WEEKLY PARTICIPATION 5: THE ℓ_1 REGULARIZER ENCOURAGES SPARSITY

Consider the problem

$$\operatorname{argmin}_{x \in \mathbb{R}} \frac{1}{2}(x-a)^2 + \lambda |x|,$$

where $\lambda > 0$ is a nonnegative constant. This is a simple example of ordinary least squares with ℓ_1-regularization.

(1) Argue that this is a convex optimization problem, and it has a unique solution, given any a. Use our rules for constructing convex functions from simpler ones.

(2) Let $s_\lambda(a)$ be the unique solution to this optimization problem, given an a. State Fermat's optimality condition as concisely as you can, using our rules for subdifferential manipulation.

(3) Use Fermat's optimality condition to find an expression for $s_\lambda(a)$, and draw a cartoon/plot of s_λ as I might in class.