$\chi(K_n) = n$, $\chi(L_n) = 2$ $\chi(S_n) = 2$

1. The chromatic numbers of the graphs K_n, L_n, S_n, and W_n satisfy which of the following orderings?

A $\chi(S_n) = \chi(W_n)$

B $\chi(K_n) \geq \chi(W_n)$

C $\chi(L_n) < \chi(S_n)$

D $\chi(S_n) > \chi(W_n)$

E $\chi(K_n) < \chi(L_n)$

$$\chi(W_n) = \begin{cases} 4, & n \text{ even} \\ 3, & n \text{ odd} \end{cases}$$

2. If the greedy coloring algorithm is used to color W_6, which of the following claims is true?

$\chi(W_6) = 4$

A Six colors may be used

both valid

B At least three colors will be used

C At least four colors will be used

D At most three colors will be used

E None of the above

3. In the binomial expansion of $(x + 1/x)^8$ the constant term is:

$$\binom{8}{k-1} x^{9-k} \left(\frac{1}{x}\right)^{k-1} = 1$$

$$\binom{8}{k-1} x^{9-k} x^{1-k} = x^0$$

$$\Rightarrow x^{9-k} \cdot x^{1-k} = x^0$$

$$9-k+1-k = 0$$

$k = 5$, 5th term $= \binom{8}{4} x^{4} \cdot x^{-4}$ $= 70$

A 0

B 8

~~C~~ 70

D 1

E None of the above

4. The course requirement at a university is to select any three from a list of seven courses. If 200 students completed the degree requirement, what is the smallest number of students that must share the same course selections?

Pigeonhole Principle

Boxes (Pigeonholes) = no. of possible selections

= (7 choose 3) = 35

Pigeons in each hole = $200/35 \simeq 5.72$ students per box

The max. out of a set of 35 boxes must be greater than this average.

∴ Atleast 6 students.

~~A~~ 6

B 7

C 2

D 0

E 10

5. How many solutions does the equation $x_1 + x_2 + x_3 = 12$ have when we impose the constraints $x_1 \geq 1$, $x_2 \geq 2$, and $x_3 \geq 3$?

$$\binom{3+6-1}{6} = \binom{8}{6} = \frac{8!}{6!\,2!} = \frac{8 \cdot 7}{2} = 28$$

A 78

B 21

~~C~~ 28

D 18

E 15

6. A furniture store recently received a shipment of twenty-five wooden tables. Out of these, ten are office tables and the rest are coffee tables. The store does a quality check before putting the tables on sale. If six of the tables are selected at random to be checked, what is the probability that at least three of those selected are coffee tables?

$$\frac{\sum_{x=3}^{6}\binom{10}{6-x}\binom{15}{x}}{\binom{25}{6}}$$

- 0.3083
- B 0.5011
- C 0.2525
- D 0.8530
- E 0.4162

7. It is known that 30% of a certain company's food processors require service while under warranty, whereas only 10% of its juicers need such service. If someone purchases both a food processor and a juicer made by this company, what is the probability that none of the machines need service while under warranty?

- A 0.40
- B 0.30
- C 0.90
- D 0.63
- E 0.70

Independence

$(1-0.30)(1-0.10)$

$= 0.70 \times 0.90$

$= 0.63$

8. Joe decided to record his favorite TV show using two recording devices while he was out with a friend. His devices do not work perfectly. Device 1 succeeds in recording a program 70% of the time and device 2 succeeds 60% of the time. What is the probability that when he gets home he is able to get exactly one recording of the TV show?

- A 1
- B 0.65
- C 0.46
- D 0.70
- E 0.60

P = P(1st succeed and 2nd doesn't) + P(2nd succeed and 1st doesn't)

$= (0.70)(0.40) + (0.30)(0.60)$

$= 0.46$

9. Wildfires are dangerous because 90% of them produce smoke. Wildfires themselves are rare and have a 1% percent chance of occurring during summer. Smoke in summer is not that rare (a 10% chance of occurring), mostly due to the increased number of outdoor barbecues. What is the probability of a wildfire in summer when there is smoke?

- A 0.9
- B 0.1
- C 0.01
- D 0.09
- E Insufficient information

$$P(\text{Fire} \mid \text{Smoke}) = \frac{P(\text{Smoke} \mid \text{Fire}) \cdot P(\text{Fire})}{P(\text{Smoke})}$$

$= 0.09$

10. A Best Buy store sells three different brands of refurbished laptops. Of the laptops, 50% are brand 1, 30% are brand 2 and 20% are brand 3. Each laptop manufacturer offers a one-year warranty on its laptops. It is known that 25% of brand 1 laptops require repair while under warranty; the corresponding percentages for brands 2 and 3 are 20% and 10%, respectively.

i) What is the probability that a randomly selected customer has a laptop that needs repair while under warranty?

ii) If a customer returns to the store with a laptop that needs warranty repair, what is the probability that it is a brand 1 laptop?

NOTE: The options below state the answers for parts i) and ii) separated by a comma.

A 0.50, 0.71

B 0.121, 0.45

☒ 0.205, 0.61

D 0.290, 0.10

E 0.113, 0.69

i) P(needs repair (R)) = P[(brand1 and repair) ~~or~~ (brand2 and repair) or (brand 3 and repair)]

$P(A_1 \cap R) = P(R|A_1) \cdot P(A_1) = 0.125$

$= P(A_1 \cap R) + P(A_2 \cap R) + P(A_3 \cap R)$

$P(R) = 0.125 + 0.060 + 0.020 = 0.205$

ii) $P(A_1|R) = \frac{P(A_1 \cap R)}{P(R)} = \frac{0.125}{0.205} = 0.61$

11. How many four-digit numbers that are divisible by 5 can be formed from the digits 1, 2, 3, 4, 5, 6 and 7 when none of the digits are repeated?

☒ 120

B 35

C 24

D 720

E None of the above

A number is divisible by 5 if it has 0 or 5 in the end (at one's place). There is only 1 5 in the given set of no's. so we fix it at ones place (1 way of doing this). Since digit should'nt repeat we have 6 ways to fill ten's place (out of 6 ~~given~~ remaining digits), 5 to fill 100th place & 4 to fill 1000th place

$= 4.5.6.1 = 120$

12. Your friend has three different email accounts. 70% of the messages come into account one, 20% come into account two and 10% come into account three. Of the messages in account one, only 1% are spam; the corresponding percentages for accounts two and three are 2% and 5%, respectively. What is the probability that a randomly selected one of your friend's messages is spam?

A 0.082

B 0.055

☒ 0.016

D 0.030

E None of the above

Law of Total probability

$P(B) = 0.01(0.70) + 0.02(0.20) + 0.05(0.10)$

$= 0.016$

13. A flu virus affects about one out of 10,000 people. There is a test to check whether the person has the flu virus. The test is quite accurate. In particular, we know that the probability that the test result is positive (suggesting the person has the flu virus), given that the person does not have the flu virus, is only 2%; the probability that the test result is negative (suggesting the person does not have the flu virus), given that the person has the flu virus, is only 1%. A random person is tested for the flu virus and the result is positive. What is the probability that the person has the flu virus?

A 0.1012

B 0.0203

C 0.0001

☒ 0.0049

E 0.5004

$P(\text{flu}|\text{test +ve}) = \frac{P(\text{test+}|\text{flu}) \cdot P(\text{flu})}{P(\text{test +ve})} = \frac{0.99 \times 0.0001}{0.020097}$

$P(\text{test +ve}) = P(\text{+ve}|\text{flu}) \cdot P(\text{flu}) + P(\text{+ve}|\text{no flu}) \cdot P(\text{no flu})$

$= 0.020097$

Ans = 0.0049

14. A biased coin shows heads with probability $\frac{2}{3}$. The coin is flipped until either two consecutive heads or two consecutive tails are obtained. What is the probability that this procedure terminates with two consecutive heads?

Winning outcomes | $(HT)^{i}\,HH$ | $T\,(HT)^{i}\,HH$

Probability ✓add | $(p(1-p))^{i}\,p^2$ | $(1-p)\times(p(1-p))^{i}\times p^2$

$$P(\text{win}) = \frac{p^2(2-p)}{1-p(1-p)} = 16/21$$

A 4/9

B 2/3

☒ 16/21

D 9/27

E None of the above

Alternate solution: Use OUTCOME TREE METHOD.

15. Suppose there are three independent events E_1, E_2, and E_3. What is $\mathbb{P}(E_3 \,|\, E1 \cap E_2)$?

A $\mathbb{P}(E_1 \cap E_2)$

☒ $\mathbb{P}(E_3)$

C $\mathbb{P}(E_1 \cap E_2 \cap E_3)$

D $\mathbb{P}(E_1 \,|\, E_2)$

E None of the above

$$P(E_3|E_1\cap E_2) = \frac{P(E_1\cap E_2\cap E_3)}{P(E_1\cap E_2)}$$ By independence

$$= \frac{P(E_1)\,P(E_2)\,P(E_3)}{P(E_1)\,P(E_2)} = P(E_3)$$

16. A certain state's license plate numbers consist of two letters followed by three numbers. What is the probability that a license plate randomly chosen from all possible license plates for this state will start with NY and end in a 3?

$$P(NY\_\_3) = \frac{100}{26^2\,100^3}$$

A $\frac{100}{26!10!}$

B $\frac{10}{26^2 10^3}$

C $\frac{100}{36^2}$

D $\frac{10}{36!}$

☒ $\frac{100}{26^2 10^3}$

17. In my garden I cross-pollinated five pairs of my flowering plants, and each pair consists of one white flowering and one orange flowering plant. It is known that this process produces offspring with orange flowers 25% of the time. If each pair produced an offspring:

i) What is the probability that there were no orange-flowered plants in the five offspring?

ii) What is the probability that there are less than two orange-flowered plants?

X = no. of orange

NOTE: Each option below gives an answer for parts i) and part ii) separated by a comma.

A 0.250, 0.501

☒ 0.237, 0.632

C 0.632, 0.250

D 0.237, 0.501

E 0.250, 0.237

i) $n=5,\ p=0.25$

$$P(X=0) = \frac{5!}{0!\,(5-0)!}\,p^0(1-p)^5 = 0.237$$

ii) $P(X=0)+P(X=1)$

$$= 0.237 + \frac{5!}{1!\,(5-1)!}\,0.25^1(1-0.25)^4 = 0.632$$

18. A manufacturing company produces parts that are good (90%), partially defective (2%), or completely defective (8%). These parts pass through an automatic inspection machine, which is able to detect and discard any part that is completely defective. What is the probability that a part is good given that it passed the inspection machine?

$P(G) = 0.90$, $P(SD) = 0.02$, $P(OD) = 0.08$

$$P(G \mid \overline{OD}) = \frac{P(G \cap \overline{OD})}{P(\overline{OD})} = \frac{P(G)}{1 - P(OD)} = \frac{.90}{1 - 0.08}$$

$= 0.978$

[A] 0.555

[☒] 0.978

[C] 0.876

[D] 0.333

[E] None of the above

19. A row of houses are randomly assigned distinct numbers between 1 and 50 (inclusive). How many houses must there be to ensure that, within the collection of observed house numbers, there are 5 consecutive numbers?

Split the numbers into 10 pigeonholes
1-5, 5-10 ....
There are atleast 41 pigeonholes
= houses.

[A] 11

[B] 50

[C] 40

[☒] 41

[E] 49

20. If n and k are positive integers with $n \geq k$, then

[A] $\binom{n-1}{k} = \binom{n}{k} + \binom{n+1}{k}$

[☒] $\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$

Pascal's Identity

[C] $\binom{n-1}{k-1} = \binom{n}{k} + \binom{n+1}{k}$

[D] $\binom{n+1}{k-1} = \binom{n}{k-1} + \binom{n+1}{k}$

[E] None of the above