Foundations of Computer Science
Lecture 1
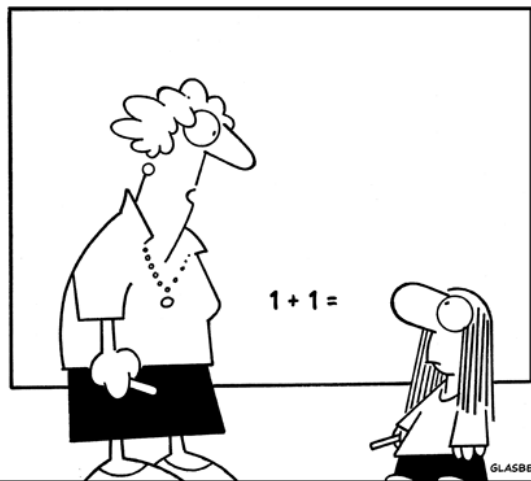
Warmup: A Taste for Discrete Math and Computing

Background

Disease spread, speed-dating, friendship networks

3 Challenge Problems

"Yes, this will be useful to you later in life."
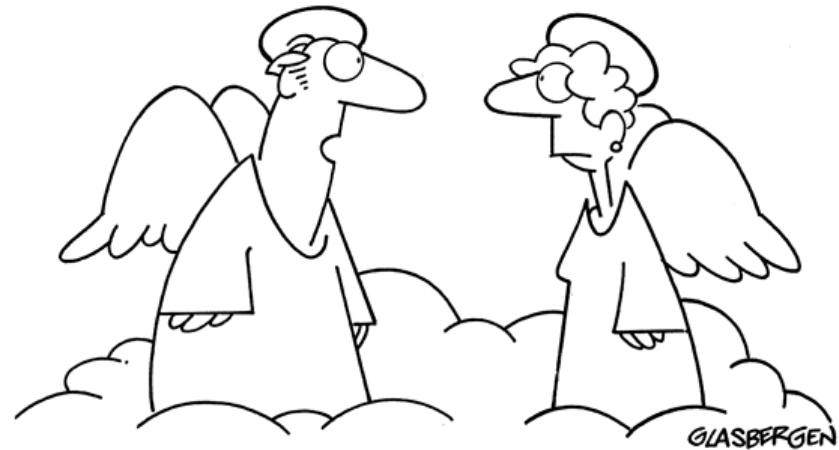
"It's important to learn math because someday you might accidentally buy a phone without a calculator."

"In 1953 you were my math teacher. You promised that algebra would come in handy someday. How much longer do I have to wait?"

(Today) Warmup: A Taste for Discrete Math and Computing

Resources and Rules

1. Web Page: www.cs.rpi.edu/∼magdon/courses/focs.html
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7. **Prerequisites:**
   CS II (data structures)
   Calc I (Calc II STRONGLY recommended)

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8. **Rules:** No food, no electronics, no cheating.

The Storyline

1. Discrete objects.

- concepts/concrete
- proof/theory/abstract
- theory of computation

our language will be mathematics . . .
. . . it will be everywhere

The Storyline

1. Discrete objects.

2. Reasoning about discrete objects

- concepts/concrete
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The Storyline

1. Discrete objects.

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3. Counting discrete objects

- concepts/concrete
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The Storyline

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5. What can we compute?

■ concepts/concrete
■ proof/theory/abstract
■ theory of computation

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The Storyline

1. Discrete objects.

2. Reasoning about discrete objects

3. Counting discrete objects

4. Randomness: probability

5. What can we compute?

6. What can we compute efficiently?

■ concepts/concrete
■ proof/theory/abstract
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Background

Programming, numbers, geometry, algebra, calculus, . . .

- What is the minimum element in the set $\{8, 9, 3, 10, 19\}$?

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- What is the minimum element in the set $\{8, 9, 3, 10, 19\}$?
- Does this set of *positive* numbers have a minimum element:
$$\{25, 97, 107, 100, 18, 33, 99, 27, 2014, 2200, 23, \ldots\}$$

Background

Programming, numbers, geometry, algebra, calculus, . . .

- What is the minimum element in the set $\{8, 9, 3, 10, 19\}$?
- Does this set of *positive* numbers have a minimum element:
$$\{25, 97, 107, 100, 18, 33, 99, 27, 2014, 2200, 23, \ldots\}$$

Any (non-empty) set containing only **positive integers** has a minimum element.

Two-Contact Ebola on a Grid

A square gets infected if two or more neighbors (N,S,E,W) are infected.
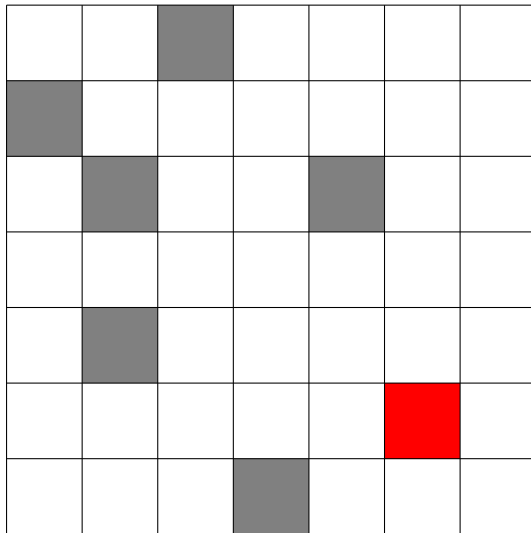
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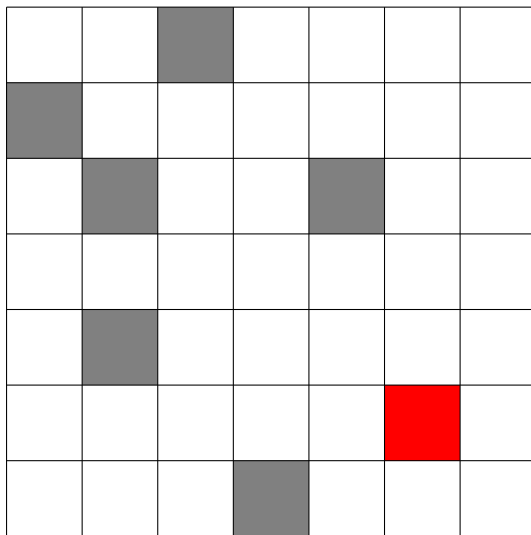
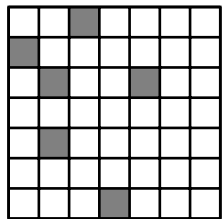- Given initial gray infections, who ultimately gets infected?
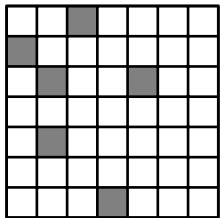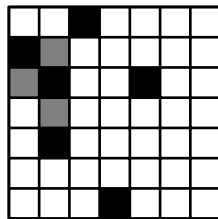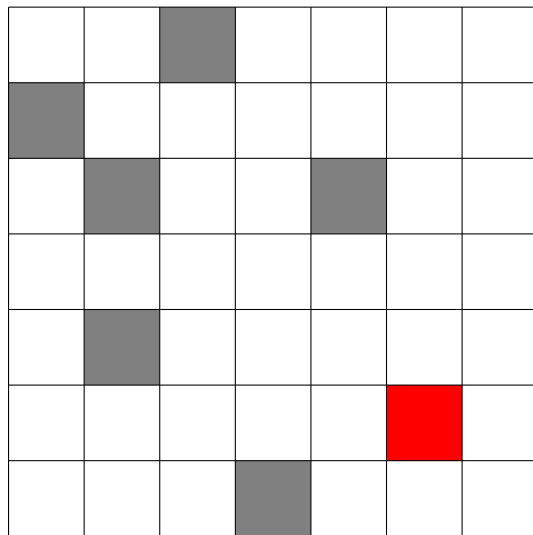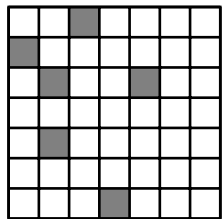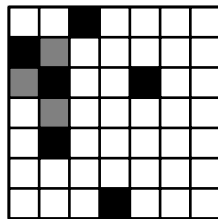
day 1

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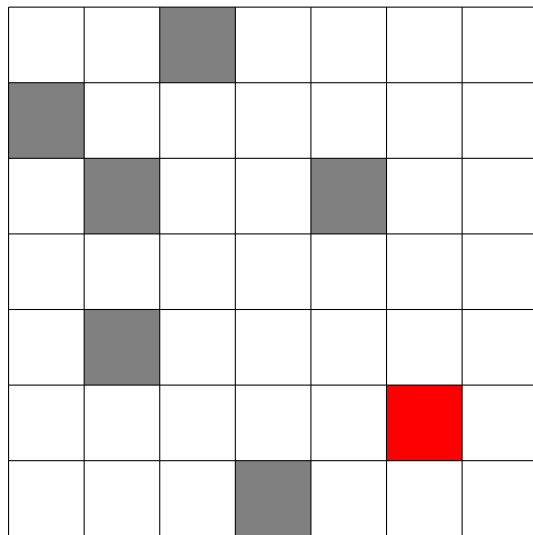
- Given initial gray infections, who ultimately gets infected?

day 1   day 2

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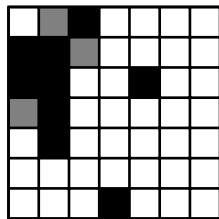
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day 1   day 2   day 3

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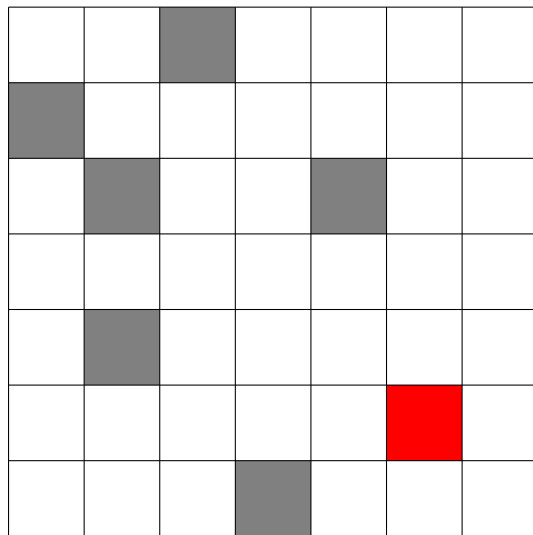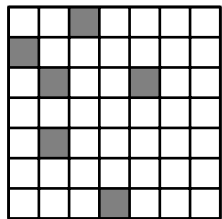day 1   day 2   day 3   day 4

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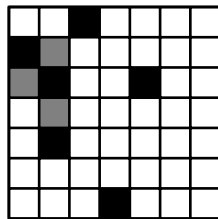|				
day 1	day 2	day 3	day 4	day 5

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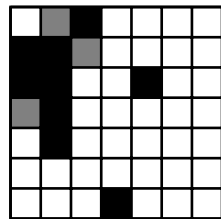

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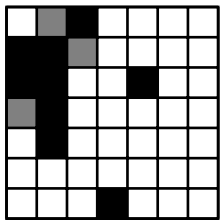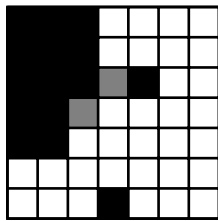
day 1   day 2   day 3   day 4   day 5   day 6

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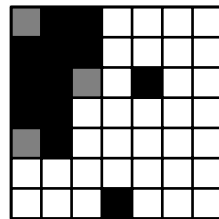- Given initial gray infections, who ultimately gets infected?

day 1   day 2   day 3   day 4   day 5   day 6   day 7

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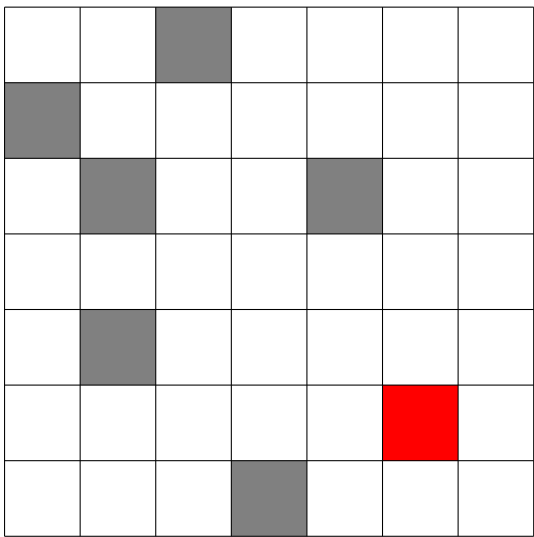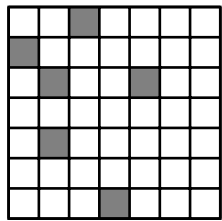

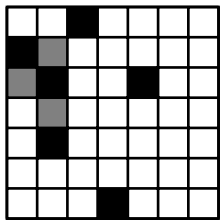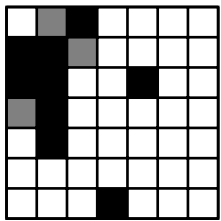| day 1 | day 2 | day 3 | day 4 | day 5 | day 6 | day 7 | day 8 |

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- Minimum infections to infect everyone?

| day 1 | day 2 | day 3 | day 4 | day 5 | day 6 | day 7 | day 8 |

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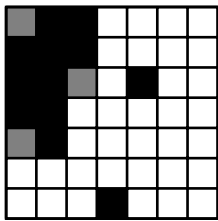- Given initial gray infections, who ultimately gets infected?

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- Given few vaccines, who to immunize?

day 1   day 2   day 3   day 4   day 5   day 6   day 7   day 8

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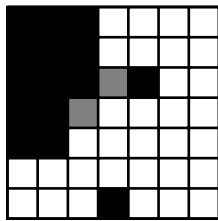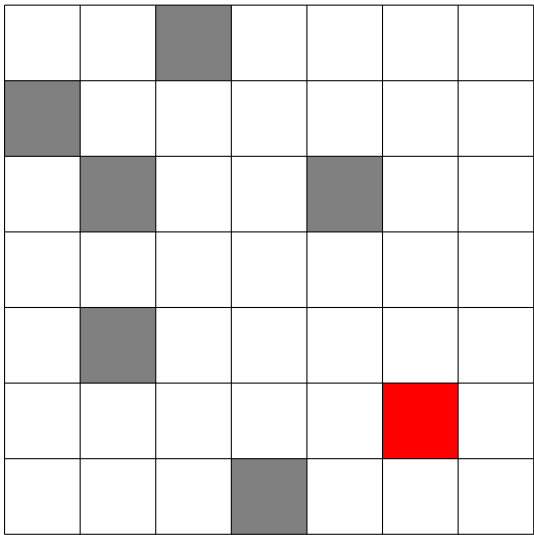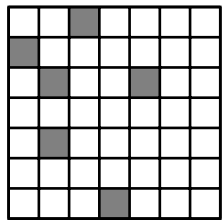- Given initial gray infections, who ultimately gets infected?

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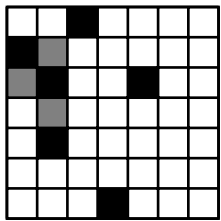| day 1 | day 2 | day 3 | day 4 | day 5 | day 6 | day 7 | day 8 |

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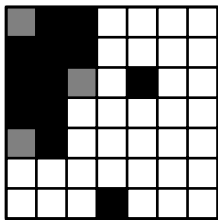- Given initial gray infections, who ultimately gets infected?

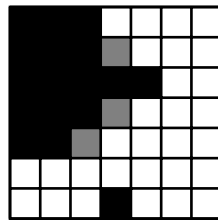- Minimum infections to infect everyone?

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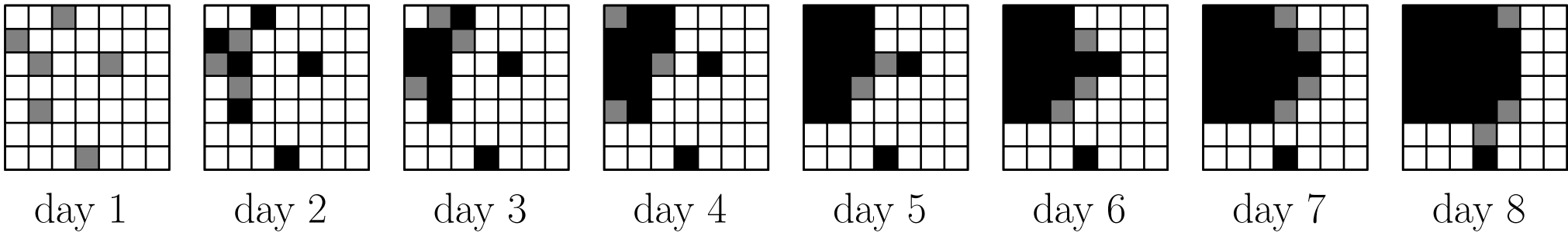
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Scheduling Speed Dates

In each round 4 people "group"-speed-date around a table. (4 rounds in all)
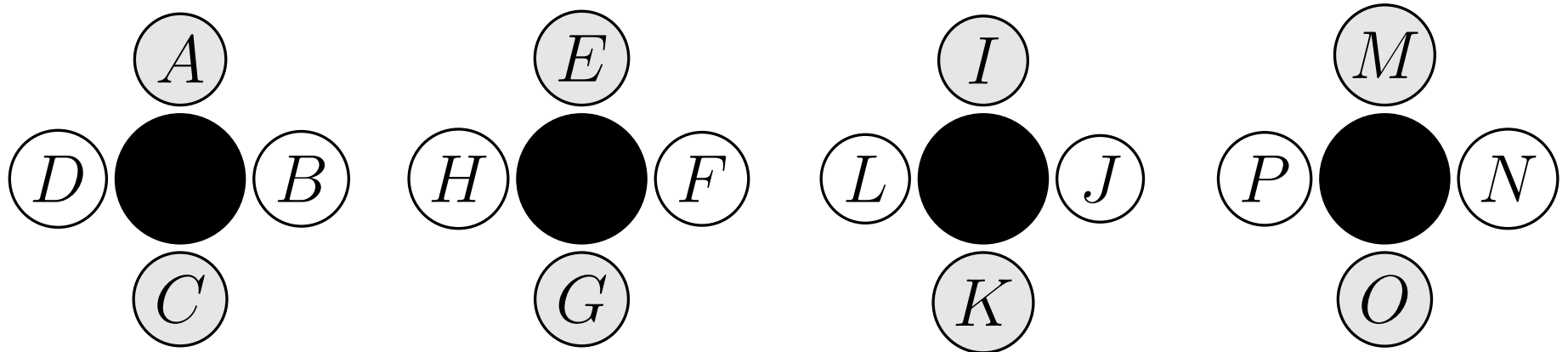
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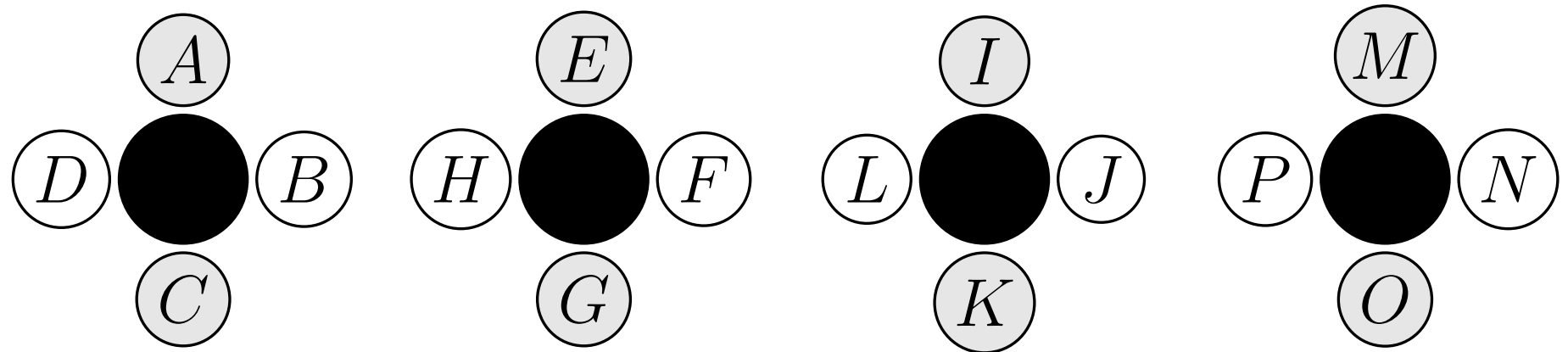
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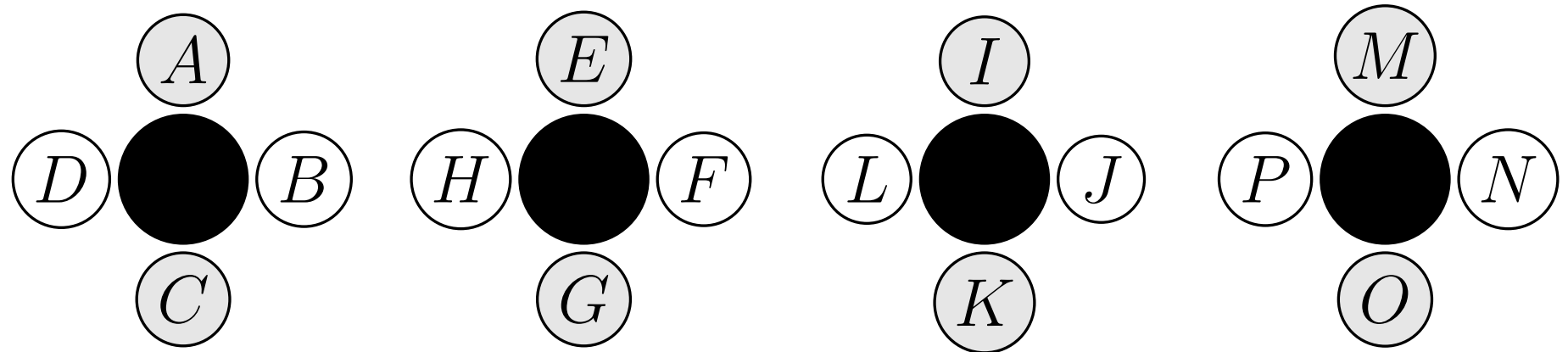
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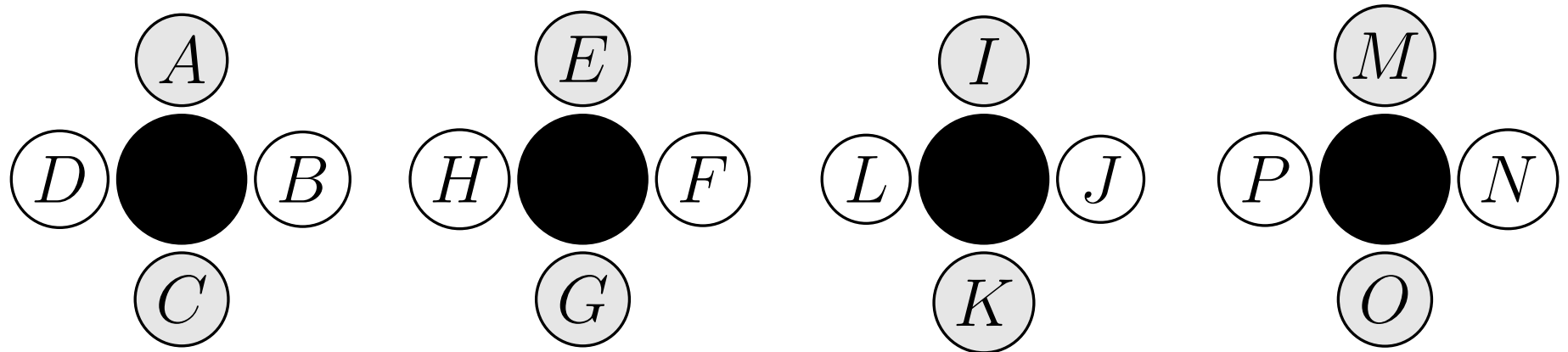
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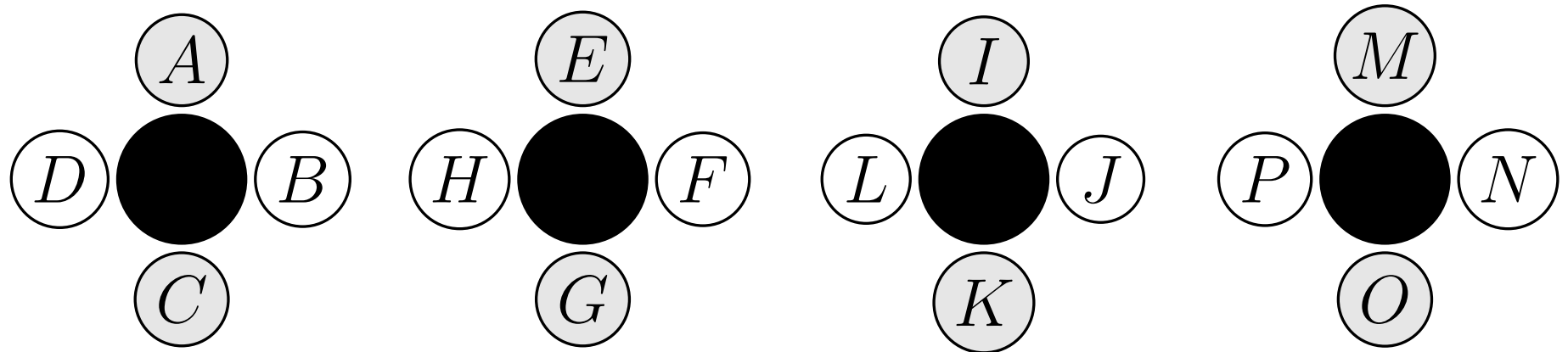
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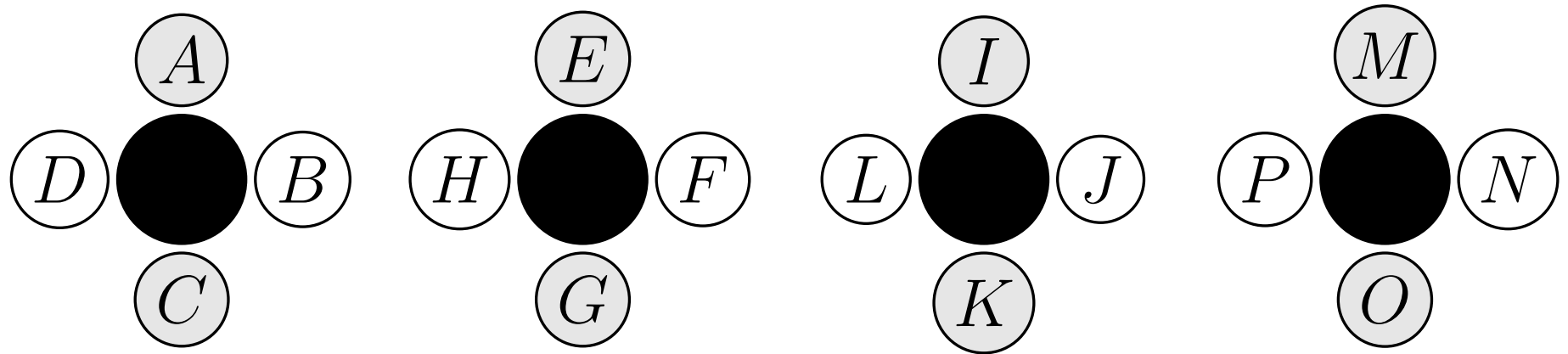
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Friendship Networks and Ads
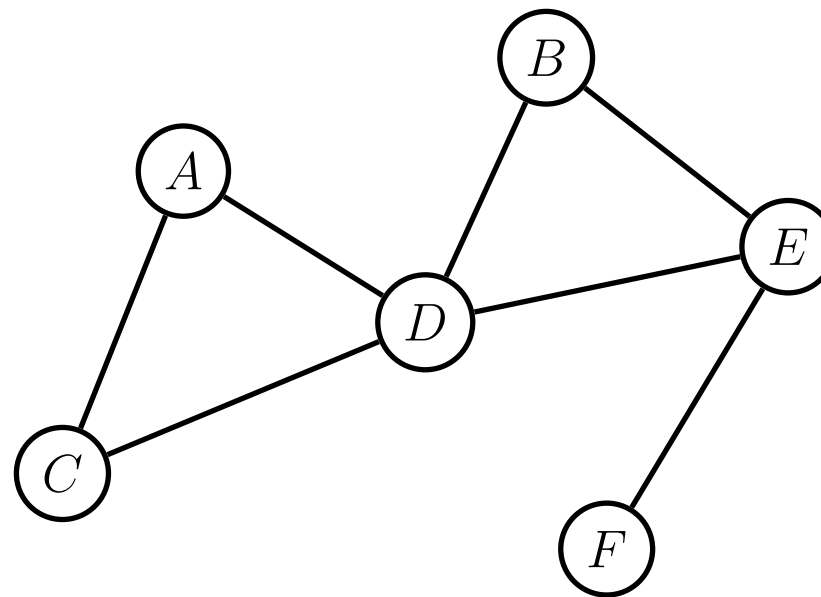
People are circles and links are friendships.

Friendship Networks and Ads

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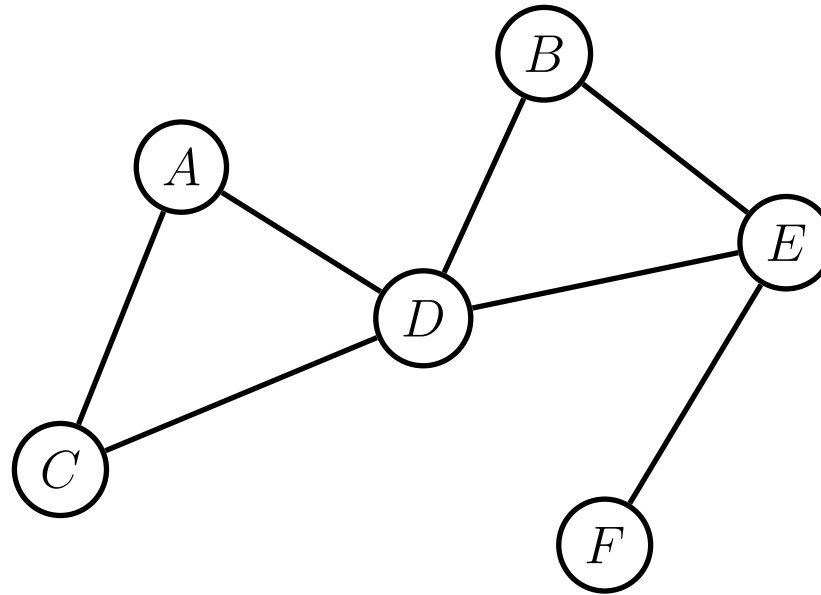
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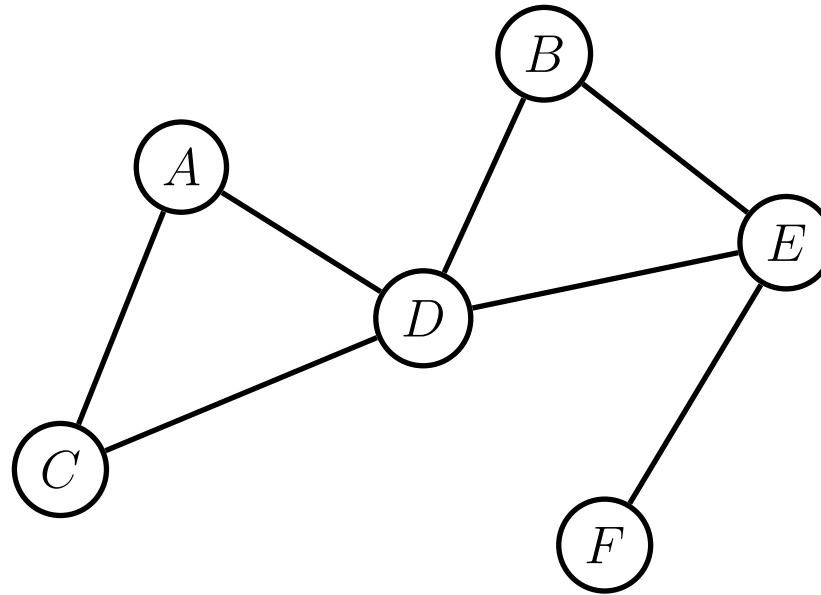
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Modeling Computers

Desktop, smartphone, fitbit, ...

What is computing?

Dominos:
$$
\begin{array}{ccc}
d_1 & d_2 & d_3 \\
\boxed{\begin{array}{c} 0 \\ \hline 100 \end{array}} &
\boxed{\begin{array}{c} 01 \\ \hline 00 \end{array}} &
\boxed{\begin{array}{c} 110 \\ \hline 11 \end{array}}
\end{array}
$$

Modeling Computers

Desktop, smartphone, fitbit, ...

What is computing?

Dominos:
$$d_1 \quad d_2 \quad d_3$$

d_1	d_2	d_3
0	01	110
100	00	11

$$d_3 d_1 d_3 \;=\; \begin{array}{|c|c|c|} \hline 110 & 0 & 110 \\ \hline 11 & 100 & 11 \\ \hline \end{array} \quad \longrightarrow \quad \begin{array}{|c|} \hline 1100110 \\ \hline 1110011 \\ \hline \end{array}$$

Domino puzzle: Want same top and bottom.

Modeling Computers

Desktop, smartphone, fitbit, . . .

What is computing?

Dominos:

$$d_1 \quad d_2 \quad d_3$$

$$\boxed{\begin{matrix} 0 \\ 100 \end{matrix}} \quad \boxed{\begin{matrix} 01 \\ 00 \end{matrix}} \quad \boxed{\begin{matrix} 110 \\ 11 \end{matrix}}$$

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Domino puzzle: Want same top and bottom.

Domino program:
  Input: dominos
  Output: sequence that works
              or
          say it can't be done

Modeling Computers

Desktop, smartphone, fitbit, . . .

We have deep questions:

1. What can we compute?

2. What *can't* we compute?

3. How fast?

Answers involve discrete math.

Computing is Mathematics

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"A programmer must *demonstrate* that their program has the required properties. If this comes as an afterthought, it is all but certain that they won't be able to meet this obligation. Only if this obligation influences the design is there hope to meet it. . .

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"The required techniques of effective reasoning are pretty formal, but as long as programming is done by people who don't master them, the software crisis will remain with us and will be considered an incurable disease. And you know what incurable diseases do: they invite the quacks and charlatans in, who in this case take the form of Software Engineering Gurus."

– Edsger Dijkstra

Polya's Mouse

"A mouse tries to escape from an old fashioned cage. After many futile attempts bouncing back-and-forth, thumping his body against the cage bars, he finally finds one place where the bars are *slightly* wider apart. The mouse, bruised and battered escapes through this small opening, and to his elation, finds freedom." – Polya

Polya's Mouse

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Connect tiles of the same letter with wires. Wires cannot cross, enter tiles, or leave the box. How can it be done? If it can't be done, why not?

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Don't be quick to dismiss either conclusion. Try this and that. Fiddle around until you understand the problem and the difficulty. Patience.

To solve such problems, "*You need brains and good luck. But, you must also sit tight and wait till you get a bright idea.*" – Polya.
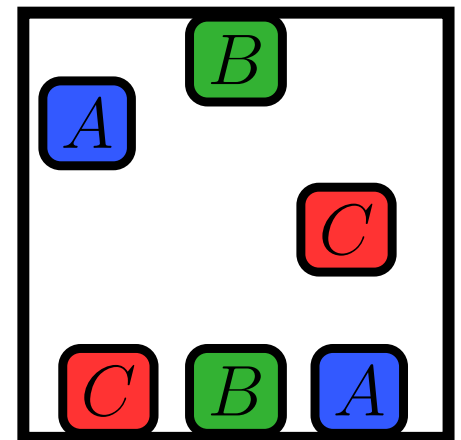
Getting Good at Discrete Math

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> 4: Prove the conjecture and make it a theorem. You now *know* something new.

Tinker, Tinker, Tinker, # Tinker!
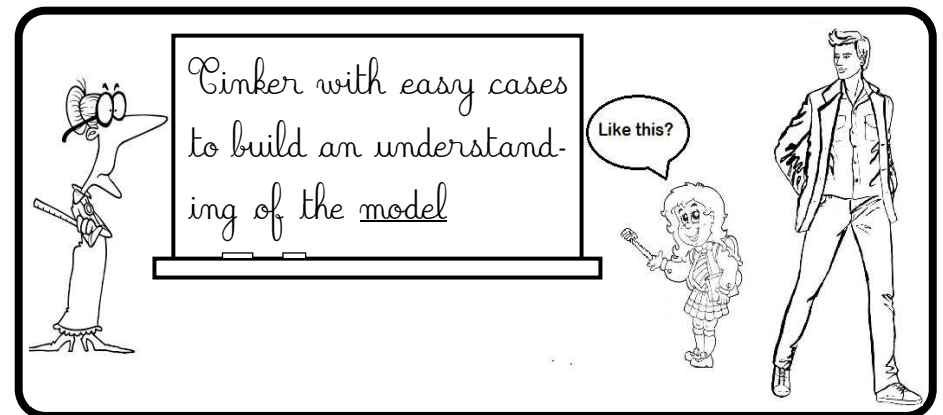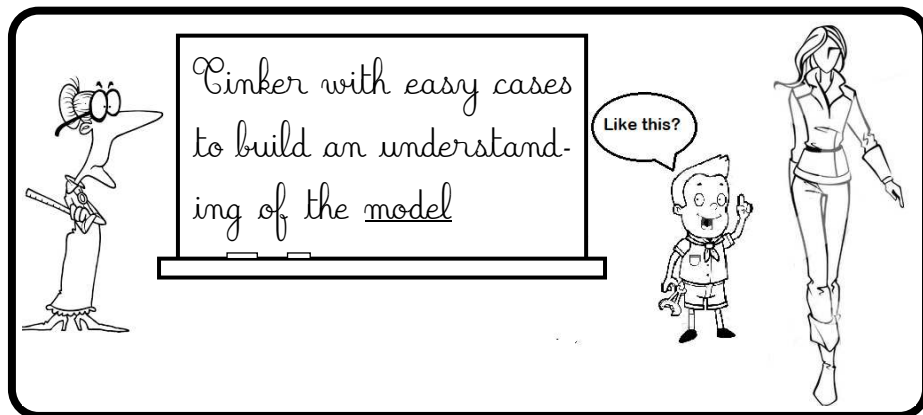
Three Challenge Problems

$10
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Three Challenge Problems

$100

Distinct subsets with the same sum

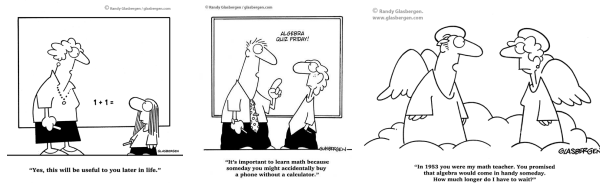|	
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1294587141921952639693619381	8634764617265724716389775433
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1474343641823476922667154474	2267353254454872616182242154
2578649763684913163429325833	4689911847578741473186337883
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3624757247737414772711372622	2446827667287451685939173534
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9893315516156422581529354454	2926718838742634774778713813
5913625989853975289562158982	3791426274497596641969142899
8313891548569672814692858479	2831727715176299968774951996
2265865138518379114874613969	3281287353463725292271916883
3477184288963424358211752214	9954744594922386766735519674
6321349612522496241515883378	3414339143545324298853248718

$10

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Three Challenge Problems
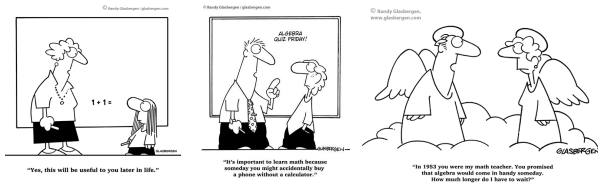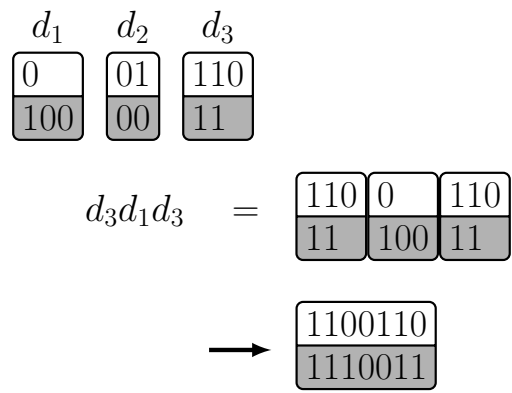
<table>
<tr><td>

$100

Distinct subsets with the same sum

</td><td>

$1,000

Domino Program

</td><td>

$10

Create the best 'math'-cartoon

</td></tr>
</table>

$100	$1,000	$10
Distinct subsets with the same sum	Domino Program	Create the best 'math'-cartoon

$100 — Distinct subsets with the same sum

57198253935679613465581556229 17964396948242132669588863933
54879458828431586966721579844 63662525317599556769444496585
47677665317542548742242577633 85454585456368989743659382744
18559243597577321258662397844 33622911862115223185668525766
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79671319617688548895942171866 28928575643552622199965984217
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47644136353239113616991835866 84152342431827875341238948588
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25786497636849131634293258333 46899118475787414731863378833
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44684637158667462589765523444 28325152413829374986146762466
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12589222637292965897854188399 98336628257346244557366383288
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87943531722131776129397762155 43448435117829128758436326522
29896942458274797691523136299 75688425627481365186151177977
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34771842889634243582117522144 99547445949223867667355519674
63213496125224962415158833788 34143391435453242988532487188

$1,000 — Domino Program

$d_1 \quad d_2 \quad d_3$

| $\frac{0}{100}$ | $\frac{01}{00}$ | $\frac{110}{11}$ |

$$d_3 d_1 d_3 \quad = \quad \boxed{\frac{110}{11}}\boxed{\frac{0}{100}}\boxed{\frac{110}{11}}$$

$$\longrightarrow \quad \frac{1100110}{1110011}$$

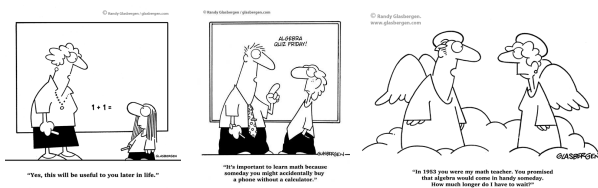**Goal:** Want same top and bottom.

Domino program:
  Input: dominos
  Output: sequence that works
          or
          say it can't be done

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