir 007, 1983, **269 dead**: autopilot took plane into Soviet airspace where it got shot down.
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- **Software errors cost the U.S. $60 billion annually in rework, lost productivity and actual damages.**

Put effort to make *sure* your program works **fully** correctly **all** the time.