QUIZ 1: <u>110 Minutes</u>

Answer **ALL** questions.

NO COLLABORATION or electronic devices. Any violations result in an F. NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

You **MUST** show **CORRECT** work to get full credit.

When in doubt, TINKER.

1	2	3	Total
150	25	25	200

1 Circle one answer per question. 15 points for each correct answer.

- (a) Let S(x) = "x is a science major" and F(x) = "x is a fan of Cosmos". Which of the following translates to "all fans of Cosmos are science majors"?
 - $\boxed{\mathbf{A}} \neg (\exists x : F(x) \land (\neg S(x)))$
 - $\boxed{\mathrm{B}} \ \forall x : (\neg F(x)) \lor (\neg S(x))$
 - $C \forall x : (\neg S(x)) \to F(x)$
 - D All of the above.
 - E None of the above.
- (b) Let p be prime. Which most accurately describes the number \sqrt{p} ?
 - A natural number.
 - B A rational number.
 - C An irrational number.
 - $\boxed{\mathrm{D}}$ It depends on the value of p.
 - E None of the above.
- (c) If x and y denote integers, which of the following propositions are true?
 - $(1) \ \forall x : (\exists y : 2x y = 0)$
 - $(2) \ \exists y : (\forall x : 2x y = 0)$
 - $(3) \ \exists y : (\forall x : xy = 0)$
 - $\boxed{\mathbf{A}}$ (1) and (2).
 - $\boxed{\mathrm{B}}$ (2) and (3).
 - $\boxed{\mathbf{C}}$ (1) and (3).
 - D All of the above.
 - E None of the above.
- (d) Let $S = \{(a,b): a,b \in \mathbb{Q} \text{ and } a < b\}$. Give the negation of the following claim:

$$\forall (a,b) \in S: (\exists t \in \mathbb{Q}: (a < t) \land (t < b)).$$

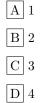
- $\boxed{\mathbf{A}} \ \exists (a,b) \in S : (\exists t \in \mathbb{Q} : (a < t) \land (t < b))$
- $\boxed{\mathbb{C}} \exists (a,b) \in S : (\exists t \in \mathbb{Q} : (a \ge t) \lor (t \ge b))$
- $\boxed{\mathsf{D}} \, \forall (a,b) \in S : (\exists t \in \mathbb{Q} : (a < t) \land (t < b))$
- E None of the above.

(e)	How many rows in the truth table of $(p \to q) \to (r \land s)$ are there where it is the case that this proposition is true?
	$oxed{A}$ 6
	B 7
	<u>C</u> 8
	E 16
(f)	Which pair is the (converse, contrapositive) of $(\forall x: (P(x) \land Q(x))) \rightarrow R(x)$?
	$\boxed{\mathbf{A}} (\exists x: (\neg P(x)) \lor (\neg Q(x))) \to \neg R(x), \neg R(x) \to (\exists x: (\neg P(x)) \lor (\neg Q(x)))$
	$ \boxed{ \mathbf{B} } \ R(x) \to \left(\forall x : P(x) \land Q(x) \right), \neg R(x) \to \left(\exists x : \left(\neg P(x) \right) \lor \left(\neg Q(x) \right) \right) $
	$\boxed{\mathbf{C}} \neg R(x) \to (\exists x : (\neg P(x)) \lor (\neg Q(x))), R(x) \to (\forall x : P(x) \land Q(x))$
	$\boxed{\mathbf{D}} \ R(x) \to (\forall x: P(x) \land Q(x)) , (\exists x: (\neg P(x)) \lor (\neg Q(x))) \to \neg R(x)$
	E None of the above.
(g)	If the proposition "If a person ever wears a green hat then they will live long or be unlucky in love" is true, under which condition can we conclude that a person has never worn a green hat?
	A They lived long and were unlucky in love.
	B They died young or were lucky in love.
	C They died young and were lucky in love.
	D All of the above.
	E None of the above.
(h)	Which type of proof is the most appropriate for proving that $11^n - 6$ is divisible by 5 if n is a natural number?
	A Contradiction.
	B Contraposition.
	C Direct.
	D Leaping Induction.
	E Weak Induction.

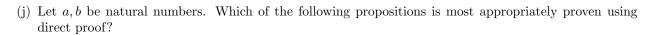
(i) You observe the following hand of cards:

S 5 P T 3 4.

Each card has a number on one side and a letter on the other. Consider the following rule: if a card has P on it, then the other side must be a 5. To verify that the rule has not been broken in the hand you have seen, which is the fewest number of cards that you must turn over?

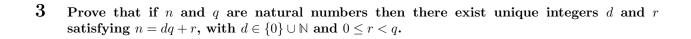


E 6



- \overline{A} There is an even number greater than ab.
- $\boxed{\mathrm{B}}(a+b)^n \geq a^n + b^n$, when n is a natural number.
- $\boxed{\mathbf{C}}$ There is a prime number greater than ab.
- D All of the above are equally appropriately proven using direct proof.
- E Direct proof is inappropriate for all the above propositions.

Prove that $2^{1/p}$, the p-th root of 2, is irrational for any integer p > 2. You may use Fermat's Last Theorem, which states that if a, b, c, n are natural numbers with n > 2, then $a^n + b^n \neq c^n$.



SCRATCH

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