QUIZ 2: 60 Minutes

Solution Set

Answer **ALL** questions.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

You **MUST** show **CORRECT** work to get full credit.

When in doubt, TINKER.

1	2	3	Total
100	**25**	**25**	**150**

1 **Circle one answer per question.** 20 **points for each correct answer.**

(a) Coach Jaimie's soccer team is craving Taco Bell. Taco Bell has 36 types of tacos and 10 types of drinks. A meal consists of a taco and a drink. How many ways are there for her eleven players to get meals if no two players are allowed to get the same meal?

Choose 11 of the 36·10 distinct meals

A $\frac{36\cdot 10}{11!}$

B $36\cdot 10$

C $\binom{36\cdot 10}{11}$

D $\binom{36\cdot 10\cdot 2}{11}$

E None of the above.

(b) Which is the smallest upper bound on the chromatic number of an r-regular n-node graph that is valid for *any* such graph?

A $r-1$

B r

C $r+1$

D $2r$

E $n-1$

Greedy coloring gives a coloring showing $\chi(G) \leq \max_{v \in G} \deg(v) + 1 = r+1$ and the triangle (2-regular) needs three colors so $r+1$ is the smallest possible

(c) This semester, 200 graduate students applied to the computer science departments at either RPI, Northeastern, or UMass Amherst. Of them, 110 applied to Northeastern and 75 applied to UMass Amherst. If 30 students applied to all three schools, 35 applied to RPI and Northeastern, 31 applied to RPI and UMass Amherst, and 40 applied to Northeastern and UMass Amherst, how many students applied to RPI?

Use PIE: $|R \cup N \cup A| = |R| + |N| + |A| - (|R \cap N| + |R \cap A| + |A \cap N|) + |R \cap N \cap A|$

so $200 = |R| + 110 + 75 - (35 + 31 + 40) + 30$

$\Rightarrow |R| = 91$

A 15

B 57

C 66

D 101

E Not enough information, or None of the above

(d) Which of the following claims are true?

(1) The number of solutions to $x_1 + \cdots + x_4 = 10$ where each variable is strictly positive is $\binom{9}{3}$.
(2) Given a set S with 10 elements, there is a bijection from 3-element subsets of S to 7-element subsets of S.
(3) $\sum_{i=1}^{n} \binom{n}{i} = 2^n - 1$.

(1) bijection between these solns and solns to $\tilde{x}_1 + \cdots + \tilde{x}_4 = 6$ with $\tilde{x}_1, \ldots, \tilde{x}_4 \geq 0$, and by goody bag method there are $\binom{6+4-1}{4-1} = \binom{9}{3}$ solns

(2) bijection maps $S \leftrightarrow S^c$

(3) By binomial theorem, $2^n - 1 = (1+1)^n - 1 = \sum_{i=0}^{n} \binom{n}{i} - 1 = \sum_{i=1}^{n} \binom{n}{i}$

A (1) and (2)

B (1) and (3)

C (2) and (3)

D All of the above

E None of the above

(e) Chek is running a monthly book club, and must choose the books for the next year. The bookclub members have suggested 40 distinct books. From these, Chek must choose 12 books, one of which will be the first book of the year. In how many ways can Chek do this?

First choose 12 of the 40 books; Next choose one of those 12 books as the first.

- A $\binom{39}{12}$
- B $\binom{40}{12}$
- C $12 \cdot \binom{40}{12}$
- D $12 \cdot \binom{38}{12}$
- E None of the above

2 **Show that, given any 10 integers $x_1, \ldots, x_{10}$ each of which is in the interval $[0, 100]$, there are two distinct subsets of the numbers that have the same subset sum.**

There are $2^{10} = 1024$ distinct subsets.
Each subset has sum at most $10 \cdot 100 = 1000$.
Using the subset sum as bins, there are at most 1000 bins, and 1024 bins.
By the pigeonhole principle, at least one pair of distinct subsets shares the same subset sum.

3 Adam and Monica share a streaming service, but live in different households. If they both attempt to access it on the same day, the service will lock both of them out for a day. To mitigate this risk, they use randomization: each independently attempts to use the service with probability p independently each day. Compute the probability that Adam is able to stream for the first time on day t.

Adam can stream if he tries and Monica does not, so he successfully streams in a try with probability $p \cdot (1-p)$.

The waiting time distribution until his first successful stream has pdf

$$\mathbb{P}(\text{Adam first streams on day } t)$$
$$= \{1 - p \cdot (1-p)\}^{t-1} \cdot p(1-p)$$
$$= \beta\,(1 - p \cdot (1-p))^{t}$$

where $\beta = \dfrac{p(1-p)}{1 - p \cdot (1-p)}$

SCRATCH

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