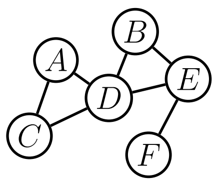
Foundations of Computer Science
Lecture 3

Making Precise Statements

Propositions
Compound Propositions and Truth Tables
Predicates and Quantifiers

Last Time

1. Sets, $\{3, 5, 11\}$
2. Sequences, 100111001
3. Graphs, 

4. Examples of basic proofs.
   - In 4 rounds of group dating, no one meets more than 12 people.
   - x^2 is even "is the same as" x is even.
   - In *any* group of 6 people there is an orgy of 3 mutual friends or a war of 3 mutual enemies.
   - **Axiom:** The Well Ordering Principle
   - $\sqrt{2}$ is not rational.

Today: Making Precise Statements

1. Making a precise statement: the proposition
2. Complicated precise statements: the compound proposition
   - Truth tables
3. Claims about many things
   - Predicates
   - Quantifiers
   - Proofs with quantifiers

Statements can be Ambiguous

1. 2+2=4.

Statements can be Ambiguous

1. 2+2=4. T

Statements can be Ambiguous

1. 2+2=4. T
2. 2+2=5.

Statements can be Ambiguous

1. 2+2=4. T
2. 2+2=5. F

Statements can be Ambiguous

1. 2+2=4. T
2. 2+2=5. F
3. You may have cake <u>OR</u> ice-cream.

Statements can be Ambiguous

1. 2+2=4. **T**
2. 2+2=5. **F**
3. You may have cake <u>OR</u> ice-cream. (Can you have both?)

Statements can be Ambiguous

1. 2+2=4. T
2. 2+2=5. F
3. You may have cake <u>OR</u> ice-cream. (Can you have both?)
4. <u>IF</u> pigs can fly <u>THEN</u> you get an A.

Statements can be Ambiguous

1. $2+2=4$. T
2. $2+2=5$. F
3. You may have cake OR ice-cream. (Can you have both?)
4. IF pigs can fly THEN you get an A. (Pigs can't fly. So, can you get an A?)

Statements can be Ambiguous

1. 2+2=4. **T**
2. 2+2=5. **F**
3. You may have cake <u>OR</u> ice-cream. (Can you have both?)
4. <u>IF</u> pigs can fly <u>THEN</u> you get an A. (Pigs can't fly. So, can you get an A?)
5. <u>EVERY</u> person has <u>A</u> soul mate.

Statements can be Ambiguous

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   - 5(a) There is a single soul mate that <u>EVERY</u> person shares.

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5. <u>EVERY</u> person has <u>A</u> soul mate.
   - 5(a) There is a single soul mate that <u>EVERY</u> person shares.
   - 5(b) <u>EVERY</u> person has their own special soul mate.

Statements can be Ambiguous

1. 2+2=4. T
2. 2+2=5. F
3. You may have cake OR ice-cream. (Can you have both?)
4. IF pigs can fly THEN you get an A. (Pigs can't fly. So, can you get an A?)
5. EVERY person has A soul mate.
   - 5(a) There is a single soul mate that EVERY person shares.
   - 5(b) EVERY person has their own special soul mate.

Why is ambiguity bad? **Proof!**

Statements can be Ambiguous

1. 2+2=4. T
2. 2+2=5. F
3. You may have cake <u>OR</u> ice-cream. (Can you have both?)
4. <u>IF</u> pigs can fly <u>THEN</u> you get an A. (Pigs can't fly. So, can you get an A?)
5. <u>EVERY</u> person has <u>A</u> soul mate.
   - 5(a) There is a single soul mate that <u>EVERY</u> person shares.
   - 5(b) <u>EVERY</u> person has their own special soul mate.

Why is ambiguity bad? **Proof!**

We asked questions of our friends to prove 5(b).

A says Sue's their soul mate;
B says Joe's their soul mate;
C says Sue's their soul mate;
D's soul mate is a red Porshe;
E says Sue's their soul mate;
F says Sam's their soul mate.

Statements can be Ambiguous

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2. 2+2=5. F
3. You may have cake <u>OR</u> ice-cream. (Can you have both?)
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Why is ambiguity bad? **Proof!**

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Pop Quiz How to prove 5(a)?

Propositions are T or F

We use the letters $p, q, r, s, \ldots$ to represent propositions.

p: Porky the pig can fly.	F
q: You got an A.	T?
r: Kilam is an American.	T?
s: 4^2 is even.	T

To get complex statements, combine basic propositions using logical connectors.

Compound Propositions

p: Porky the pig can fly.	F
q: You got an A.	T?
r: Kilam is an American.	T?
s: 4^2 is even.	T

Connector	Symbol	An example in words
NOT	$\neg p$	

Compound Propositions

p: Porky the pig can fly. F

q: You got an A. T?

r: Kilam is an American. T?

s: 4^2 is even. T

Connector	Symbol	An example in words
NOT	$\neg p$	IT IS NOT THE CASE THAT (Porky the pig can fly)

Compound Propositions

p: Porky the pig can fly. F

q: You got an A. T?

r: Kilam is an American. T?

s: 4^2 is even. T

Connector	Symbol	An example in words
NOT	$\neg p$	IT IS NOT THE CASE THAT (Porky the pig can fly)
AND	$p \wedge q$	

Compound Propositions

p: Porky the pig can fly.	F
q: You got an A.	T?
r: Kilam is an American.	T?
s: 4^2 is even.	T

Connector	Symbol	An example in words
NOT	$\neg p$	IT IS NOT THE CASE THAT (Porky the pig can fly)
AND	$p \wedge q$	(Porky the pig can fly) AND (You got an A)

Compound Propositions

p: Porky the pig can fly. F

q: You got an A. T?

r: Kilam is an American. T?

s: 4^2 is even. T

Connector	Symbol	An example in words
NOT	$\neg p$	IT IS NOT THE CASE THAT (Porky the pig can fly)
AND	$p \wedge q$	(Porky the pig can fly) AND (You got an A)
OR	$p \vee q$	

Compound Propositions

p:	Porky the pig can fly.	F
q:	You got an A.	T?
r:	Kilam is an American.	T?
s:	4^2 is even.	T

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Compound Propositions

p: Porky the pig can fly. F

q: You got an A. T?

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Connector	Symbol	An example in words
NOT	$\neg p$	IT IS NOT THE CASE THAT (Porky the pig can fly)
AND	$p \wedge q$	(Porky the pig can fly) AND (You got an A)
OR	$p \vee q$	(Porky the pig can fly) OR (You got an A)
IF... THEN...	$p \to q$	

Compound Propositions

p: Porky the pig can fly. F

q: You got an A. T?

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Connector	Symbol	An example in words
NOT	$\neg p$	IT IS NOT THE CASE THAT (Porky the pig can fly)
AND	$p \wedge q$	(Porky the pig can fly) AND (You got an A)
OR	$p \vee q$	(Porky the pig can fly) OR (You got an A)
IF...THEN...	$p \to q$	IF (Porky the pig can fly) THEN (You got an A)

Negation (NOT), $\neg p$

The negation $\neg p$ is T when p is F, and the negation $\neg p$ is F when p is T.

Negation (NOT), $\neg p$

The negation $\neg p$ is T when p is F, and the negation $\neg p$ is F when p is T.

"Porky the pig can fly" is F

Negation (NOT), $\neg p$

The negation $\neg p$ is T when p is F, and the negation $\neg p$ is F when p is T.

"Porky the pig can fly" is F

So,

IT IS NOT THE CASE THAT (Porky the pig can fly) is T

Conjunction (AND), $p \wedge q$

Both p and q must be T for $p \wedge q$ to be T; otherwise $p \wedge q$ is F.

Conjunction (AND), $p \wedge q$

Both p and q must be T for $p \wedge q$ to be T; otherwise $p \wedge q$ is F.

"Porky the pig can fly" is F

Conjunction (AND), $p \wedge q$

Both p and q must be T for $p \wedge q$ to be T; otherwise $p \wedge q$ is F.

"Porky the pig can fly" is F

We don't know whether "You got an A".

Conjunction (AND), $p \wedge q$

Both p and q must be T for $p \wedge q$ to be T; otherwise $p \wedge q$ is F.

"Porky the pig can fly" is F

We don't know whether "You got an A".

It does not matter.

Conjunction (AND), $p \wedge q$

Both p and q must be T for $p \wedge q$ to be T; otherwise $p \wedge q$ is F.

"Porky the pig can fly" is F

We don't know whether "You got an A".

It does not matter.

(Porky the pig can fly) $\wedge$ (You got an A) is F

Disjunction (OR), $p \vee q$

Both p and q must be F for $p \vee q$ to be F; otherwise $p \vee q$ is T.

Disjunction (OR), $p \vee q$

Both p and q must be F for $p \vee q$ to be F; otherwise $p \vee q$ is T.

"Porky the pig can fly" is F

Disjunction (OR), $p \vee q$

Both p and q must be F for $p \vee q$ to be F; otherwise $p \vee q$ is T.

“Porky the pig can fly” is F

We don’t know whether “You got an A”.

Disjunction (OR), $p \vee q$

Both p and q must be F for $p \vee q$ to be F; otherwise $p \vee q$ is T.

"Porky the pig can fly" is F

We don't know whether "You got an A".

Now it matters.

Disjunction (OR), $p \vee q$

Both p and q must be F for $p \vee q$ to be F; otherwise $p \vee q$ is T.

"Porky the pig can fly" is F

We don't know whether "You got an A".

Now it matters.

(Porky the pig can fly) $\vee$ (You got an A) is T or F

(Depends on whether you got an A.)

Disjunction (OR), $p \vee q$

Both p and q must be F for $p \vee q$ to be F; otherwise $p \vee q$ is T.

"Porky the pig can fly" is F

We don't know whether "You got an A".

Now it matters.

(Porky the pig can fly) $\vee$ (You got an A) is T or F

(Depends on whether you got an A.)

Pop Quiz: "You can have cake" OR "You can have ice-cream." Can you have both?

Truth Tables

p	
F	
T	

Truth Tables

p	$\neg p$
F	T
T	F

Truth Tables

p	q	$\neg p$
F	F	T
F	T	T
T	F	F
T	T	F

Truth Tables

p	q	$\neg p$	$p \wedge q$
F	F	T	
F	T	T	
T	F	F	
T	T	F	

Truth Tables

p	q	$\neg p$	$p \wedge q$
F	F	T	F
F	T	T	F
T	F	F	F
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Truth Tables

p	q	$\neg p$	$p \wedge q$	$p \vee q$
F	F	T	F	
F	T	T	F	
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Truth Tables

p	q	$\neg p$	$p \wedge q$	$p \vee q$
F	F	T	F	F
F	T	T	F	T
T	F	F	F	T
T	T	F	T	T

Truth Tables

p	q	$\neg p$	$p \wedge q$	$p \vee q$
F	F	T	F	F
F	T	T	F	T
T	F	F	F	T
T	T	F	T	T

The truth table defines the "meaning" of these logical connectors.

Implication (IF... THEN...), $p \to q$

<u>IF</u> "Porky the pig can fly" <u>THEN</u> "You got an *A*." (T/F?)

Implication (IF... THEN...), $p \rightarrow q$

<u>IF</u> "Porky the pig can fly" <u>THEN</u> "You got an A." (T/F?)

Suppose T. Since pigs can't fly, does it mean you can't get an A?

Implication (IF. . . THEN. . .), $p \to q$

<u>IF</u> "Porky the pig can fly" <u>THEN</u> "You got an A." (T/F?)

Suppose T. Since pigs can't fly, does it mean you can't get an A?

<u>IF</u> "n^2 is even", <u>THEN</u> "n is even." (T)

Implication (IF... THEN...), $p \rightarrow q$

<u>IF</u> "Porky the pig can fly" <u>THEN</u> "You got an A." (T/F?)

Suppose T. Since pigs can't fly, does it mean you can't get an A?

<u>IF</u> "n^2 is even", <u>THEN</u> "n is even." (T)

Suppose n^2 is even. Can we conclude $n \neq 5$?

Implication (IF… THEN…), $p \to q$

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Suppose n^2 is even. Can we conclude $n \neq 5$?

<u>IF</u> "it rained last night" <u>THEN</u> "the grass is wet." (T)

Implication (IF... THEN...), $p \to q$

IF "Porky the pig can fly" THEN "You got an A." (T/F?)

Suppose T. Since pigs can't fly, does it mean you can't get an A?

IF "n^2 is even", THEN "n is even." (T)

Suppose n^2 is even. Can we conclude $n \neq 5$?

IF "it rained last night" THEN "the grass is wet." (T)

$$\begin{aligned} p &: \text{ it rained last night} \\ q &: \text{ the grass is wet} \end{aligned}$$

$$p \to q$$

Implication (IF. . . THEN. . .), $p \to q$

IF "Porky the pig can fly" THEN "You got an A." (T/F?)

Suppose T. Since pigs can't fly, does it mean you can't get an A?

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IF "it rained last night" THEN "the grass is wet." (T)

$$\begin{aligned} p &: \text{ it rained last night} \\ q &: \text{ the grass is wet} \end{aligned}$$

$$p \to q$$

What does it *mean* for this common-sense implication to be true?

Implication (IF... THEN...), $p \to q$

IF "Porky the pig can fly" THEN "You got an A." (T/F?)

Suppose T. Since pigs can't fly, does it mean you can't get an A?

IF "n^2 is even", THEN "n is even." (T)

Suppose n^2 is even. Can we conclude $n \neq 5$?

IF "it rained last night" THEN "the grass is wet." (T)

$$\begin{aligned} p &: \text{ it rained last night} \\ q &: \text{ the grass is wet} \end{aligned}$$

$$p \to q$$

What does it *mean* for this common-sense implication to be true?
What can you conclude? Did it rain last night? Is the grass wet?

Adding New Information to a True Implication: p is T

IF "it rained last night" THEN "the grass is wet."

$$p : \text{it rained last night}$$
$$q : \text{the grass is wet}$$

$$p \to q$$

Weather report in morning paper: rain last night. ← new information

Adding New Information to a True Implication: p is T

IF "it rained last night" THEN "the grass is wet."

$$p \; : \; \text{it rained last night}$$
$$q \; : \; \text{the grass is wet}$$

$$p \to q$$

Weather report in morning paper: rain last night. ← new information

IF (it rained last night) THEN (the grass is wet)	T
It rained last night (from the weather report)	T
Is the grass wet?	**YES!**

Adding New Information to a True Implication: p is T

IF "it rained last night" THEN "the grass is wet."

$$p : \text{it rained last night}$$
$$q : \text{the grass is wet}$$

$$p \rightarrow q$$

Weather report in morning paper: rain last night. ← new information

IF (it rained last night) THEN (the grass is wet)	T		$p \rightarrow q$	T
It rained last night (from the weather report)	T		p	T
Is the grass wet?	**YES!**	$\therefore$	q	T

For a **true** implication $p \rightarrow q$, when p is T, you can conclude q is T.

Adding New Information to a True Implication: q is T

IF "it rained last night" THEN "the grass is wet."

$$\begin{aligned} p &: \text{ it rained last night} \\ q &: \text{ the grass is wet} \end{aligned}$$

$$p \to q$$

While picking up the morning paper, you see wet grass. ← new information

Adding New Information to a True Implication: q is T

IF "it rained last night" THEN "the grass is wet."

$$\begin{aligned} p &: \text{ it rained last night} \\ q &: \text{ the grass is wet} \end{aligned}$$

$$p \rightarrow q$$

While picking up the morning paper, you see wet grass. ← new information

IF (it rained last night) THEN (the grass is wet)	T
The grass is wet (from walking outside)	T
Did it rain last night?	

Adding New Information to a True Implication: q is T

IF "it rained last night" THEN "the grass is wet."

$$\begin{aligned} p &: \text{ it rained last night} \\ q &: \text{ the grass is wet} \end{aligned}$$

$$p \rightarrow q$$

While picking up the morning paper, you see wet grass. ← new information

IF (it rained last night) THEN (the grass is wet)	T		$p \rightarrow q$	T
The grass is wet (from walking outside)	T		q	T
Did it rain last night?	☹	$\therefore$	p	T or F

For a true implication $p \rightarrow q$, when q is T, you cannot conclude p is T.

Adding New Information to a True Implication: p is F

IF "it rained last night" THEN "the grass is wet."

$$\begin{aligned} p &: \text{ it rained last night} \\ q &: \text{ the grass is wet} \end{aligned}$$

$$p \to q$$

Weather report in morning paper: no rain last night. $\leftarrow$ new information

Adding New Information to a True Implication: p is F

IF "it rained last night" THEN "the grass is wet."

$$p : \text{it rained last night}$$
$$q : \text{the grass is wet}$$

$$p \to q$$

Weather report in morning paper: no rain last night. ← new information

IF (it rained last night) THEN (the grass is wet)	T		$p \to q$	T
It rained last night (from the weather report)	F		p	F
Is the grass wet?		$\therefore$	q	T or F

For a **true** implication $p \to q$, when p is F, you **cannot** conclude q is F.

Adding New Information to a True Implication: q is F

IF "it rained last night" THEN "the grass is wet."

$$
\begin{aligned}
p &: \text{ it rained last night} \\
q &: \text{ the grass is wet}
\end{aligned}
$$

$$p \to q$$

While picking up the paper, you see dry grass. ← new information

Adding New Information to a True Implication: q is F

IF "it rained last night" THEN "the grass is wet."

$$p : \text{it rained last night}$$
$$q : \text{the grass is wet}$$

$$p \to q$$

While picking up the paper, you see dry grass. ← new information

IF (it rained last night) THEN (the grass is wet)	T	$p \to q$	T
It grass is wet (from walking outside)	F	q	F
Did it rain last night?		$\therefore\ p$	F

For a **true** implication $p \to q$, when q is F, you can conclude p is F.

Implication: Inferences When New Information Comes

For a **true** implication $p \to q$:

When p is `T`, you can conclude that q is `T`.

When q is `T`, you **cannot** conclude p is `T`.

When p is `F`, you **cannot** conclude q is `F`.

When q is `F`, you can conclude p is `F`.

Implication: Inferences When New Information Comes

For a **true** implication $p \to q$:

When p is T, you can conclude that q is T.

When q is T, you **cannot** conclude p is T.

When p is F, you **cannot** conclude q is F.

When q is F, you can conclude p is F.

$$\text{IF } \underbrace{\text{(Porky the pig can fly)}}_{\text{F}} \text{ THEN } \underbrace{\text{(You got an } A\text{)}}_{\text{can be T or F (phew)}}$$

Falsifying "IF (it rained last night) THEN (the grass is wet)"

- You are a scientist collecting data to *verify* that the implication is valid (true).

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- **One night it rained. That morning the grass was dry.** ← new information

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- You are a scientist collecting data to *verify* that the implication is valid (true).
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- What do you think about the implication now?

Falsifying "IF (it rained last night) THEN (the grass is wet)"

- You are a scientist collecting data to *verify* that the implication is valid (true).
- **One night it rained. That morning the grass was dry.** ← new information
- What do you think about the implication now?

This is a *falsifying scenario*.

IF (it rains) THEN (the grass is wet) ← not T

$p \to q$ is F *only* when p is T and q is F. In all other cases $p \to q$ is T.

Implication is *Extremely* Important, $p \to q$

All these are $p \to q$ ($p =$ "it rained last night" and $q =$ "the grass is wet"):

If it rained last night then the grass is wet. IF p THEN q

Implication is *Extremely* Important, $p \to q$

All these are $p \to q$ ($p =$ "it rained last night" and $q =$ "the grass is wet"):

If it rained last night then the grass is wet.	IF p THEN q
It rained last night implies the grass is wet.	p IMPLIES q

Implication is *Extremely* Important, $p \to q$

All these are $p \to q$ ($p =$ "it rained last night" and $q =$ "the grass is wet"):

If it rained last night then the grass is wet.	IF p THEN q
It rained last night implies the grass is wet.	p IMPLIES q
It rained last night only if the grass is wet.	p ONLY IF q

Implication is *Extremely* Important, $p \to q$

All these are $p \to q$ ($p =$ "it rained last night" and $q =$ "the grass is wet"):

If it rained last night then the grass is wet.	IF p THEN q
It rained last night implies the grass is wet.	p IMPLIES q
It rained last night only if the grass is wet.	p ONLY IF q
The grass is wet if it rained last night.	q IF p

Implication is *Extremely* Important, $p \to q$

All these are $p \to q$ ($p =$ "it rained last night" and $q =$ "the grass is wet"):

If it rained last night then the grass is wet.	IF p THEN q
It rained last night implies the grass is wet.	p IMPLIES q
It rained last night only if the grass is wet.	p ONLY IF q
The grass is wet if it rained last night.	q IF p
The grass is wet whenever it rains.	q WHENEVER p

Truth Tables:

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$\boldsymbol{p \to q}$
F	F	T	F	F	T
F	T	T	F	T	T
T	F	F	F	T	F
T	T	F	T	T	T

Example: IF (you are hungry OR you are thirsty) THEN you visit the cafeteria

$$(p \vee q) \rightarrow r \qquad \text{where} \qquad \begin{array}{l} p : \text{you are hungry} \\ q : \text{you are thirsty} \\ r : \text{you visit the cafeteria} \end{array}$$

	p	q	r	
1.	F	F	F	
2.	F	F	T	
3.	F	T	F	
4.	F	T	T	
5.	T	F	F	
6.	T	F	T	
7.	T	T	F	
8.	T	T	T	

Example: IF (you are hungry OR you are thirsty) THEN you visit the cafeteria

$$(p \vee q) \rightarrow r \qquad \text{where} \qquad \begin{array}{l} p: \text{you are hungry} \\ q: \text{you are thirsty} \\ r: \text{you visit the cafeteria} \end{array}$$

	p	q	r	$(p \vee q)$
1.	F	F	F	F
2.	F	F	T	F
3.	F	T	F	T
4.	F	T	T	T
5.	T	F	F	T
6.	T	F	T	T
7.	T	T	F	T
8.	T	T	T	T

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$(p \vee q) \rightarrow r$ where

p : you are hungry
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	p	q	r	$(p \vee q)$	$\boldsymbol{(p \vee q) \rightarrow r}$
1.	F	F	F	F	T
2.	F	F	T	F	T
3.	F	T	F	T	F
4.	F	T	T	T	T
5.	T	F	F	T	F
6.	T	F	T	T	T
7.	T	T	F	T	F
8.	T	T	T	T	T

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	p	q	r	$\boldsymbol{(p \vee q) \rightarrow r}$
1.	F	F	F	T
2.	F	F	T	T
3.	F	T	F	F
4.	F	T	T	T
5.	T	F	F	F
6.	T	F	T	T
7.	T	T	F	F
8.	T	T	T	T

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$(p \vee q) \rightarrow r$ where

p : you are hungry
q : you are thirsty
r : you visit the cafeteria

- *You are thirsty:* q is T.

	p	q	r	$\boldsymbol{(p \vee q) \rightarrow r}$
1.	F	F	F	T
2.	F	F	T	T
3.	F	T	F	F
4.	F	T	T	T
5.	T	F	F	F
6.	T	F	T	T
7.	T	T	F	F
8.	T	T	T	T

Example: IF (you are hungry OR you are thirsty) THEN you visit the cafeteria

$(p \vee q) \rightarrow r$ where

p : you are hungry
q : you are thirsty
r : you visit the cafeteria

- *You are thirsty:* q is T. In both cases r is T. (you visit the cafeteria)

	p	q	r	$(\boldsymbol{p} \vee \boldsymbol{q}) \rightarrow \boldsymbol{r}$
1.	F	F	F	T
2.	F	F	T	T
3.	F	T	F	F
4.	F	T	T	T
5.	T	F	F	F
6.	T	F	T	T
7.	T	T	F	F
8.	T	T	T	T

Example: IF (you are hungry OR you are thirsty) THEN you visit the cafeteria

$$(p \vee q) \rightarrow r \qquad \text{where} \qquad \begin{array}{l} p : \text{you are hungry} \\ q : \text{you are thirsty} \\ r : \text{you visit the cafeteria} \end{array}$$

- *You are thirsty:* q is T. In both cases r is T. (you visit the cafeteria)
- *You did visit the cafeteria:* r is T.

	p	q	r	$\boldsymbol{(p \vee q) \rightarrow r}$
1.	F	F	F	T
2.	F	F	T	T
3.	F	T	F	F
4.	F	T	T	T
5.	T	F	F	F
6.	T	F	T	T
7.	T	T	F	F
8.	T	T	T	T

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$$(p \vee q) \rightarrow r \qquad \text{where} \qquad \begin{array}{l} p: \text{you are hungry} \\ q: \text{you are thirsty} \\ r: \text{you visit the cafeteria} \end{array}$$

- *You are thirsty:* q is T. In both cases r is T. (you visit the cafeteria)
- *You did visit the cafeteria:* r is T.
  Are you hungry? We don't know.
  Are you thirsty? We don't know.
  (You accompanied your hungry friend (row 2).)

	p	q	r	$(p \vee q) \rightarrow r$
1.	F	F	F	T
2.	F	F	T	T
3.	F	T	F	F
4.	F	T	T	T
5.	T	F	F	F
6.	T	F	T	T
7.	T	T	F	F
8.	T	T	T	T

Example: IF (you are hungry OR you are thirsty) THEN you visit the cafeteria

$(p \vee q) \rightarrow r$ where

p : you are hungry
q : you are thirsty
r : you visit the cafeteria

- *You are thirsty:* q is T. In both cases r is T. (you visit the cafeteria)
- *You did visit the cafeteria:* r is T.
  Are you hungry? We don't know.
  Are you thirsty? We don't know.
  (You accompanied your hungry friend (row 2).)
- *You did not visit the cafeteria:* r is F.

	p	q	r	$\boldsymbol{(p \vee q) \rightarrow r}$
1.	F	F	F	T
2.	F	F	T	T
3.	F	T	F	F
4.	F	T	T	T
5.	T	F	F	F
6.	T	F	T	T
7.	T	T	F	F
8.	T	T	T	T

Example: IF (you are hungry OR you are thirsty) THEN you visit the cafeteria

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- *You are thirsty:* q is T. In both cases r is T.
  (you visit the cafeteria)
- *You did visit the cafeteria:* r is T.
  Are you hungry? We don't know.
  Are you thirsty? We don't know.
  (You accompanied your hungry friend (row 2).)
- *You did not visit the cafeteria:* r is F.
  p and q are both F.
  (You are neither hungry nor thirsty.)

	p	q	r	$(p \vee q) \rightarrow r$
1.	F	F	F	T
2.	F	F	T	T
3.	F	T	F	F
4.	F	T	T	T
5.	T	F	F	F
6.	T	F	T	T
7.	T	T	F	F
8.	T	T	T	T

Equivalent Compound Statements

p	q	$p \to q$	$\neg q \to \neg p$	$\neg p \vee q$	$q \to p$
F	F	T	T	T	T
F	T	T	T	T	F
T	F	F	F	F	T
T	T	T	T	T	T
		rains $\to$ wet grass	dry grass $\to$ no rain	no rain $\vee$ wet grass	wet grass $\to$ rain

$$p \to q \overset{\text{eqv}}{\equiv} \neg q \to \neg p \overset{\text{eqv}}{\equiv} \neg p \vee q$$

Equivalent Compound Statements

p	q	$p \to q$	$\neg q \to \neg p$	$\neg p \vee q$	$q \to p$
F	F	T	T	T	T
F	T	T	T	T	F
T	F	F	F	F	T
T	T	T	T	T	T
		rains $\to$ wet grass	dry grass $\to$ no rain	no rain $\vee$ wet grass	wet grass $\to$ rain

$$p \to q \stackrel{\text{eqv}}{\equiv} \neg q \to \neg p \stackrel{\text{eqv}}{\equiv} \neg p \vee q$$

Order is very important: $p \to q$ and $q \to p$ **do not** mean the same thing.

IF I'm dead, THEN my eyes are closed **vs.** IF my eyes are closed, THEN I'm dead

Equivalent Compound Statements

p	q	$p \to q$	$\neg q \to \neg p$	$\neg p \vee q$	$q \to p$
F	F	T	T	T	T
F	T	T	T	T	F
T	F	F	F	F	T
T	T	T	T	T	T
		rains $\to$ wet grass	dry grass $\to$ no rain	no rain $\vee$ wet grass	wet grass $\to$ rain

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Order is very important: $p \to q$ and $q \to p$ **do not** mean the same thing.

IF I'm dead, THEN my eyes are closed **vs.** IF my eyes are closed, THEN I'm dead

Pop Quiz 3.5. Compound propositions are used for program control flow, especially IF... THEN....

```
if(x > 0 || (y > 1 && x < y))
  Execute some instructions.
```

```
if(x > 0 || y > 1)
  Execute some instructions.
```

Use truth-tables to show that both do the same thing. Which do you prefer and why?

Proving an Implication: Reasoning Without Facts

IF (n^2 is even) THEN (n is even).

p : n^2 is even
q : n is even
$p \rightarrow q$

p	q	$p \rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

What is n? How to prove?

Proving an Implication: Reasoning Without Facts

$$\text{IF } (n^2 \text{ is even}) \text{ THEN } (n \text{ is even}).$$

$p: \; n^2$ is even
$q: \; n$ is even
$p \rightarrow q$

p	q	$p \rightarrow q$
F	F	T
F	T	T
$\boxed{\text{T}}$	$\boxed{\text{F}}$	$\boxed{\text{F}}$
T	T	T

What is n? How to prove?

We must show that the highlighted row *cannot* occur.

Proving an Implication: Reasoning Without Facts

$$\text{IF } (n^2 \text{ is even}) \text{ THEN } (n \text{ is even}).$$

$p: \; n^2$ is even
$q: \; n$ is even

$p \to q$

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F	T	T
T	F	F
T	T	T

What is n? How to prove?

We must show that the highlighted row *cannot* occur.

In this row, q is F: $n = 2k + 1$.

Proving an Implication: Reasoning Without Facts

$$\text{IF } (n^2 \text{ is even}) \text{ THEN } (n \text{ is even}).$$

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q : n is even
$p \to q$

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

What is n? How to prove?

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In this row, q is F: $n = 2k + 1$.

$$n^2 = (2k+1)^2 = 2(2k^2 + 2k) + 1$$

Proving an Implication: Reasoning Without Facts

$$\text{IF } (n^2 \text{ is even}) \text{ THEN } (n \text{ is even}).$$

$p: \ n^2$ is even
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What is n? How to prove?

We must show that the highlighted row *cannot* occur.

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p *cannot* be T.

Proving an Implication: Reasoning Without Facts

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p : n^2 is even
q : n is even

$p \to q$

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

What is n? How to prove?

We must show that the highlighted row *cannot* occur.

In this row, q is F: $n = 2k + 1$.

$$n^2 = (2k+1)^2 = 2(2k^2 + 2k) + 1$$

p *cannot* be T. This row cannot happen: $p \to q$ is always T. ■

Quantifiers

<u>EVERY</u> person has <u>A</u> soulmate.

Kilam has <u>some</u> gray hair.

Quantifiers

EVERY person has A soulmate.

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Any map can be colored with 4 colors with adjacent countries having different colors.

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<u>Every</u> even integer $n > 2$ is the sum of 2 primes *(Goldbach, 1742)*.
<u>Someone</u> broke this faucet.
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These statements are more complex because of *quantifiers*:

EVERY; A; SOME; ANY; ALL; THERE EXISTS.

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Someone broke this faucet.
There exists a creature with blue eyes and blonde hair.
All cars have four wheels.

These statements are more complex because of *quantifiers*:

EVERY; A; SOME; ANY; ALL; THERE EXISTS.

Compare:

My Ford Escort has four wheels;
ALL cars have four wheels.

Predicates Are Like Functions

ALL cars have four wheels

Predicates Are Like Functions

ALL cars have four wheels

Define *predicate* $P(c)$ and its *domain*

$$C = \{c | c \text{ is a car}\} \qquad \leftarrow \text{ set of cars}$$

Predicates Are Like Functions

ALL cars have four wheels

Define *predicate* $P(c)$ and its *domain*

$$\begin{aligned} C &= \{c | c \text{ is a car}\} & \leftarrow \text{ set of cars} \\ P(c) &= \text{"car } c \text{ has four wheels"} \end{aligned}$$

Predicates Are Like Functions

ALL cars have four wheels

Define *predicate* $P(c)$ and its *domain*

$$\begin{aligned} C &= \{c | c \text{ is a car}\} &\qquad\leftarrow \text{ set of cars} \\ P(c) &= \text{“car } c \text{ has four wheels”} \end{aligned}$$

“for all c in C, the statement $P(c)$ is true.”

$$\forall c \in C : P(c).$$

($\forall$ means “for all”)

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	Predicate	Function
	$P(c)$ = "car c has four wheels"	$f(x) = x^2$

Predicates Are Like Functions

ALL cars have four wheels

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$$\begin{aligned} C &= \{c | c \text{ is a car}\} && \leftarrow \text{ set of cars} \\ P(c) &= \text{“car } c \text{ has four wheels”} \end{aligned}$$

“for all c in C, the statement $P(c)$ is true.”

$$\forall c \in C : P(c).$$

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	Predicate	Function
Input	$P(c)$ = “car c has four wheels” parameter $c \in C$	$f(x) = x^2$ parameter $x \in \mathbb{R}$

Predicates Are Like Functions

ALL cars have four wheels

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$$\begin{aligned} C &= \{c \mid c \text{ is a car}\} && \leftarrow \text{ set of cars} \\ P(c) &= \text{“car } c \text{ has four wheels”} \end{aligned}$$

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Input	parameter $c \in C$	parameter $x \in \mathbb{R}$
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	$P(c)$ = “car c has four wheels”	$f(x) = x^2$
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Example	P(Jen's VW) = “car ‘Jen's VW’ has four wheels”	$f(5) = 25$

Predicates Are Like Functions

$$\textsc{all} \text{ cars have four wheels}$$

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	Predicate	Function
	$P(c) = $ "car c has four wheels"	$f(x) = x^2$
Input	parameter $c \in C$	parameter $x \in \mathbb{R}$
Output	**statement** $P(c)$	**value** $f(x)$
Example	$P(\text{Jen's VW}) = $ "car 'Jen's VW' has four wheels"	$f(5) = 25$
	$\forall c \in C : P(c)$	$\forall x \in \mathbb{R},\ f(x) \geq 0$

Predicates Are Like Functions

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	$P(c)$ = “car c has four wheels”	$f(x) = x^2$
Input	parameter $c \in C$	parameter $x \in \mathbb{R}$
Output	**statement** $P(c)$	**value** $f(x)$
Example	$P(\text{Jen's VW})$ = “car ‘Jen’s VW’ has four wheels”	$f(5) = 25$
	$\forall c \in C : P(c)$	$\forall x \in \mathbb{R},\ f(x) \geq 0$
Meaning	For all $c \in C$, the statement $P(c)$ is T.	For all $x \in \mathbb{R}$, $f(x)$ is ≥ 0.

There EXISTS a Creature with Blue eyes and Blonde Hair

Define *predicate* $Q(a)$ and its *domain*

$$A = \{a | a \text{ is a creature}\} \qquad \leftarrow \text{ set of creatures}$$

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"there exists a in A for which the statement $Q(a)$ is true."

$$\exists a \in A : Q(a).$$

($\exists$ means "there exists")

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$$\begin{aligned} A &= \{a \mid a \text{ is a creature}\} && \leftarrow \text{ set of creatures} \\ Q(a) &= \text{“}a \text{ has blue eyes and blonde hair”} \end{aligned}$$

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$$\exists a \in A : \underbrace{(G(a) \wedge H(a))}_{\text{compound predicate}}$$

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Define *predicate* $Q(a)$ and its *domain*

$$\begin{aligned} A &= \{a | a \text{ is a creature}\} & \leftarrow \text{ set of creatures} \\ Q(a) &= \text{“}a \text{ has blue eyes and blonde hair”} \end{aligned}$$

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$$\exists a \in A : (\underbrace{G(a) \wedge H(a)}_{\text{compound predicate}})$$

(When the domain is understood, we don't need to keep repeating it. We write $\exists a : Q(a)$, or $\exists a : (G(a) \wedge H(a))$.)

Negating Quantifiers

IT IS NOT THE CASE THAT(There is creature with blue eyes and blonde hair)

Negating Quantifiers

IT IS NOT THE CASE THAT(There is creature with blue eyes and blonde hair)

Same as: "All creatures don't have blue eyes and blonde hair"

$$\neg\Big(\exists a \in A : Q(a)\Big) \quad \overset{\text{eqv}}{\equiv} \quad \forall a \in A : \neg Q(a)$$

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IT IS NOT THE CASE THAT(All cars have four wheels)

Negating Quantifiers

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IT IS NOT THE CASE THAT(All cars have four wheels)

Same as: “There is a car which does not have four wheels”

$$\neg\Big(\forall c \in C : P(c)\Big) \overset{\text{eqv}}{\equiv} \exists c \in C : \neg P(c)$$

Negating Quantifiers

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Same as: "There is a car which does not have four wheels"

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When you take the negation inside the quantifier and negate the predicate, you must switch quantifiers: $\forall \to \exists$, $\exists \to \forall$

Every Person Has a Soul Mate

Define domains and a predicate.

$$A = \{a \mid a \text{ is an person}\}.$$

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When quantifiers are mixed, the order in which they appear is important for the meaning. Order generally cannot be switched.

Proofs with Quantifiers

Claim 1. $\forall n > 2$: IF n is even, THEN n is a sum of two primes. *(Goldbach, 1742)*

Claim 2. $\exists (a, b, c) \in \mathbb{N}^3 : a^2 + b^2 = c^2$.

($(a, b, c) \in \mathbb{N}^3$ means triples of natural numbers)

Claim 3. $\neg\exists (a, b, c) \in \mathbb{N}^3 : a^3 + b^3 = c^3$.

Claim 4. $\forall (a, b, c) \in \mathbb{N}^3 : a^3 + b^3 \neq c^3$.

Think about what it would take to prove these claims.