QUIZ 1: 110 Minutes

Answer **ALL** questions.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

You **MUST** show **CORRECT** work to get full credit.

When in doubt, TINKER.

1	2	3	Total
150	25	25	200

1 **Circle one answer per question.** 15 **points for each correct answer.**

(a) Let $S(x)$ = "x is a science major" and $F(x)$ = "x is a fan of Cosmos". Which of the following translates to "all fans of Cosmos are science majors"?

- A $\neg(\exists x : F(x) \wedge (\neg S(x)))$
- B $\forall x : (\neg F(x)) \vee (\neg S(x))$
- C $\forall x : (\neg S(x)) \rightarrow F(x)$
- D All of the above.
- E None of the above.

(b) Let p be prime. Which most accurately describes the number $\sqrt{p}$?

- A A natural number.
- B A rational number.
- C An irrational number.
- D It depends on the value of p.
- E None of the above.

(c) If x and y denote integers, which of the following propositions are true?

(1) $\forall x : (\exists y : 2x - y = 0)$
(2) $\exists y : (\forall x : 2x - y = 0)$
(3) $\exists y : (\forall x : xy = 0)$

- A (1) and (2).
- B (2) and (3).
- C (1) and (3).
- D All of the above.
- E None of the above.

(d) Let $S = \{(a, b) : a, b \in \mathbb{Q} \text{ and } a < b\}$. Give the negation of the following claim:

$$\forall (a, b) \in S : (\exists t \in \mathbb{Q} : (a < t) \wedge (t < b)).$$

- A $\exists (a, b) \in S : (\exists t \in \mathbb{Q} : (a < t) \wedge (t < b))$
- B $\forall (a, b) \in S : (\forall t \in \mathbb{Q} : (a \geq t) \vee (t \geq b))$
- C $\exists (a, b) \in S : (\exists t \in \mathbb{Q} : (a \geq t) \vee (t \geq b))$
- D $\forall (a, b) \in S : (\exists t \in \mathbb{Q} : (a < t) \wedge (t < b))$
- E None of the above.

(e) How many rows in the truth table of $(p \to q) \to (r \wedge s)$ are there where it is the case that this proposition is true?

A 6

B 7

C 8

D 9

E 16

(f) Which pair is the (converse, contrapositive) of $(\forall x : (P(x) \wedge Q(x))) \to R(x)$?

A $(\exists x : (\neg P(x)) \vee (\neg Q(x))) \to \neg R(x), \quad \neg R(x) \to (\exists x : (\neg P(x)) \vee (\neg Q(x)))$

B $R(x) \to (\forall x : P(x) \wedge Q(x))\,, \quad \neg R(x) \to (\exists x : (\neg P(x)) \vee (\neg Q(x)))$

C $\neg R(x) \to (\exists x : (\neg P(x)) \vee (\neg Q(x)))\,, \quad R(x) \to (\forall x : P(x) \wedge Q(x))$

D $R(x) \to (\forall x : P(x) \wedge Q(x))\,, \quad (\exists x : (\neg P(x)) \vee (\neg Q(x))) \to \neg R(x)$

E None of the above.

(g) If the proposition "If a person ever wears a green hat then they will live long or be unlucky in love" is true, under which condition can we conclude that a person has never worn a green hat?

A They lived long and were unlucky in love.

B They died young or were lucky in love.

C They died young and were lucky in love.

D All of the above.

E None of the above.

(h) Which type of proof is the most appropriate for proving that $11^n - 6$ is divisible by 5 if n is a natural number?

A Contradiction.

B Contraposition.

C Direct.

D Leaping Induction.

E Weak Induction.

(i) You observe the following hand of cards:

S 5 P T 3 4.

Each card has a number on one side and a letter on the other. Consider the following rule: if a card has P on it, then the other side must be a 5. To verify that the rule has not been broken in the hand you have seen, which is the fewest number of cards that you must turn over?

A 1

B 2

C 3

D 4

E 6

(j) Let a, b be natural numbers. Which of the following propositions is most appropriately proven using direct proof?

A There is an even number greater than ab.

B $(a+b)^n \geq a^n + b^n$, when n is a natural number.

C There is a prime number greater than ab.

D All of the above are equally appropriately proven using direct proof.

E Direct proof is inappropriate for all the above propositions.

2 **Prove that $2^{1/p}$, the p-th root of 2, is irrational for any integer $p > 2$. You may use Fermat's Last Theorem, which states that if a, b, c, n are natural numbers with $n > 2$, then $a^n + b^n \neq c^n$.**

3 **Prove that if n and q are natural numbers then there exist unique integers d and r satisfying $n = dq + r$, with $d \in \{0\} \cup \mathbb{N}$ and $0 \leq r < q$.**

SCRATCH

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