# **QUIZ 3:** <u>110 Minutes</u>



Answer **ALL** questions.

NO COLLABORATION or electronic devices. Any violations result in an F. NO questions allowed during the test. Interpret and do the best you can.

# GOOD LUCK!

You **MUST** show **CORRECT** work to get full credit.

When in doubt, TINKER.

1	2	3	Total
150	25	25	200

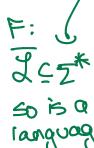
### 1 Circle one answer per question. 15 points for each correct answer.

- (a) Let X and Y be independent random variables taking values in a set with n elements. What is the probability of the event  $\mathbf{X} = \mathbf{Y}$ ?
  - $|\mathbf{A}| 1/n$
  - $|\mathbf{B}| \, 2/n$
  - $|C| 1/n^2$
  - $D 2/n^2$
  - E Not enough information/none of the above
- Assure WLOGI Host set is \$1,..., nf and IP(x=i)=pi > then by independence  $\mathbb{P}(X=Y) = \sum_{i=1}^{n} \mathbb{P}(X=Y=i)$ 
  - = = P(X=i)P(Y=i)
  - = 2 Pi distribution of X
- (b) If X and Y are independent and have the same variance, which of these have the same variance as X?
  - (I)  $(\mathbf{X} \mathbf{Y})/\sqrt{2}$
  - (II) X
  - (III)  $(\mathbf{X} + \mathbf{Y})/2$
  - A 1, II
  - B II, III
  - |C|I
  - |D|II
  - |E|III

- $\sigma(X-A) = + \sigma_{S}(X-A)$  $=\frac{1}{2}[\sigma^2(x) + \sigma^2(4)]$  $\sigma^{2}(-X) = \frac{1}{2} \left[ \frac{\partial^{2}(X)}{\partial x^{2}(X)} = \sigma^{2}(X) \right]$   $\sigma^{2}(X+y) = \frac{1}{4} \left[ \frac{\partial^{2}(X+y)}{\partial x^{2}(X)} = \sigma^{2}(X) + \sigma^{2}(Y) \right]$ 
  - - = +[90s(X)] = +0s(X)
- (c) Which of the following is true? (All complements are taken in  $\Sigma^*$ .)
  - A The complement of a regular language may not be a regular language F: Regular language
  - B The complement of a language may not be a language
  - C The set of all infinite-length binary strings has smaller cardinality than the set of valid C programs
  - D The set of questions that can be asked using the English language is countable
  - E | All of the above claims are true



T: Here is an injection from these questions to 5\* (use ASCII encoding for example)



- (d) Roll a six-sided die until you get a number greater than two. What is the expected number of rolls you need? X = # rolls to get a 3,4,5,or 6has waiting time distrib w/ param  $p = \frac{4}{6} = \frac{2}{3}$ 
  - A)1.5
  - B 2
  - C 3.5
  - $D \mid 4$
  - $\mid \mathbf{E} \mid$  None of the above.
- (e) Which of the following is not countable?

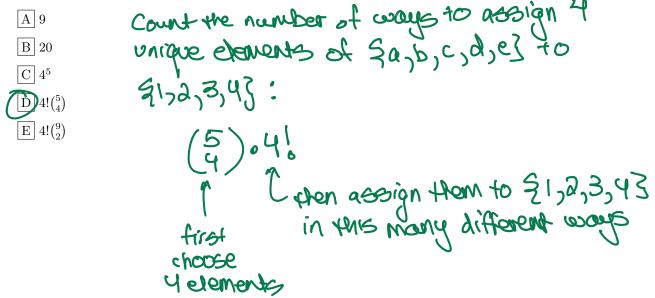
50 EX= 1 = 3

- B The set of real solutions to the equation  $\sin(x) = 0$
- C The set of all graphs (use the definition of a graph in terms of vertices and edges)
- The set of all languages (use the definition of "language" from Chapter 23)
- E The set of prime numbers ( ) countable countable can describe each graph w/ a

- (f) Which of the following strings is not in the language given by the regular expression ( $\{0\} \bullet \{11\}^*$ ) $\cup \{01\}^*$ ?
  - $A \in \mathcal{L}$  is in  $9013^{16}$

  - C 01012 is in 5013\*
  - D 011114 is in 903. 9113\*
  - E All of the above are in the language

the number of bins that contain exactly $k$ balls. What $k$ balls is the exactly $k$ balls. What $k$ balls in $k$ balls. What $k$ balls is $k$ balls in $k$ ball
to $\{a,b,c,d,e\}$ .  Det of ways to assign 4  3 of $\{a,b,c,d,e\}$ to
the air, and the caps land randomly on the heads of the get back their own hats. Given that both the variance by to find an upper bound on the probability that $\mathbf{X}$ is $\mathbf{P}(\mathbf{X}-\mathbf{I} \gg 49)$



- (j) Kofi is considering vacationing in the Bahamas and is attempting to determine how many days of rain he should expect. His friend tells him that he can expect to wait 5 days between rainy days in the Bahamas. How many days of rain should Kofi expect during a 2 week stay in the Bahamas?
  - $A 1\frac{1}{5}$
  - $(B) 2\frac{4}{5}$
  - $C 3\frac{1}{2}$
  - $\boxed{\mathrm{D}}$  4
  - E None of the above

Let p be the probability that it rains in a given day. The expected waiting time for rain is L=5, so p=L. The number of days of rain in two weeks has distribution Binomial (14, p), so has expectation  $14p=\frac{14}{5}=2\frac{4}{5}$ 

2 For a fair coin, compute the expected number of flips to get 2 heads in a row.

Let X be number of flips to get 2 heads in a row.

By total expectation,

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and

E[X/rolla head]= 1 + 1. TP(rolla head)

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and

3 Prove or disprove: the union of uncountably many languages is a language.

Each larguage is a subset of  $\Xi^*$ , so regardless of the number of them, their union is also a subset of  $\Xi^*$ , so it is a larguage.

## SCRATCH

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