MIDTERM: 90 Minutes

Last Name: _____

First Name: _____

RIN: _____

Section: _____

Answer **ALL** questions.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.
SUBMIT your crib sheet, with your name written on it.

GOOD LUCK!

You **MUST** show **CORRECT** work to get full credit.

When in doubt, TINKER.

1	2	3	4	Total
120	25	25	25	195

1 Circle one answer per question. 15 points for each correct answer.

(a) Which method of proof is most appropriate for establishing that $(a+b)^n \geq a^n + b^n$ when a, b are non-negative reals and $n \in \mathbb{N}$?

A Direct

B Contraposition

C Contradiction

D Induction

E Structural Induction

(b) Which is true of the quantity $\frac{n^4+3n}{n^2+7}$?

A It is $o(n^2)$

B It is $\omega(n^2)$

C It is $o(n)$

D It is $\omega(n)$

E None of the above.

(c) Which of these properties does the gcd have?

(I) If ℓ is a positive integer linear combination of m and n, then $\gcd(m,n)|\ell$
(II) If $d_1|m$ and $d_2|n$ and d_1 and d_2 are relatively prime, then $d_1 d_2 | \gcd(m,n)$
(III) If k is a natural number, $\gcd(m^k, n^k) = \gcd(m,n)^k$

A I and II.

B I and III.

C II and III.

D II only.

E III only.

(d) It is now 7pm. Where is the hour hand in 2023^3 hours?

A 3pm.

B 8pm.

C 12am.

D 2am.

E None of the above.

(e) Which of these claims are true?

(I) A connected graph on n vertices with average degree strictly less than 2 is a tree.
(II) There does not exist a friend network with 7 friends, each of whom knows 3 friends.
(III) $K_{4,5}$ is a regular graph.

A All of them.

B None of them.

C I and II.

D I and III.

E II and III.

(f) What is the most precise asymptotic behavior of $S = \sum_{i=1}^{n} \exp(i)$?

A $S \in O(\exp(n))$

B $S \in O(\exp(n^2))$

C $S \in \Theta(\exp(n))$

D $S \in \Theta(\exp(n^2))$

E None of the above.

(g) Compute $\sum_{i=1}^{2n}(1+3i)$

A $2n + 6n^2$

B $3n + 6n^2$

C $5n + 6n^2$

D $6n + 5n^2$

E None of the above.

(h) Let G be a connected planar graph on 10 vertices with 15 edges. How many faces does G have?

A 7

B 9

C 11

D 12

E None of the above.

2 **Let $\mathcal{P}$ be a recursively defined set: $(1,0) \in \mathcal{P}$ and $(x,y) \in \mathcal{P} \to (x+1, y+2) \in \mathcal{P}$. Prove that every point $(x,y) \in \mathcal{P}$ satisfies $y = 2x - 2$.**

3 **Show that when k is a natural number, $2^k - 1$ and $2^k + 1$ are relatively prime.**

4 **Prove that given a graph $G = (V, E)$ with n vertices, one can partition V into two sets so that every vertex in a set has at least half of its neighbors in the same set.**

SCRATCH

SCRATCH

SCRATCH