QUIZ 2: 110 Minutes

Answer **ALL** questions.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

You **MUST** show **CORRECT** work to get full credit.

When in doubt, TINKER.

1	2	3	Total
150	25	25	200

1 Circle one answer per question. 15 points for each correct answer.

(a) Let C_n be the cycle graph on n vertices. What is $\chi(C_n)$?

A 2

B 3

C 4

D None of the above.

E Not enough information.

(b) How many functions $f : \{1, \ldots, 5\} \to \{1, \ldots, 10\}$ are *strictly* increasing?

A $\binom{14}{4}$

B $\binom{14}{5}$

C $\binom{15}{5}$

D $\binom{15}{9}$

E None of the above.

(c) A social network has seven people Ⓐ, Ⓑ, ..., Ⓖ. Summing up the number of friends of each person in the network gives 26. How many *different* such social networks are there?

A $7 \cdot 13$

B $\frac{13!}{6!}$

C $2^{\binom{7}{2}/26}$

D $\binom{\binom{7}{2}}{26}$

E None of the above.

(d) A vase contains r red balls and b blue balls. A ball is chosen at random from the vase, its color is noted, and it is returned to the vase together with d more balls of the same color. This is repeated indefinitely. What is the probability that the second ball is blue?

A $\frac{b}{b+r+d}$

B $\frac{b}{b+r}$

C $\frac{b+d}{b+r+d}$

D $\frac{b+d}{b+r}$

E None of the above.

(e) You randomly choose two gloves, without replacement, from a drawer with seven pairs of gloves. What is the probability that the gloves are a matching pair?

A $1/14$

B $1/13$

C $7/\binom{14}{2}$

D $14/\binom{14}{2}$

E None of the above

(f) The first round of a karate tournament pairs the fighters into groups of two. If there are 16 fighters, how many ways are there of forming the first round?

A $\frac{16!}{8!2}$

B $\binom{16}{2}\binom{14}{2}\cdots\binom{2}{2}$

C $\binom{21}{7}$

D $\binom{21}{16}$

E None of the above.

(g) Of 50 students in Ethics, Statistics, and Circuits and Systems, the number failing each combination of courses is shown. How many students passed all the courses?

E	S	C	ES	EC	SC	ESC
10	5	5	2	4	2	1

A 28

B 31

C 35

D 37

E None of the above

(h) A bag has 4 coins: two 2-headed coins, a 2-tailed coin, and a regular fair coin. Randomly pick a coin and place it on the table. You see a heads facing up. What is the probability that the side facing down is heads?

A $1/4$

B $1/2$

C $3/4$

D $4/5$

E None of the above

(i) Independently generate a length five binary string $b_0 b_1 \cdots b_4$ with $\mathbb{P}[b_i = 0] = 1/2$ for each i. What is the probability that the string is sorted from low to high?

A $5/32$

B $3/16$

C $1/8$

D $1/2$

E None of the above

(j) A bucket contains twenty dates. Fifteen of these dates have had their seed removed. A sly dog eats five dates, selected at random. Subsequently, a date is randomly selected from the remaining dates. What is the probability that this date contains a seed?

A $\sum_{k=0}^{5} \frac{\binom{5}{k}}{\binom{20}{5}} \frac{5-k}{15}$

B $\sum_{k=0}^{5} \frac{\binom{5}{k}}{\binom{20}{k}} \frac{5-k}{15}$

C $\sum_{k=0}^{5} \frac{\binom{5}{k}}{\binom{20}{5}} \frac{5-k}{20}$

D $\sum_{k=0}^{5} \frac{\binom{5}{k}}{\binom{20}{k}} \frac{5-k}{20}$

E None of the above

2 **Three monkeys A, B, C have a 6-shooter pistol loaded with 2 bullets. Starting with A, each spins the bullet-wheel to a chamber uniformly randomly selected from the six chambers and shoots their foot. They repeat this process cyclically in the order A, B, then C, until there are no bullets left. Compute probabilities p_A, p_B, p_C for each monkey to be the first shot.**

3 **Label the vertices of K_{15} with the integers $1, \ldots, 15$, and construct a graph G by adding a vertex 16 and adding edges connecting it to vertices $5, 10,$ and 15. How many perfect matchings are there on G?**

SCRATCH

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