

QUIZ 3: 110 Minutes

Answer **ALL** questions.

NO COLLABORATION or electronic devices. Any violations result in an F.

NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

You **MUST** show **CORRECT** work to get full credit.

When in doubt, **TINKER**.

1	2	3	Total
150	25	25	200

1 Circle one answer per question. 15 points for each correct answer.

- (a) Let \mathbf{X} and \mathbf{Y} be independent random variables taking values in a set with n elements. What is the probability of the event $\mathbf{X} = \mathbf{Y}$?

- ☐ A $1/n$
☐ B $2/n$
☐ C $1/n^2$
☐ D $2/n^2$
☐ E Not enough information/none of the above

- (b) If \mathbf{X} and \mathbf{Y} are independent and have the same variance, which of these have the same variance as \mathbf{X} ?

- (I) $(\mathbf{X} - \mathbf{Y})/\sqrt{2}$
(II) $-\mathbf{X}$
(III) $(\mathbf{X} + \mathbf{Y})/2$

- ☐ A I, II
☐ B II, III
☐ C I
☐ D II
☐ E III

- (c) Which of the following is true? (All complements are taken in Σ^* .)

- ☐ A The complement of a regular language may not be a regular language
☐ B The complement of a language may not be a language
☐ C The set of all infinite-length binary strings has smaller cardinality than the set of valid C programs
☐ D The set of questions that can be asked using the English language is countable
☐ E All of the above claims are true

(d) Roll a six-sided die until you get a number greater than two. What is the expected number of rolls you need?

☐ A 1.5

☐ B 2

☐ C 3.5

☐ D 4

☐ E None of the above.

(e) Which of the following is not countable?

☐ A The Cartesian product $A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$ of two countable sets

☐ B The set of real solutions to the equation $\sin(x) = 0$

☐ C The set of all graphs (use the definition of a graph in terms of vertices and edges)

☐ D The set of all languages (use the definition of “language” from Chapter 23)

☐ E The set of prime numbers

(f) Which of the following strings is not in the language given by the regular expression $(\{0\} \bullet \{11\}^*) \cup \{01\}^*$?

☐ A ε

☐ B 001

☐ C 0101

☐ D 01111

☐ E All of the above are in the language

- (g) You toss m balls randomly into n bins. Let \mathbf{X} be the number of bins that contain exactly k balls. What is $\mathbb{E}[\mathbf{X}]$?

☐ A $n \frac{k!(m-k)!}{m!}$

☐ B n/k

☐ C $n \binom{m}{k} \left(\frac{1}{n}\right)^k \left(1 - \frac{1}{n}\right)^{m-k}$

☐ D $n \binom{\frac{m}{k}}{\frac{m}{n}}$

☐ E None of the above.

- (h) 100 student throw their graduation caps into the air, and the caps land randomly on the heads of the students. Let \mathbf{X} be the number of students who get back their own hats. Given that both the variance and mean of \mathbf{X} are 1, use Chebyshev's inequality to find an upper bound on the probability that \mathbf{X} is at least 50.

☐ A 1/100

☐ B 1/50

☐ C 1/49

☐ D 1/32

☐ E None of the above

- (i) Count the number of injections from $\{1, 2, 3, 4\}$ to $\{a, b, c, d, e\}$.

☐ A 9

☐ B 20

☐ C 4^5

☐ D $4! \binom{5}{4}$

☐ E $4! \binom{9}{2}$

- (j) Kofi is considering vacationing in the Bahamas and is attempting to determine how many days of rain he should expect. His friend tells him that he can expect to wait 5 days between rainy days in the Bahamas. How many days of rain should Kofi expect during a 2 week stay in the Bahamas?

☐ A $1\frac{1}{5}$

☐ B $2\frac{4}{5}$

☐ C $3\frac{1}{2}$

☐ D 4

☐ E None of the above

2 For a fair coin, compute the expected number of flips to get 2 heads in a row.

3 Prove or disprove: the union of uncountably many languages is a language.

SCRATCH

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