QUIZ 1: 120 Minutes

Answer **ALL** questions.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

Circle at most one answer per question.
10 points for each correct answer.

You **MUST** show **CORRECT** work to get full credit.

When in doubt, TINKER.

Total
200

1. $\sqrt{3}$ is what kind of number?

A A natural number.

B A rational number.

C An irrational number.

D An integer.

E None of the above.

2. Find the correct expression for the recurrence given by $A_0 = 1$ and $A_n = 3(A_{n-1} + 1) - 1$ when $n \geq 1$.

A $A_n = 2 \cdot 3^n - 1$

B $A_n = 3 \cdot 2^n - 1$

C $A_n = 5 \cdot 3^n - 4$

D $A_n = 3 \cdot 4^n - 2$

E None of the above

3. Which of the following is equivalent to the proposition $\forall x : (\neg \exists y : R(x, y))$?

A $\exists x : \forall y : \neg R(x, y)$

B $\forall x : \forall y : \neg R(x, y)$

C $\forall x : \exists y : R(x, y)$

D $\exists x : \forall y : \neg R(x, y)$

E None of the above.

4. An integer $n \in \mathbb{Z}$ has an odd square, that is n^2 is odd. Which claim is true?

A n is positive.

B n^2 is divisible by 3.

C n is odd.

D n is divisible by 3.

E None of the above claims are true.

5. S is recursively defined as follows: $1 \in S$, $2 \in S$, and if $a, b \in S$, then $ab + 1 \in S$. Which of the following is *not* true about S?

A S contains all the primes.

B $51 \in S$.

C All powers of 2 are in S.

D Given an element $x \in S$ that is not 1 or 2, the pair (a, b) that satisfies $ab + 1 = x$ is unique.

E All of the above are true.

6. Which of the following captures the proposition "For p to be true, it is sufficient that q be true"?

A $p \to q$

B $q \to p$

C $p \leftrightarrow q$

D $\neg q \to \neg p$

E None of the above.

7. All that we know of P is that $P(1), P(2), P(3)$ are true and $P(n) \to P(3n)$. We can conclude that P is true for which of the following values of n?

A 12

B 51

C 162

D 300

E All of the above.

8. Which of the following is *not* equivalent to $p \leftrightarrow q$?

A $(\neg p \to \neg q) \wedge (\neg q \to \neg p)$

B $(p \to q) \wedge (\neg q \to \neg p)$

C $(\neg p \vee q) \wedge (\neg q \vee p)$

D $(p \to q) \wedge (\neg p \to \neg q)$

E All of the above are equivalent.

9. Which of the following is the negation of "There is a student who got As on all the assignments and attended all lectures, but did not pass FOCS"? Let $A(x)$ denote "x got As on all assignments", $L(x)$ denote "x attended all lectures", and $P(x)$ denote "x passed FOCS".

A $\forall x : A(x) \wedge L(x) \wedge P(x)$

B $\exists x : A(x) \wedge L(x) \to P(x)$

C $\forall x : P(x) \to A(x) \wedge L(x)$

D $\forall x : \neg(A(x) \wedge L(x)) \vee P(x)$

E None of the above.

10. Which proof technique is most appropriate for showing that the product of any two consecutive integers is even?

A Direct.

B Leaping Induction.

C Contrapositive.

D Contradiction.

E None of the above.

11. Which proof technique is most appropriate for showing that $p_k \leq 2^{2^k}$, where p_k is the kth prime?

A Direct.

B Contraposition.

C Strong Induction.

D Contradiction.

E None of the above.

12. Consider the recursively defined function $f(n) = f(n/2)$ when $n \in \mathbb{N}$ is even and larger than 1, and $f(n) = f(n-1) + 1$ when $n \in \mathbb{N}$ is odd and larger than 3. How many base cases are needed so this function is well-defined on $\mathbb{N}$?

A It is already well-defined.

B 1

C 2

D 3

E None of the above.

13. What is the difference between using Induction versus Strong Induction to prove $P(n)$ for $n \geq 1$?

A The base cases are different.

B Induction is usually easier than Strong Induction.

C In Induction you prove $P(n+1)$. In Strong Induction you prove $P(n+2)$.

D In Induction you assume $P(n)$. In Strong Induction you assume $P(1) \wedge P(2) \wedge \cdots \wedge P(n)$.

E There is no difference between the two methods.

14. Which would be the worst choice of proof technique for establishing $n^8 \leq 2^n$ when $n \geq 80$?

A Leaping Induction.

B Strong Induction.

C Weak Induction.

D Direct.

E All of the above are equally suitable methods.

15. Which proof technique should be used to show that there are no rational solutions to $x^2 - 4x + 1 = 0$?

A Direct.

B Contrapositive.

C Contradiction.

D Induction.

E None of the above.

16. What are the first four terms A_0, A_1, A_2, A_3 in the recurrence $A_n = \begin{cases} 1 & n = 0; \\ 3A_{n-1} + 2 & n \geq 1. \end{cases}$

A $1, 5, 17, 53$

B $1, 5, 8, 11$

C $1, 3, 6, 9$

D $1, 3, 8, 12$

E None of the above

17. Let $A = \{7k \mid k \in \mathbb{N}\}$ and $B = \{3k \mid k \in \mathbb{N}\}$. Which statement is true?

A $A \cap B = \emptyset$

B $A \cap B$ has more than one element

C $A \subseteq B$

D $B \subseteq A$

E A and B contain only odd numbers.

18. How many lines are in the truth table for the proposition $p \to q \vee r$?

A 2

B 6

C 8

D 16

E None of the above

19. Which is the appropriate proof technique for the claim: n^7 is odd $\to n$ is odd ?

A Direct.

B Contrapositive.

C Contradiction.

D Induction.

E None of the above.

20. For which of the domains $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$ is the following statement true: $\forall x : (\exists y : x^2 > y)$?

A $\mathbb{N}$

B $\mathbb{N}$ and $\mathbb{Z}$

C $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$

D $\mathbb{Q}$ and $\mathbb{R}$

E None of the above are correct.