

**CSCI 4530/6530 Advanced Computer Graphics**

<https://www.cs.rpi.edu/~cutler/classes/advancedgraphics/S25/>

# **Lecture 14: The Rendering Equation & Irradiance Caching & Photon Mapping**

# *The Light of Mies van der Rohe*

Henrik Wann Jensen  
SIGGRAPH 2000



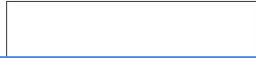
RENDERED USING DALI - HENRIK WANN JENSEN 2000



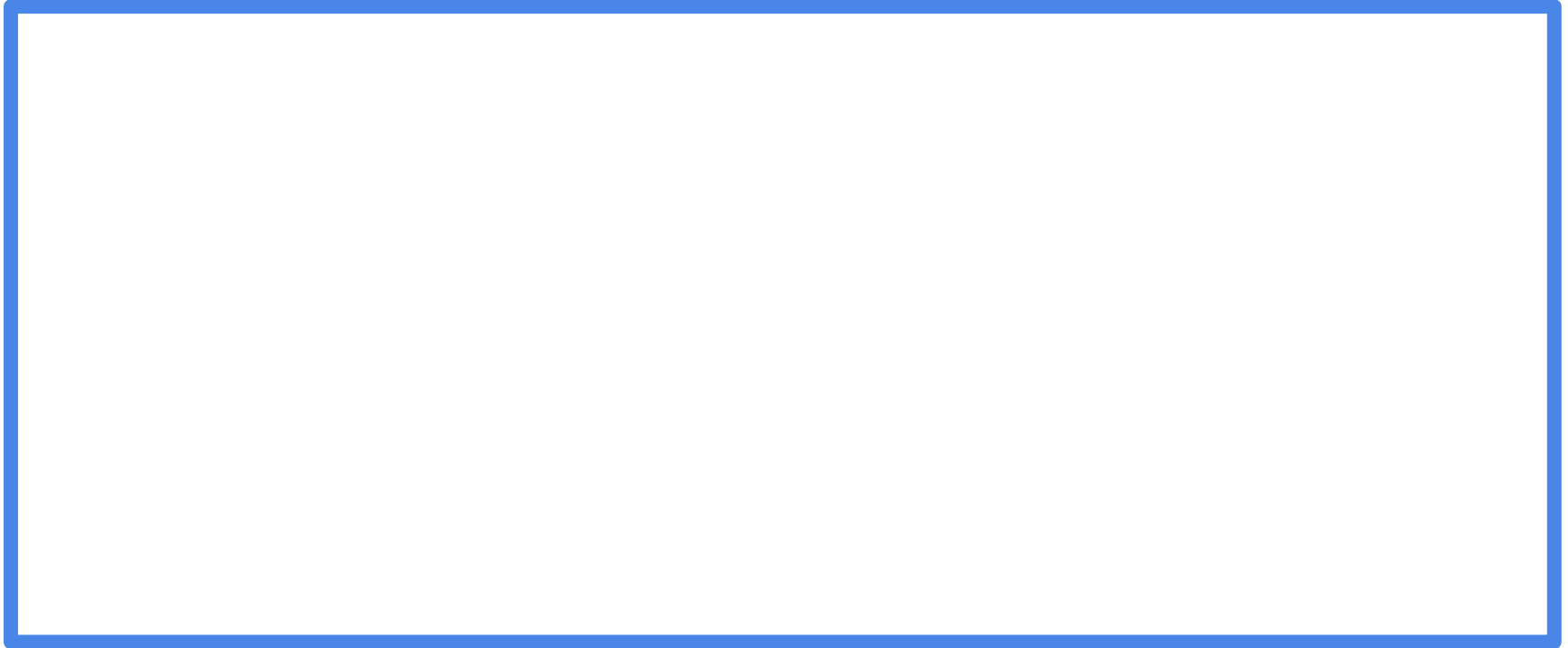
# Worksheet: Progressive Radiosity

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grey wall



Perform 5 iterations of progressive refinement radiosity.



undistributed @ iter 1

( 0 , 0 , 0 )

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# Final Project Brainstorming

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- Each student should post two different project ideas on the forum.  
For each idea:
  - Describe the idea, motivation, & an example of a potential result
  - A significant/interesting technical implementation
- Have you already decided on one idea?
- Do you already have a partner? Who? (e.g. name of classmate)
- If you have not decided on an idea and/or a partner everyone must post one idea
- **Part 1: Due Friday 2/28 @ 11:59pm**
- Teams of 2 strongly recommended  
(individuals & teams >2 require instructor permission)
- **Projects from prior terms are on the website**



## Part 2: Peer Feedback

*Due Monday 3/10 @ 11:59pm*

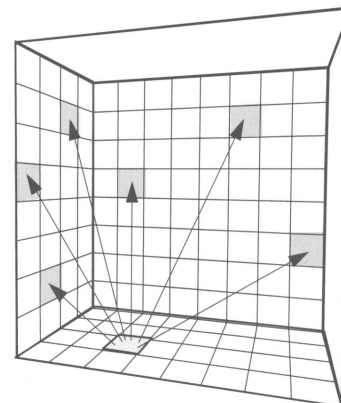
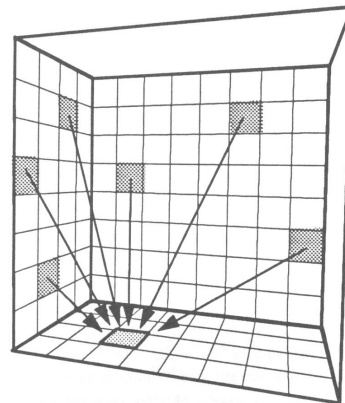
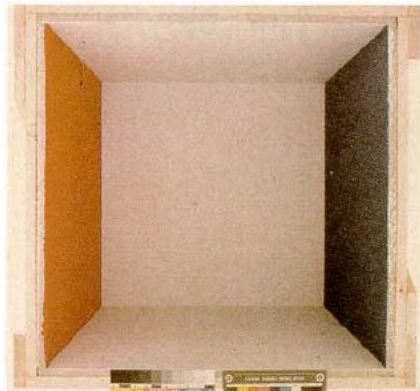
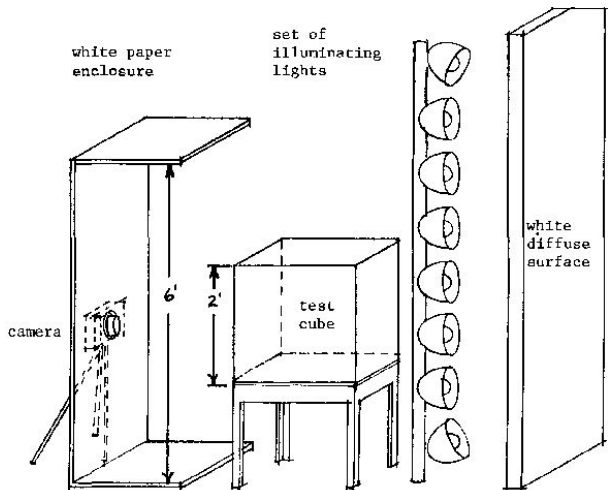
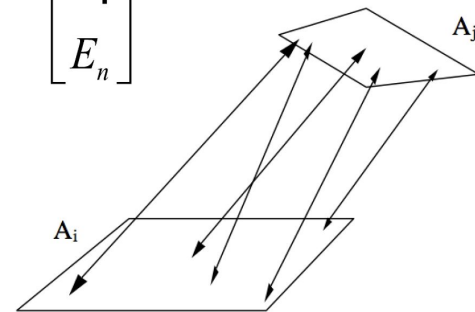
Reply to 3 of your  
classmates' ideas posts!

# Last Time?

- Cornell Box
- Form Factors
- Radiosity Matrix
- Progressive Radiosity (Incremental Solver)

$$\begin{bmatrix} 1 - \rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1 - \rho_2 F_{22} & & \\ \vdots & & \ddots & \\ -\rho_n F_{n1} & \cdots & \cdots & 1 - \rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

solve for  $B_i$





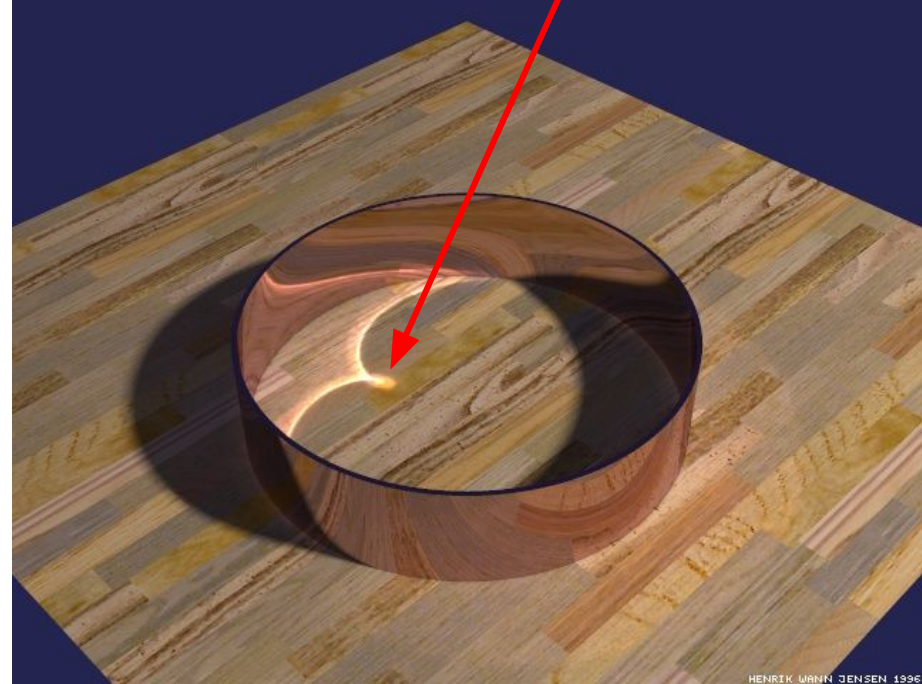
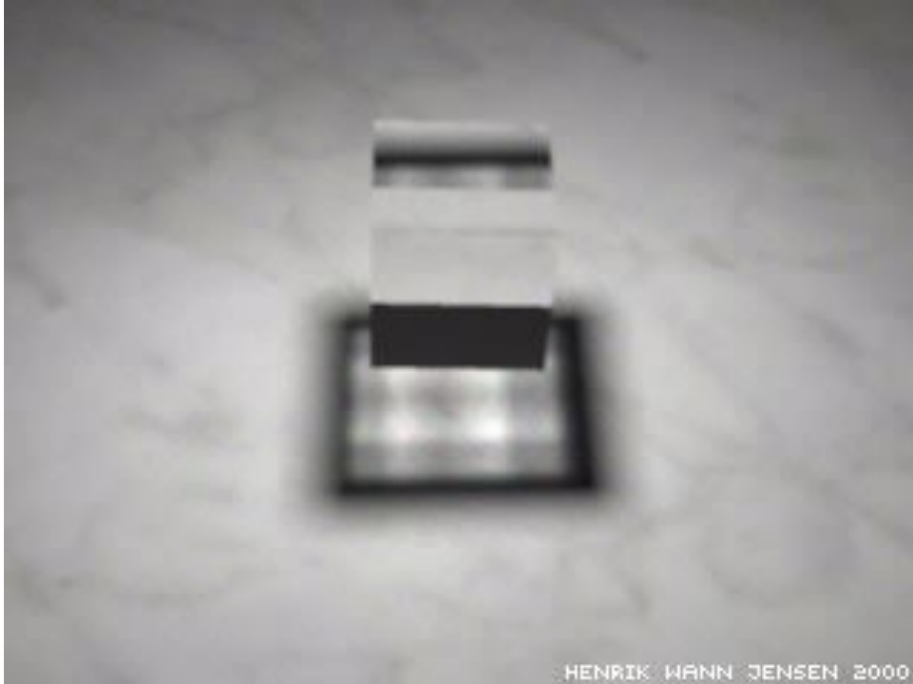
# Today

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- Worksheet: Progressive Radiosity
- **The Rendering Equation**
- Ray Casting vs. Ray Tracing vs. Monte-Carlo Ray Tracing vs. Path Tracing
- Irradiance Caching
- Photon Mapping
- Papers for Today
- Ray Grammar
- Papers for Next Time

# Is this Traditional Ray Tracing?

*We don't know that this spot is extra bright.  
We don't know what directions to cast rays  
from diffuse surface to find this mirror caustic.*



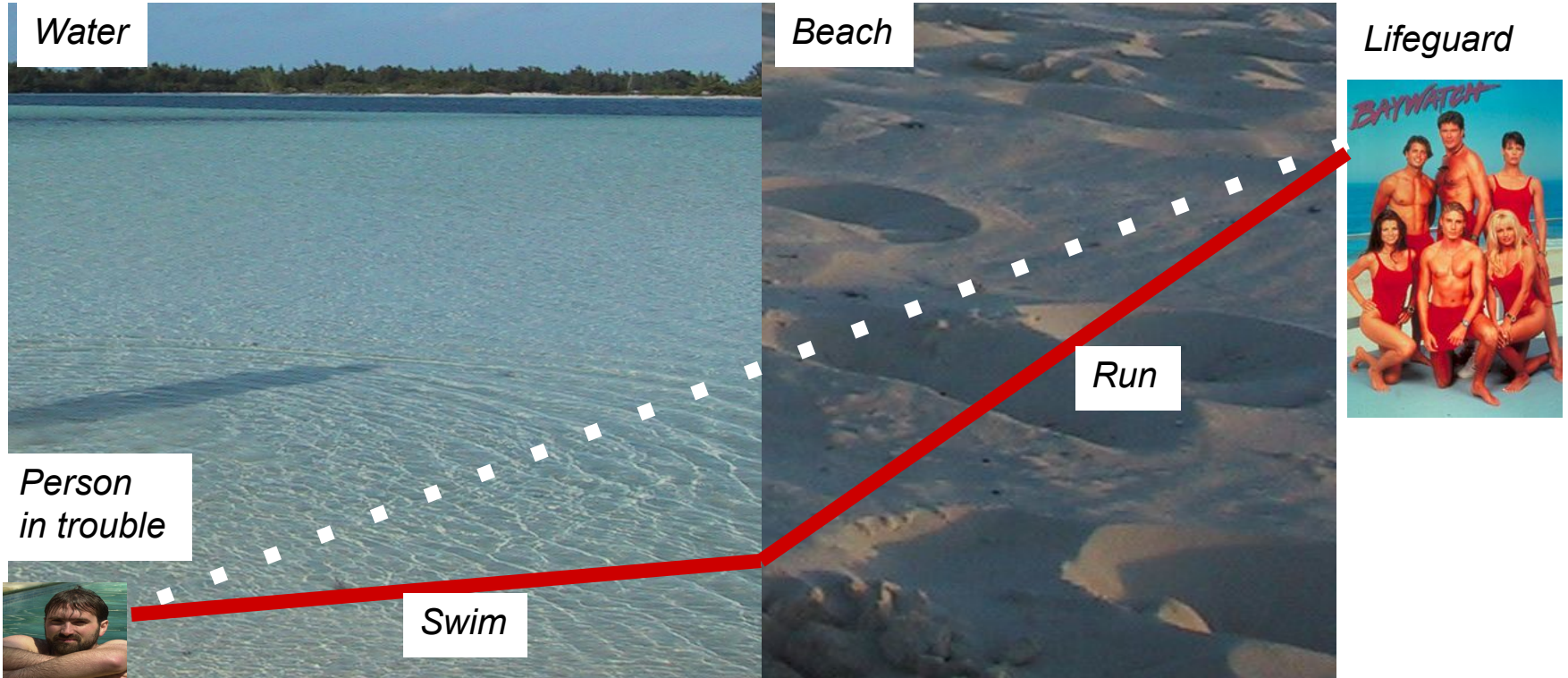
*Images by Henrik Wann Jensen*

*No. Refraction and complex reflections for illumination are not handled properly in traditional (backward) ray tracing.*



# Refraction and the Lifeguard Problem

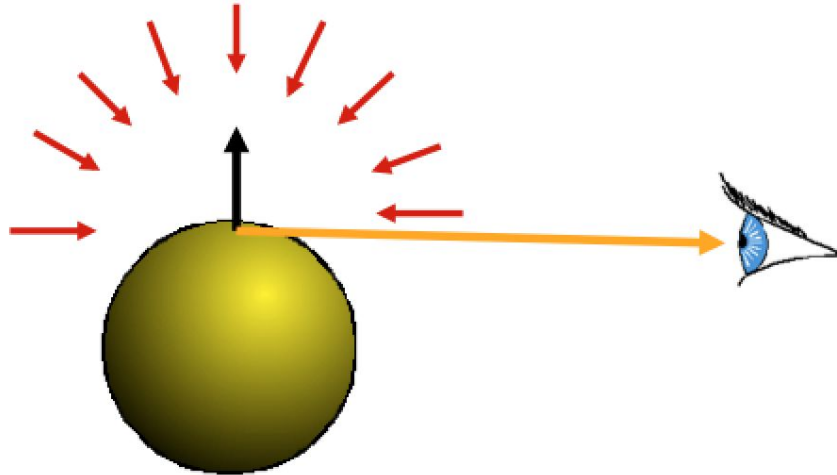
- Running is faster than swimming: What is the optimal rescue path?



# The Rendering Equation

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- Clean mathematical framework for light-transport simulation
- At each point, outgoing **light in one direction** is the integral of **incoming light in all directions** multiplied by reflectance property



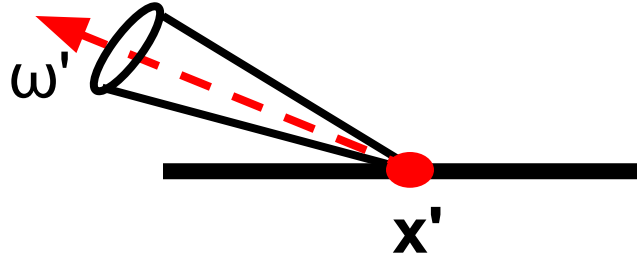
# The Rendering Equation, *Kajiya, SIGGRAPH 1986*

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# The Rendering Equation

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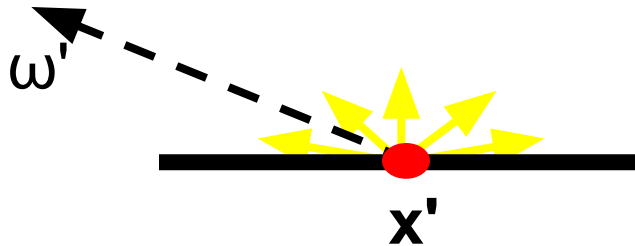


$$L(x', \omega') = E(x', \omega') + \int \rho_{x'}(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$

$L(x', \omega')$  is the radiance from a point on a surface in a given direction  $\omega'$

# The Rendering Equation

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$$L(x', \omega') = E(x', \omega') + \int \rho_{x'}(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$

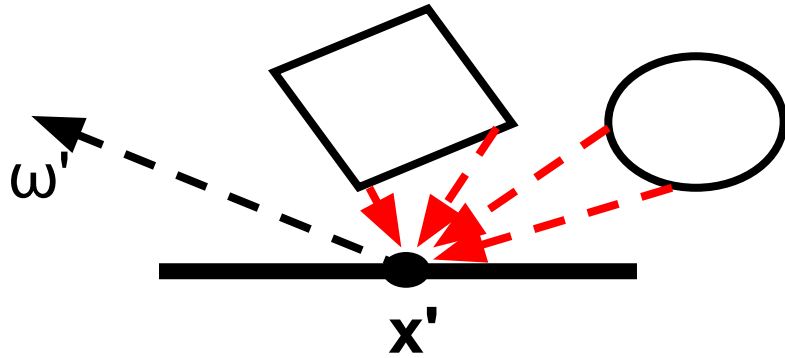


$E(x', \omega')$  is the emitted radiance from a point:  $E$  is non-zero only if  $x'$  is emissive (a light source)



# The Rendering Equation

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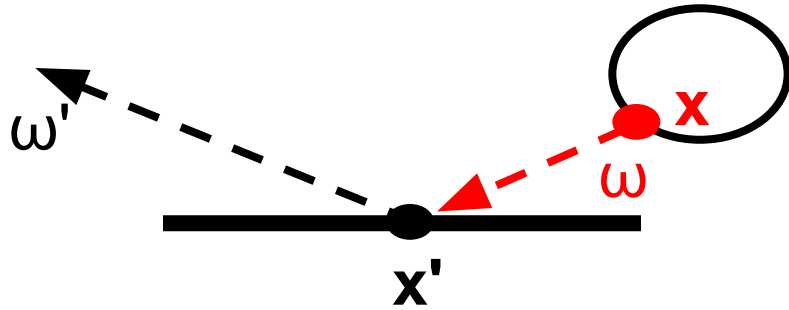


$$L(x', \omega') = E(x', \omega') + \underbrace{\int \rho_{x'}(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA}_{\text{Sum the contribution from all of the other surfaces in the scene}}$$

Sum the contribution from all of the other surfaces in the scene

# The Rendering Equation

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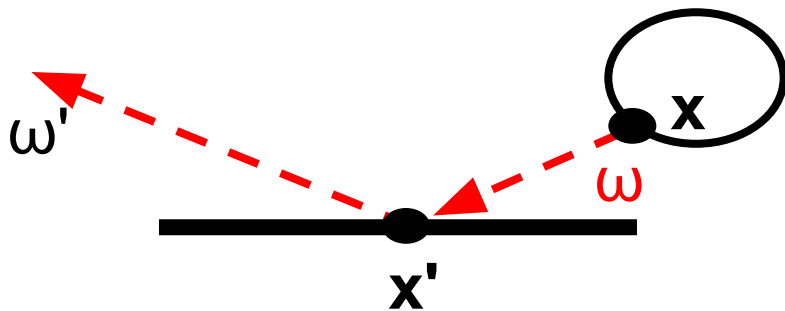


$$L(x', \omega') = E(x', \omega') + \int \rho_{x'}(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$

For each  $x$ , compute  $L(x, \omega)$ , the radiance at point  $x$  in the direction  $\omega$  (from  $x$  to  $x'$ )

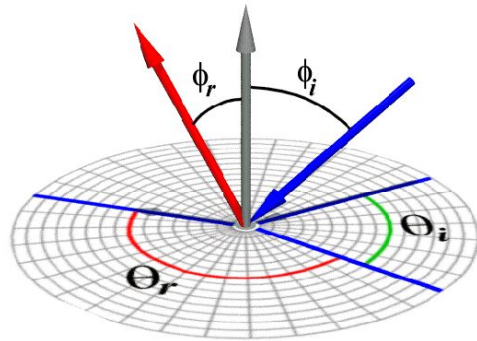
# The Rendering Equation

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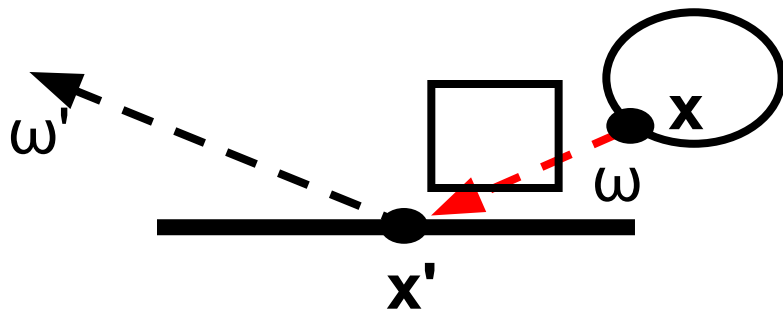
$$L(x', \omega') = E(x', \omega') + \int \rho_{x'}(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$

scale the contribution by  $\rho_{x'}(\omega, \omega')$ , the reflectivity (BRDF) of the surface at  $x'$



# The Rendering Equation

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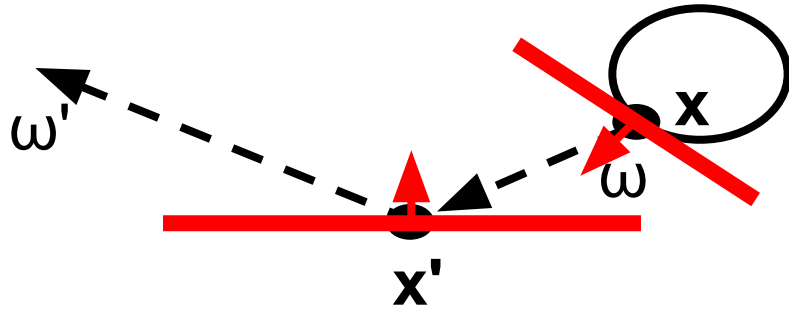


$$L(x', \omega') = E(x', \omega') + \int \rho_{x'}(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$

For each  $x$ , compute  $V(x, x')$ ,  
the visibility between  $x$  and  $x'$ :  
1 when the surfaces are unobstructed  
along the direction  $\omega$ , 0 otherwise

# The Rendering Equation

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$$L(x', \omega') = E(x', \omega') + \int \rho_{x'}(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$

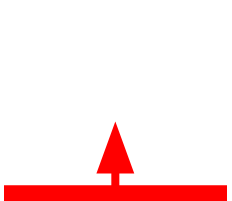
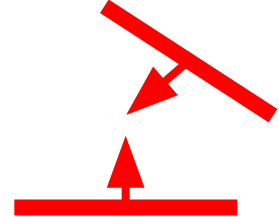
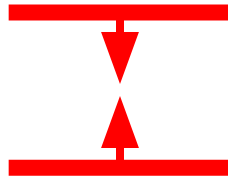
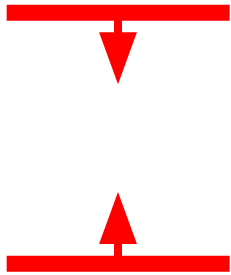
For each  $x$ , compute  $G(x, x')$ , which describes the on the geometric relationship between the two surfaces at  $x$  and  $x'$



# Intuition about Geometry Term $G(x,x')$ ?

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- Which arrangement of two surfaces will yield the greatest transfer of light energy? Why?



# Rendering Equation $\rightarrow$ Radiosity

*Different rendering techniques can be expressed as a subset/approximation of the full rendering equation.*

$$L(x', \omega') = E(x', \omega') + \int \rho_{x'}(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$



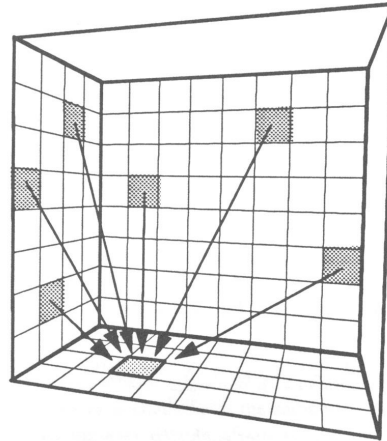
Radiosity assumption:  
perfectly diffuse surfaces (not directional)

$$B_{x'} = E_{x'} + \rho_{x'} \int B_x G(x, x') V(x, x')$$



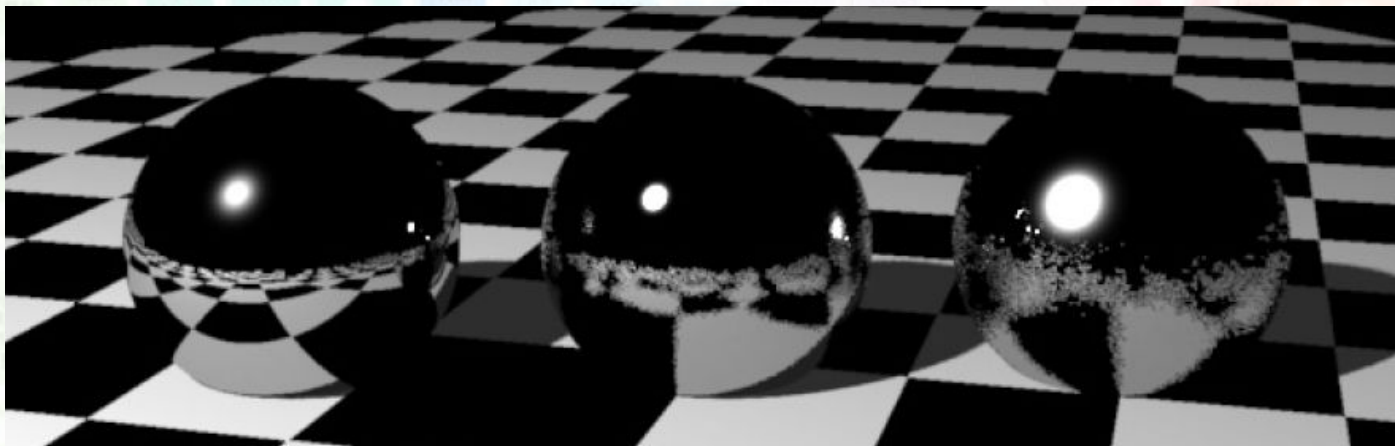
discretize

$$B_i = E_i + \rho_i \sum_{j=1}^n F_{ij} B_j$$

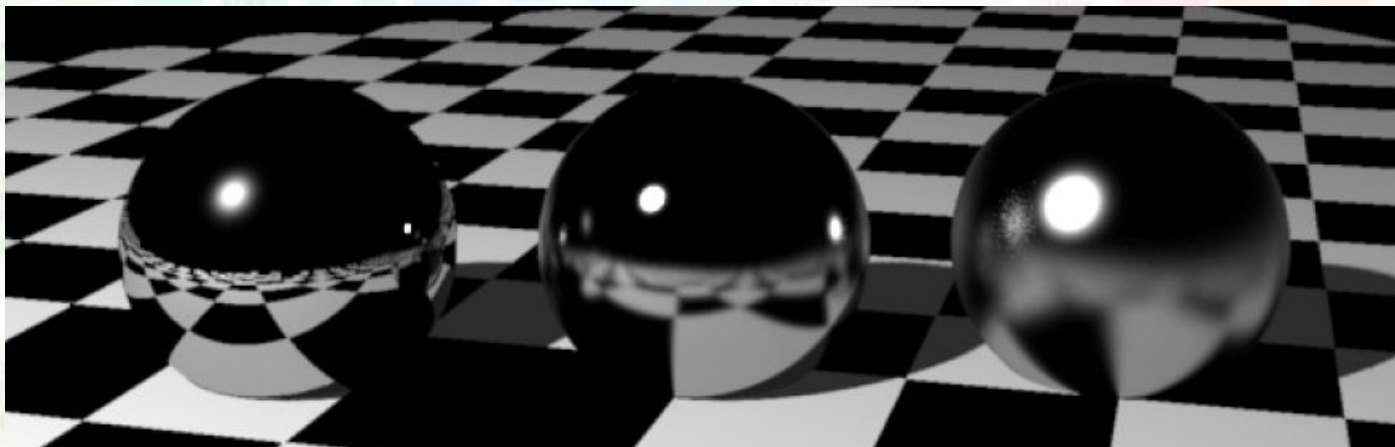


# Questions?

1 glossy  
sample  
per pixel



256 glossy  
samples  
per pixel



# Today

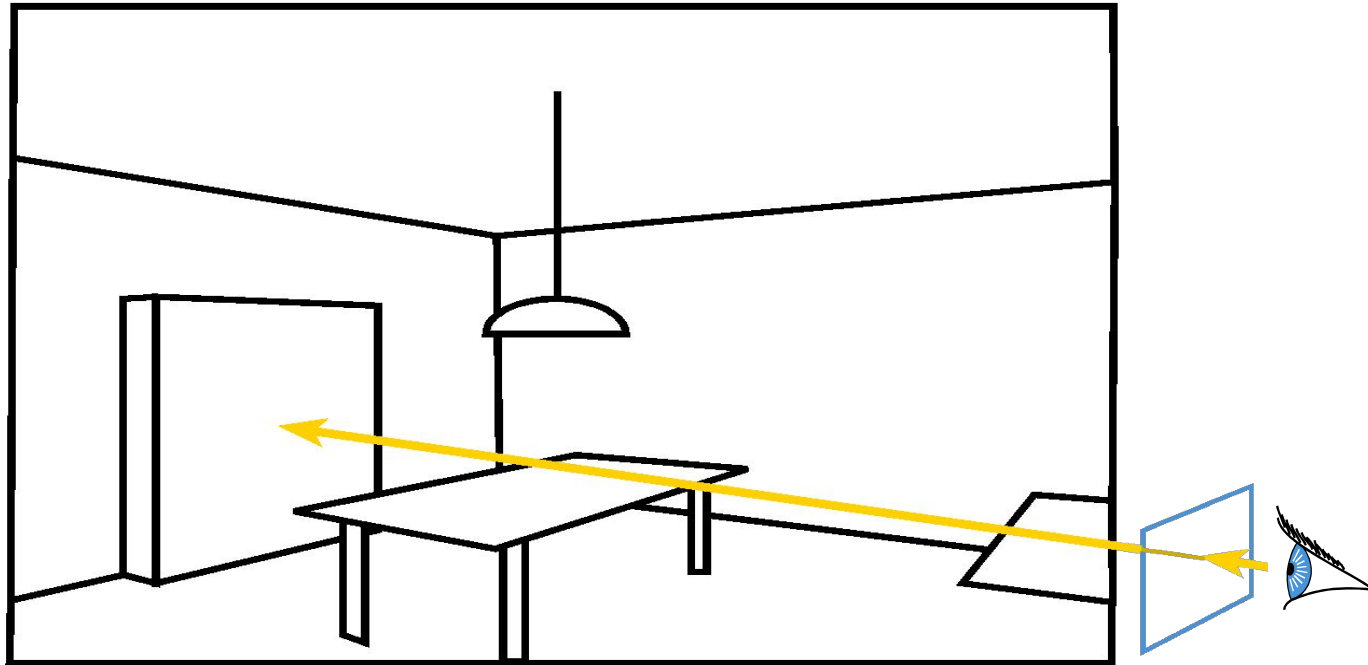
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- Worksheet: Progressive Radiosity
- The Rendering Equation
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# Ray Casting

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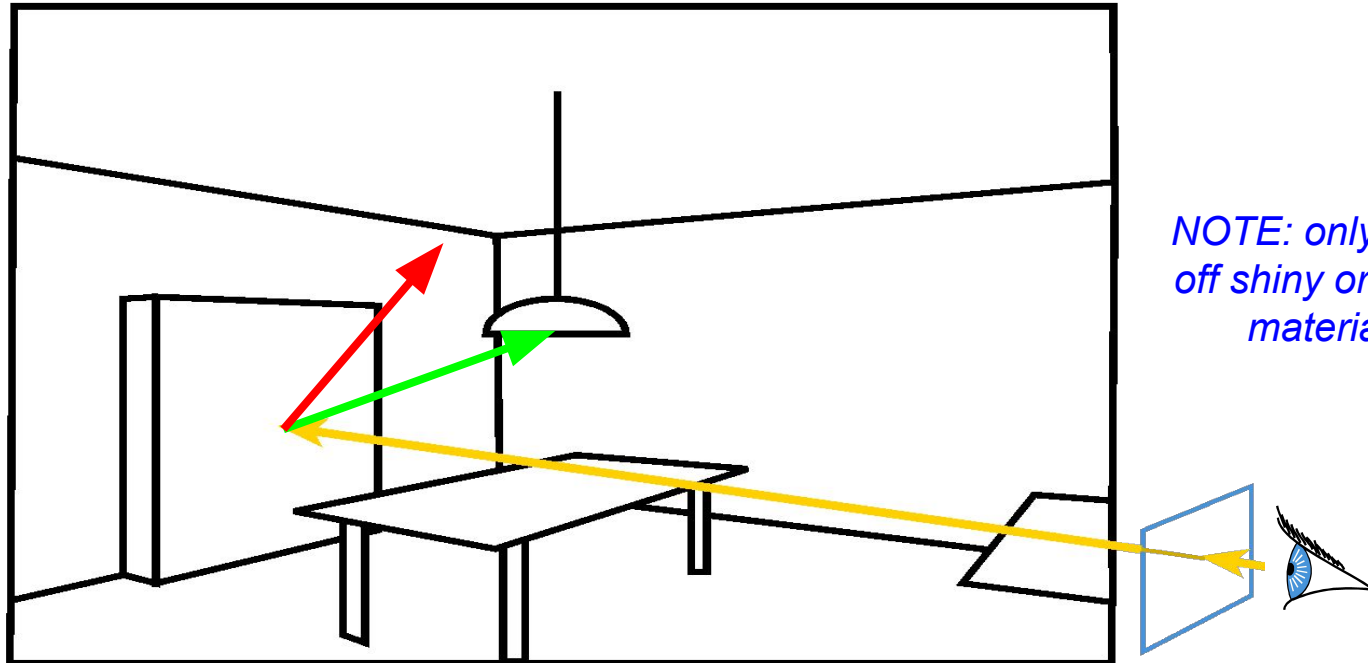
- Cast a ray from the eye through each pixel





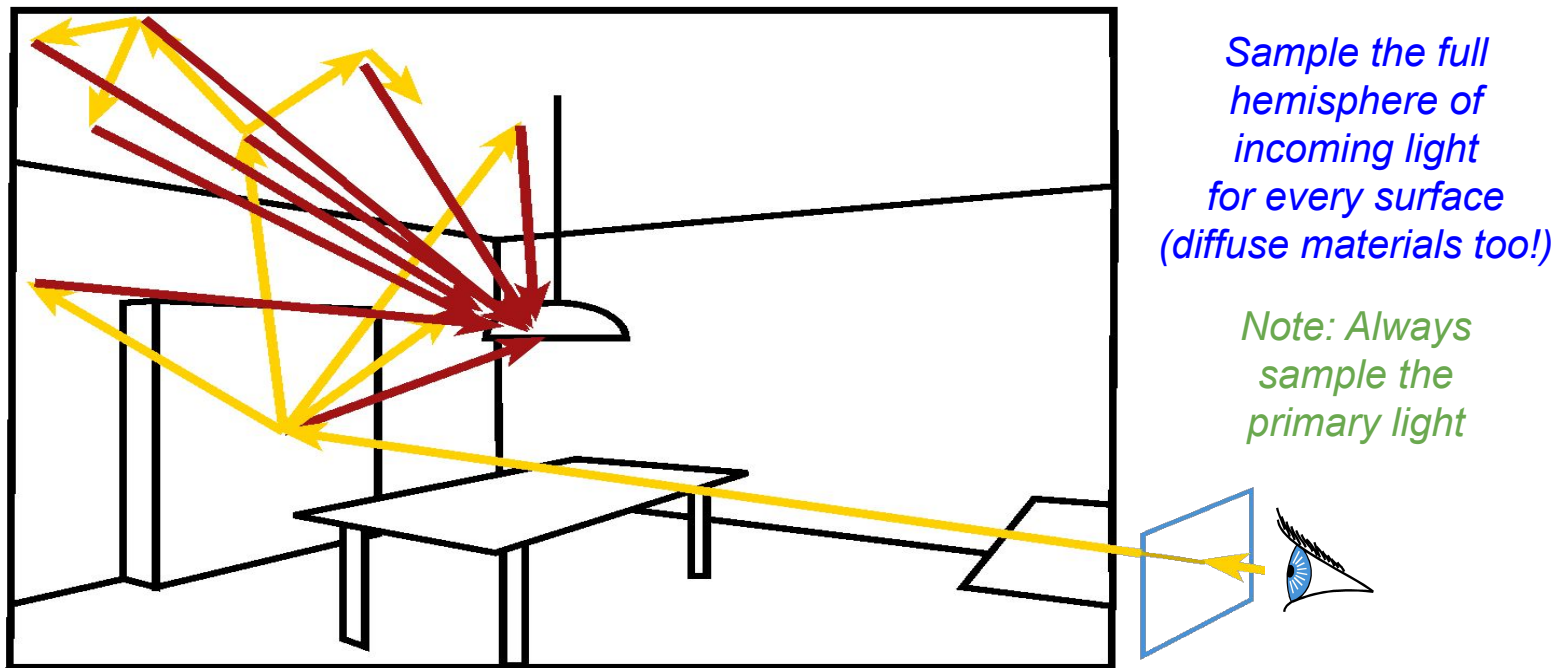
# Ray Tracing

- Cast a ray from the eye through each pixel
- Trace secondary rays (shadow, reflection, refraction)



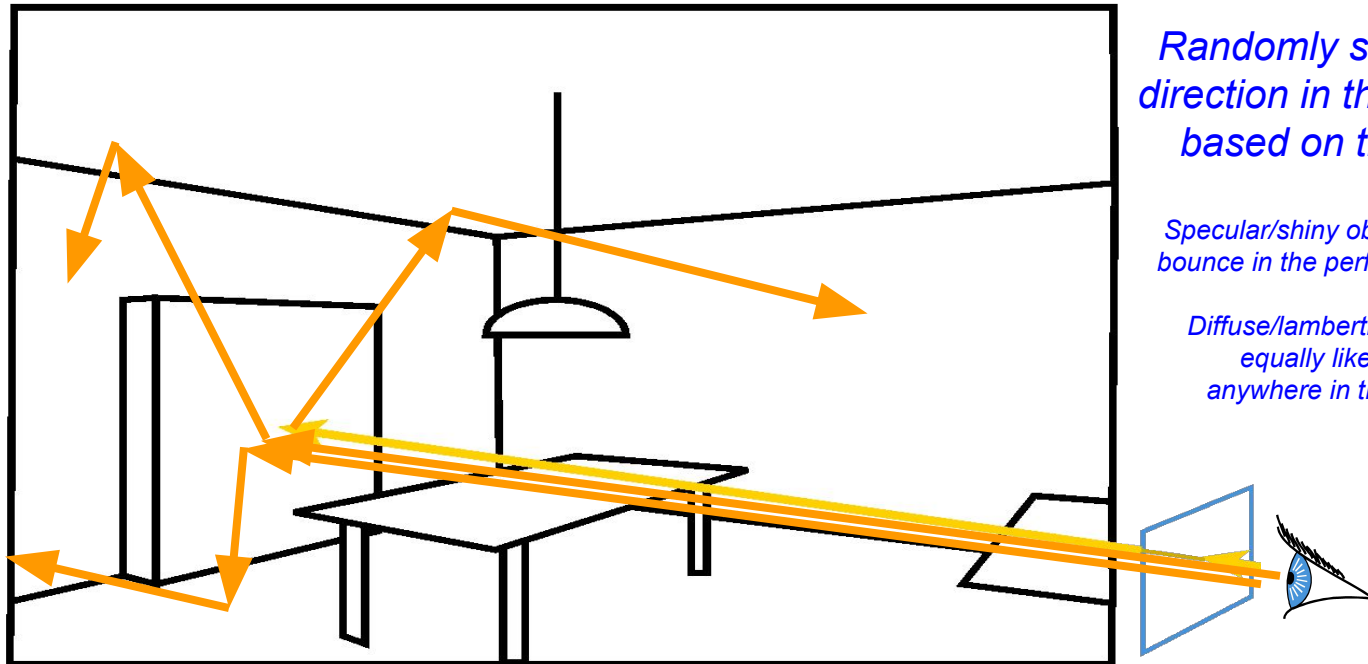
# Monte Carlo Ray Tracing

- Cast a ray from the eye through each pixel
- *Cast lots and lots of random rays to accumulate radiance contribution*
  - *Recurse to solve the full Rendering Equation*



# (Monte Carlo) Path Tracing

- Trace *only one* secondary ray per recursion
- But send many primary rays per pixel - NOTE: anti-aliasing for free!

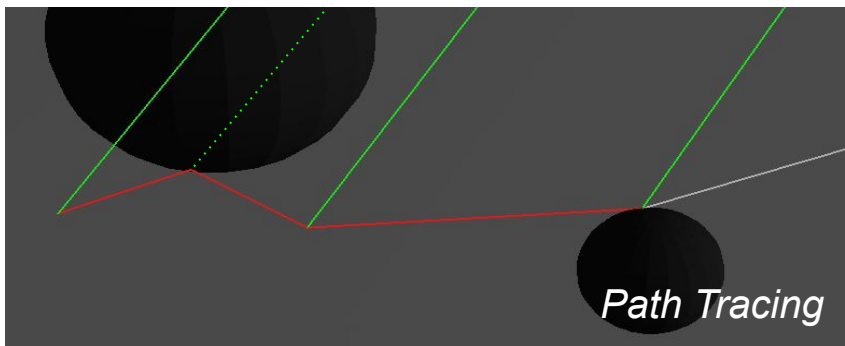
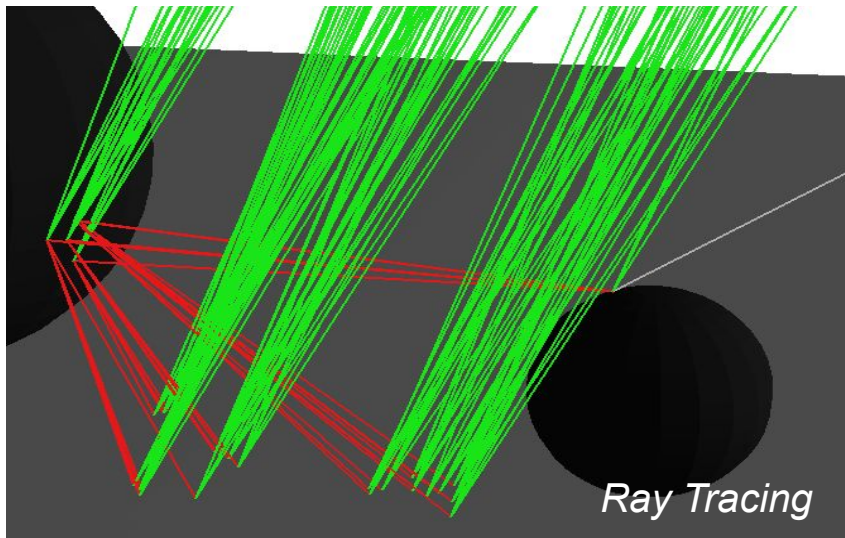


*Randomly select one ray direction in the hemisphere based on the material.*

*Specular/shiny objects most likely to bounce in the perfect mirror direction.*

*Diffuse/lambertian/matte objects equally likely to bounce anywhere in the hemisphere.*

# Ray Tracing vs. Path Tracing



2 bounces  
5 glossy samples  
5 shadow samples

How many rays cast per pixel?

1 main ray + 5 shadow rays +  
5 glossy rays + 5x5 shadow rays +  
5\*5 glossy rays + 5x5x5 shadow rays  
= 186 rays

How many 3 bounce paths can we trace per pixel  
for the same cost?

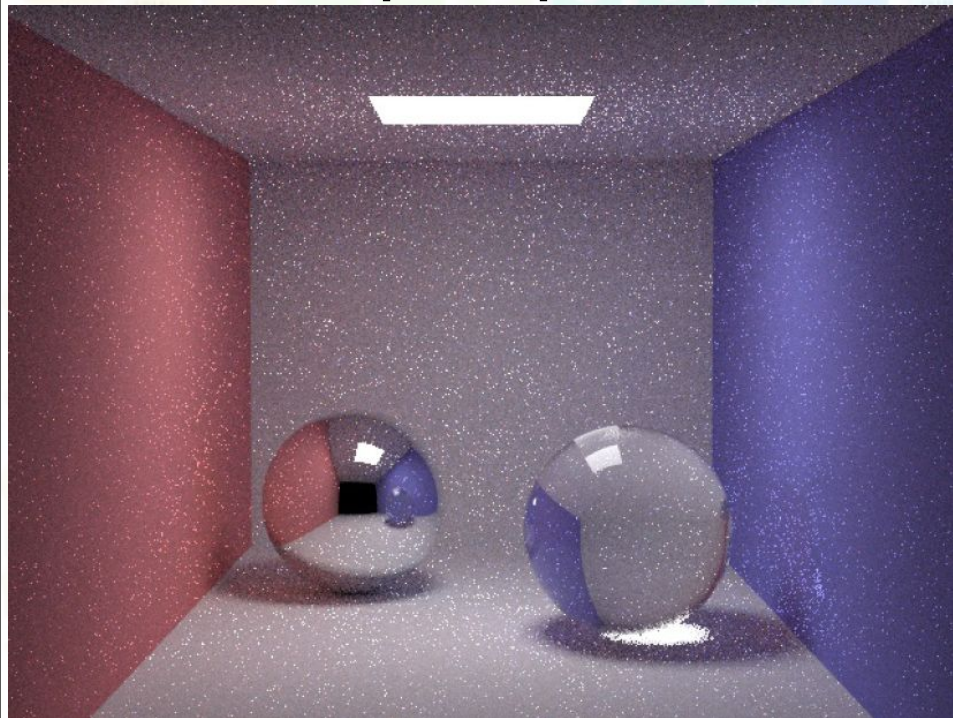
186 rays / 8 ray casts per path  
= ~23 paths

*Which will probably have less error?*

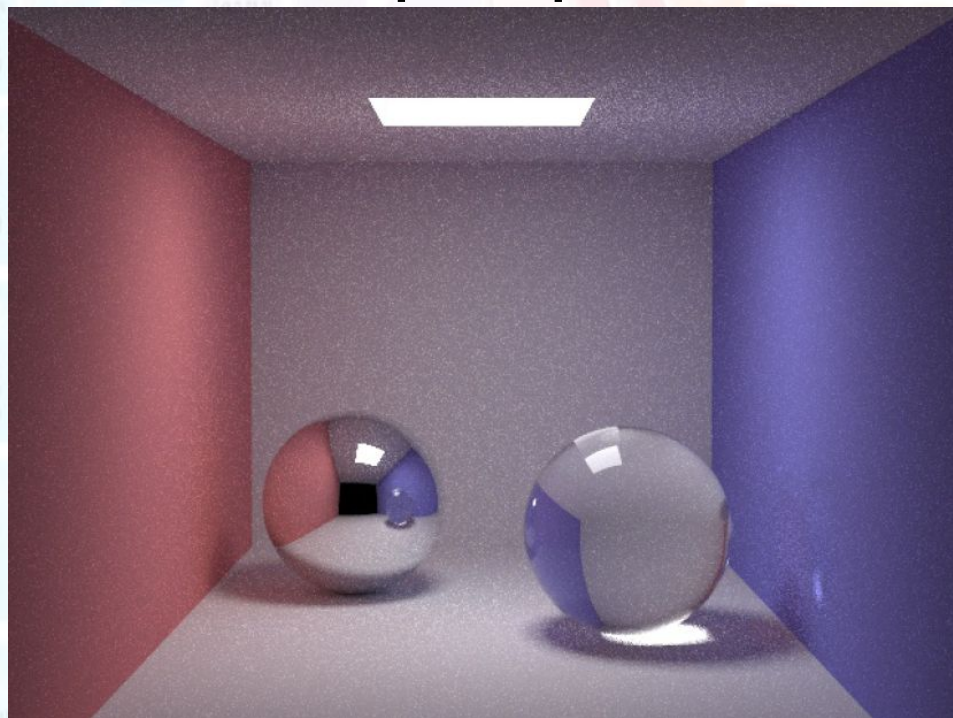
# Questions?

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**10 paths/pixel**



**100 paths/pixel**



*Images from Henrik Wann Jensen*



# Today

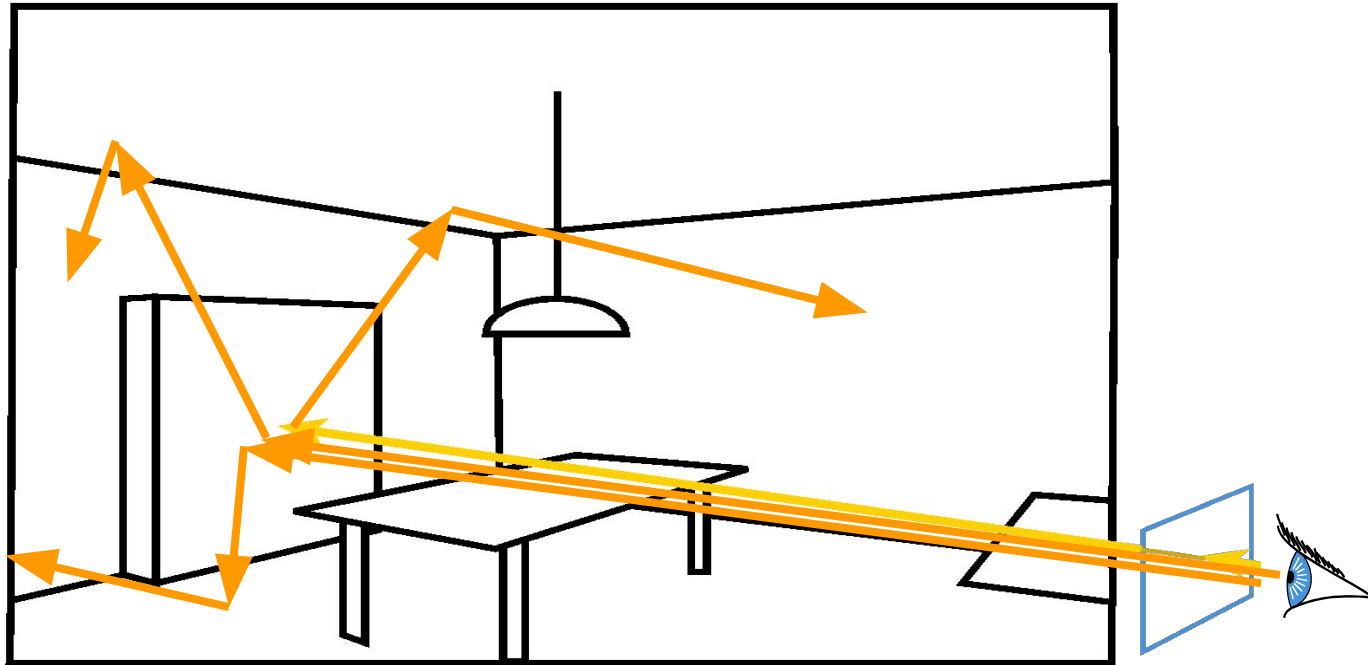
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# Problem: Path Tracing is Costly

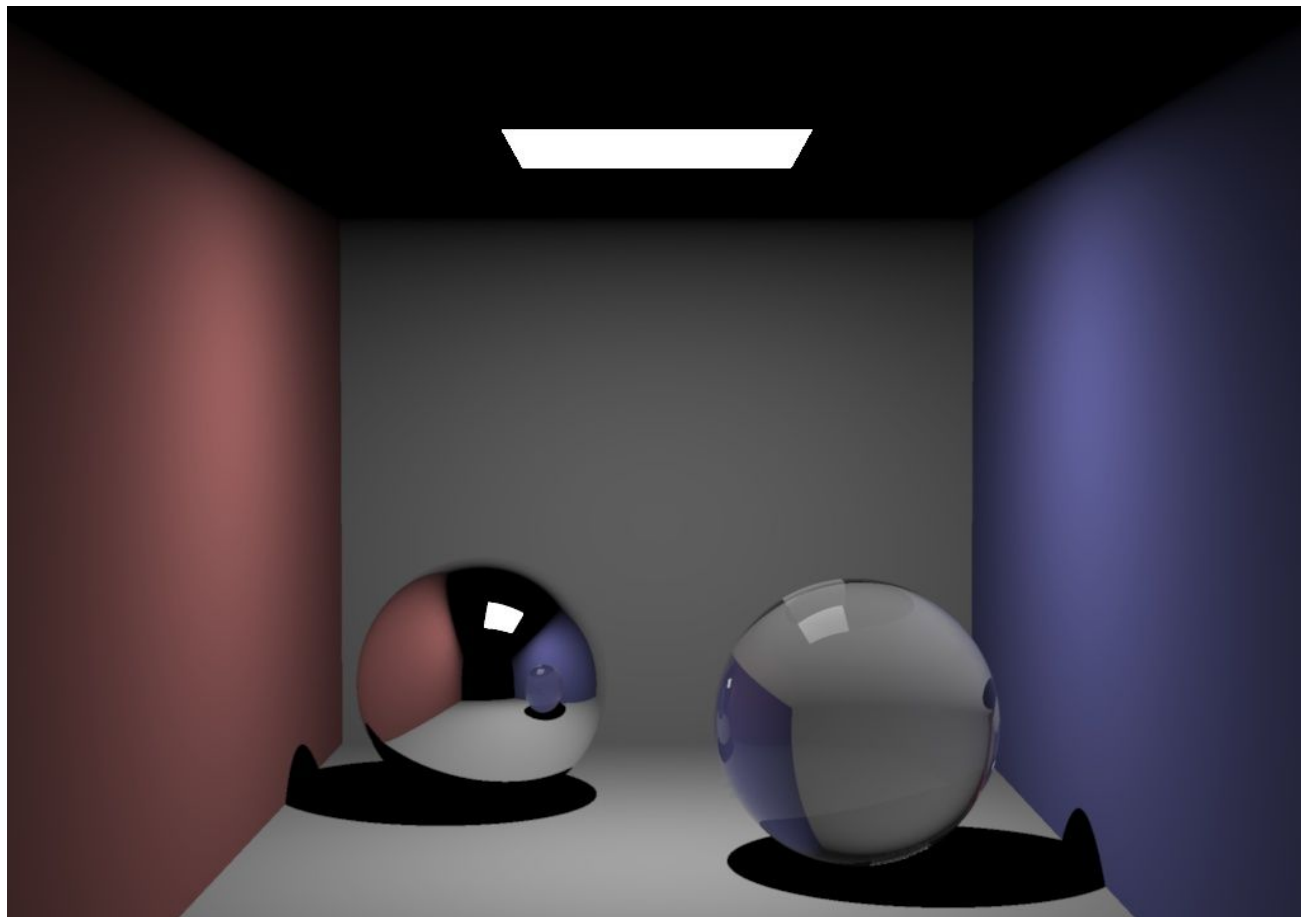
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- Requires tons of rays per pixel



# Direct Illumination

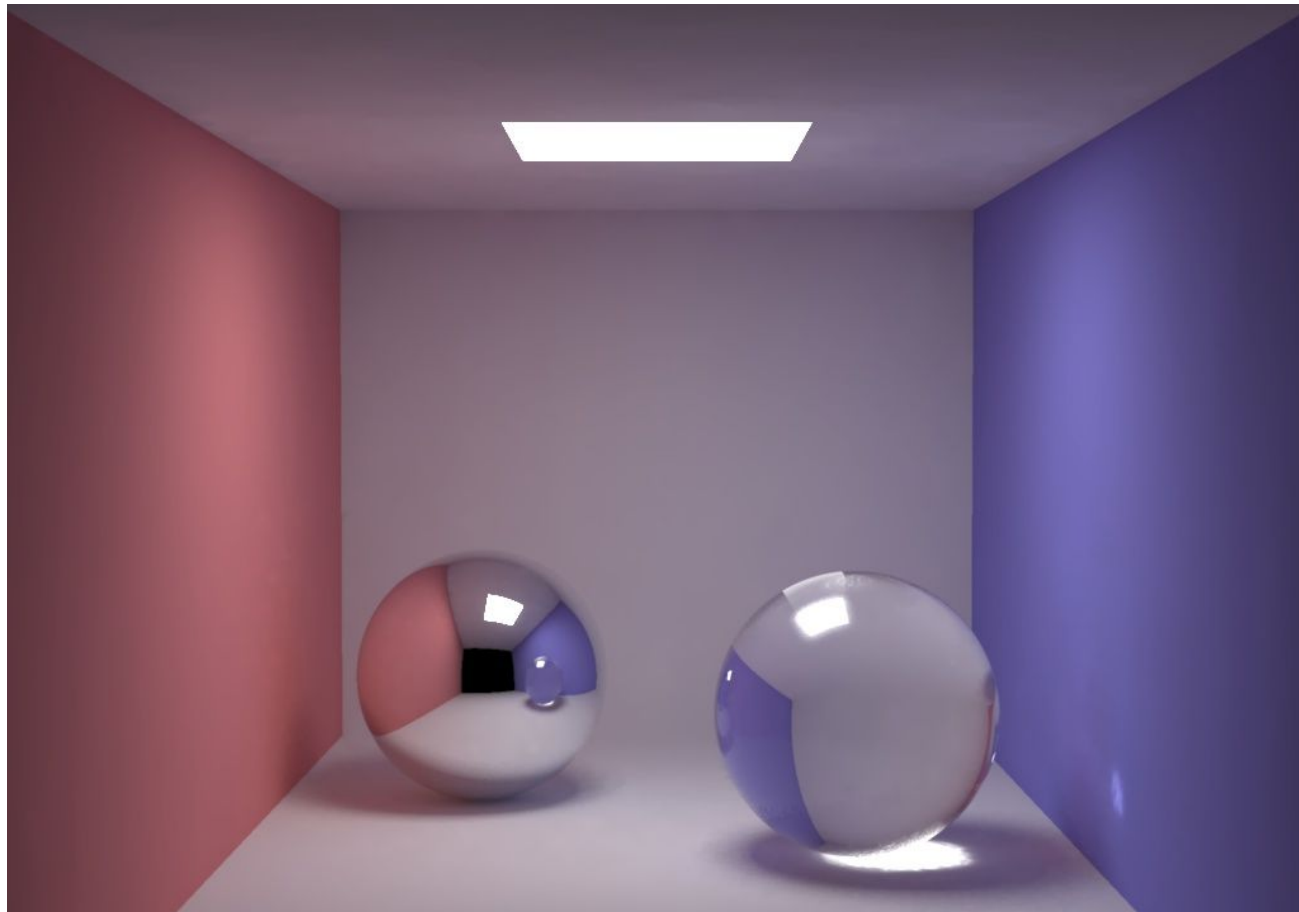
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*Henrik Wann Jensen*

# Global Illumination

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*Henrik Wann Jensen*

# Global - Direct = Indirect Illumination

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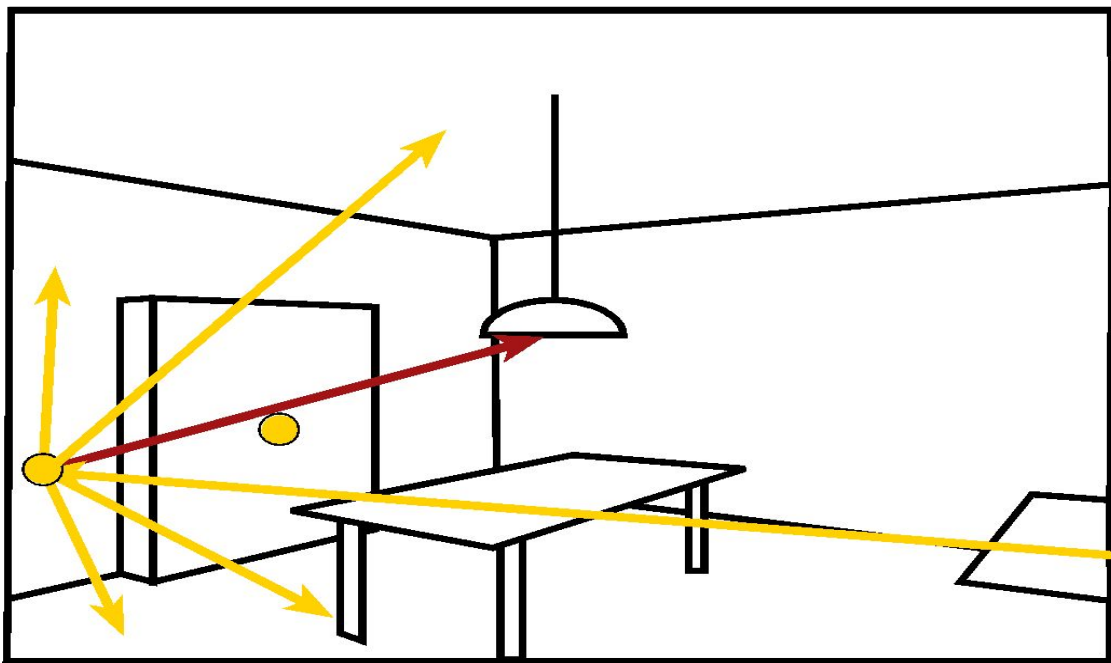


*Notice that indirect illumination is very smooth, except at discontinuities!*

*Henrik Wann Jensen*

# Irradiance Cache

- The indirect illumination is smooth
- So let's store and re-use the computed indirect illumination

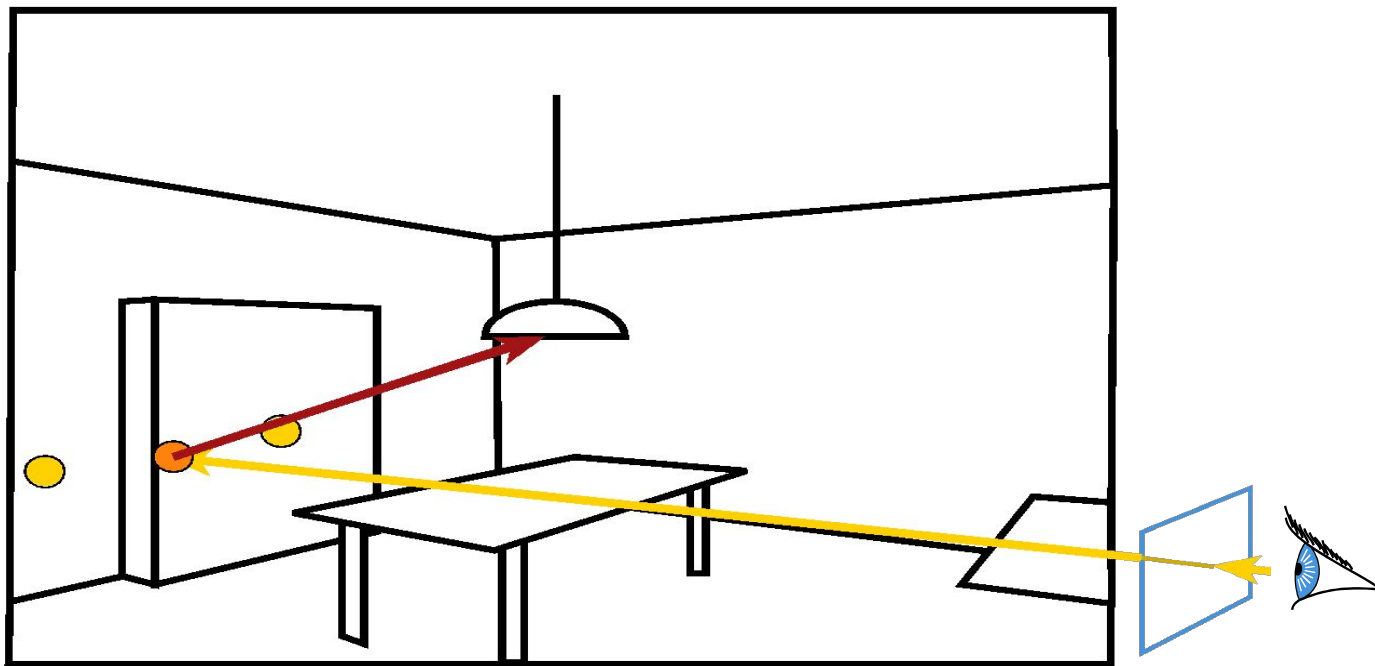


**irradiance** is the  
radiant flux received by  
a surface per unit area.

Units: ( $\text{W}/\text{m}^2$ )  
watt per square meter

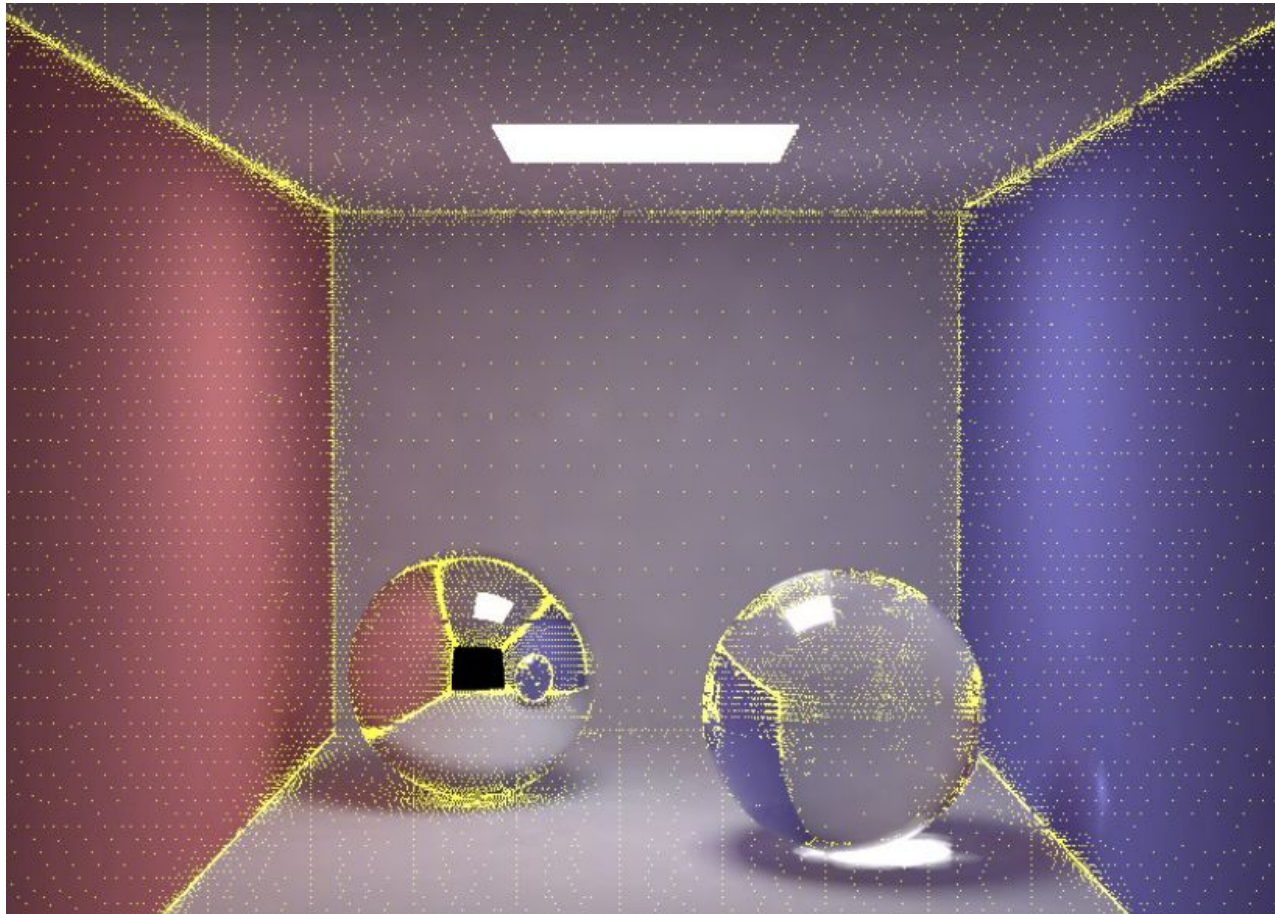
# Irradiance Cache

- Interpolate nearby cached irradiance values, *when they are available*
- Always re-calculate direct lighting (e.g., shadows from primary light)



# Irradiance Cache

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*Henrik Wann Jensen*



# Questions?

- Why do we need “good” random numbers?
- With a fixed random sequence, we see the structure in the error.

*Henrik Wann Jensen*



# Today

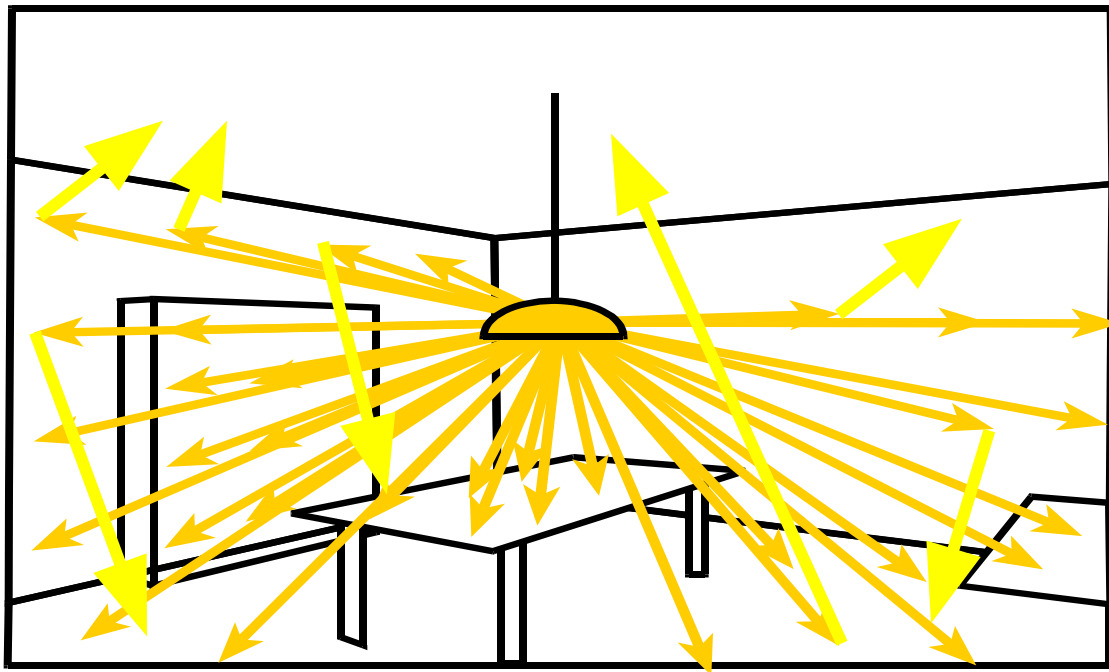
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# Photon Mapping

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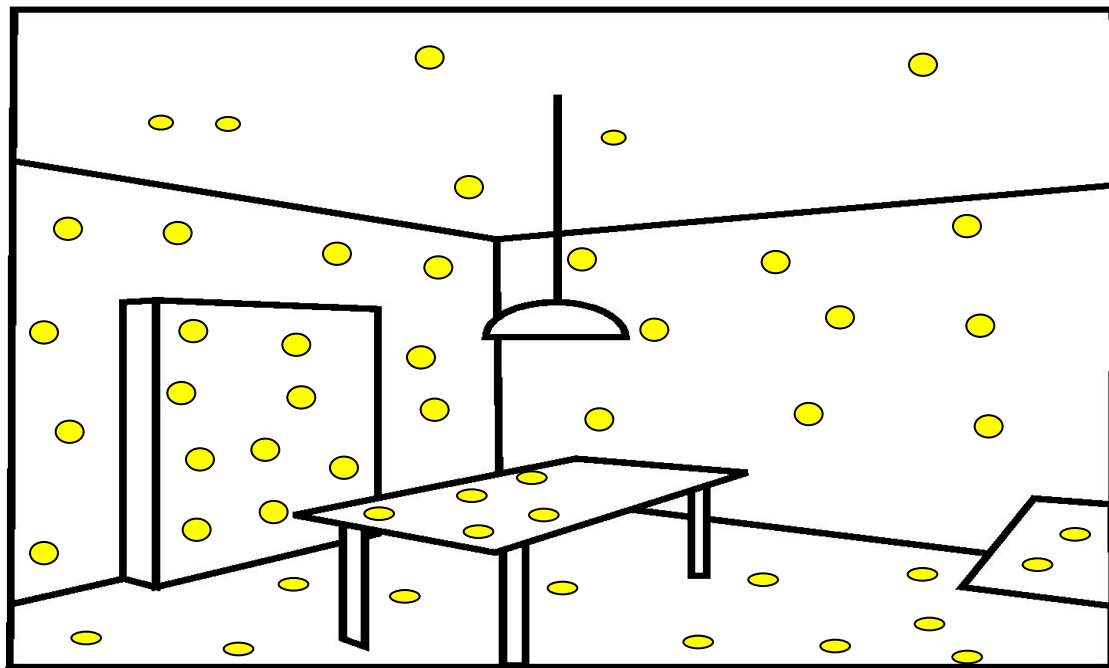
- Preprocess: Cast rays from light sources
  - *NOTE: This is independent of camera/eye viewpoint*



# Photon Mapping

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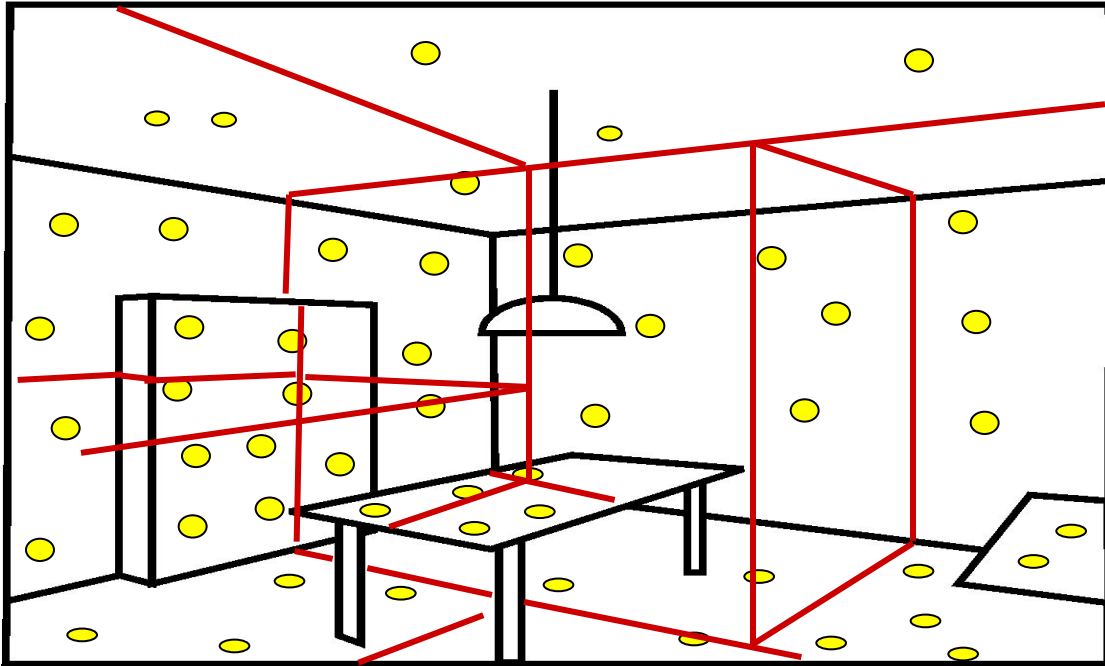
- Preprocess: Cast rays from light sources
- Store photons: position + light power + incoming direction



# Using a kD-tree to storing the Photon Map

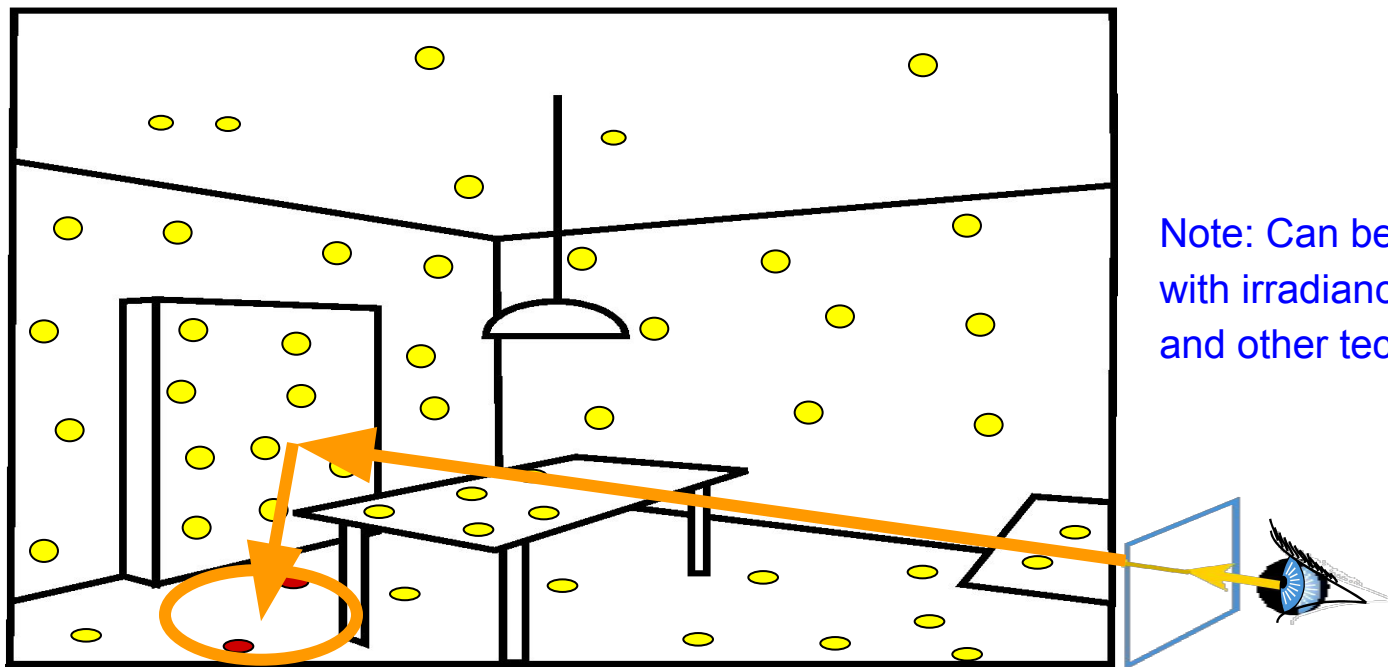
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- Efficiently store photons for fast access
- Use hierarchical spatial structure (e.g., in a kd-tree)



# Rendering with Photon Map

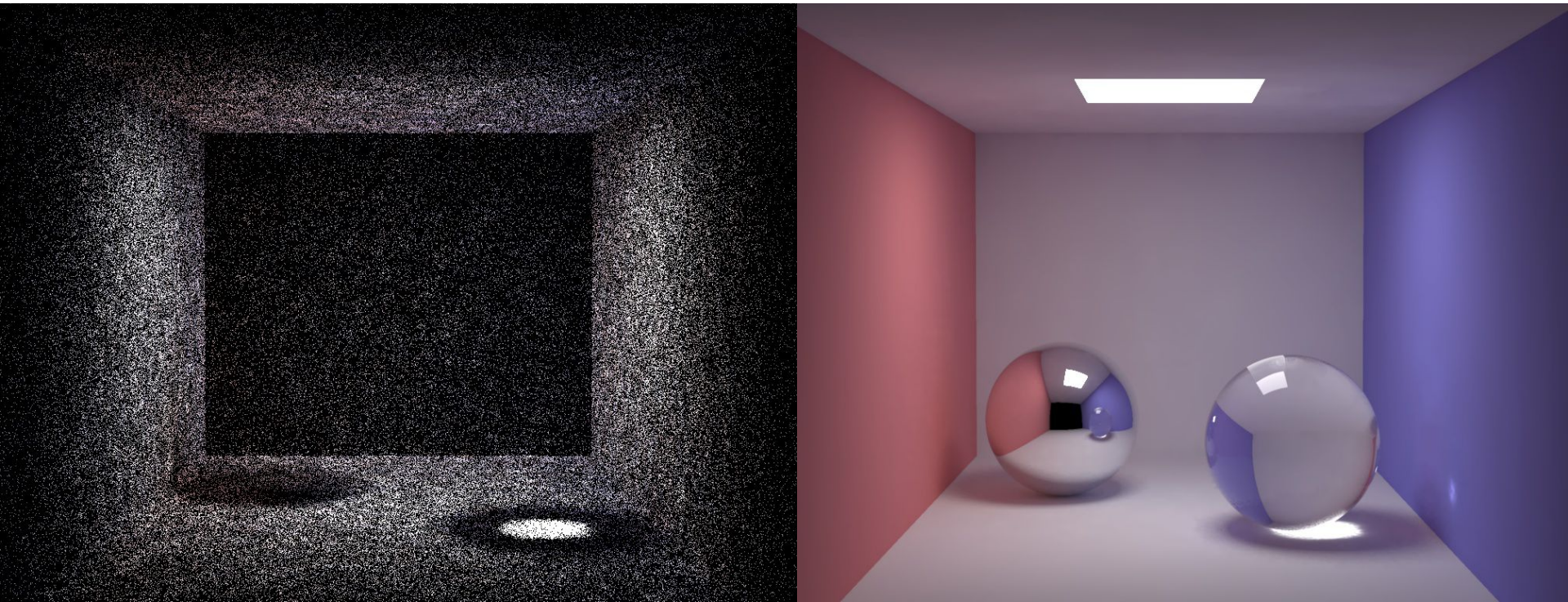
- We still trace primary rays and bounce once based on the material
- For secondary rays: Estimate irradiance using  $k$  closest photons





# Photon Map Results

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*Henrik Wann Jensen*



# Today

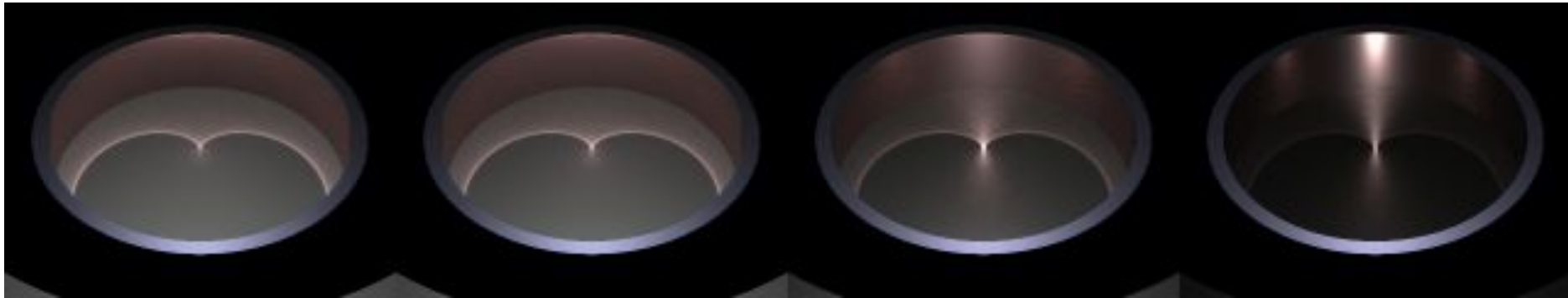
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# Reading for Today

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- “Rendering Caustics on Non-Lambertian Surfaces”,  
Henrik Wann Jensen, *Graphics Interface* 1996.

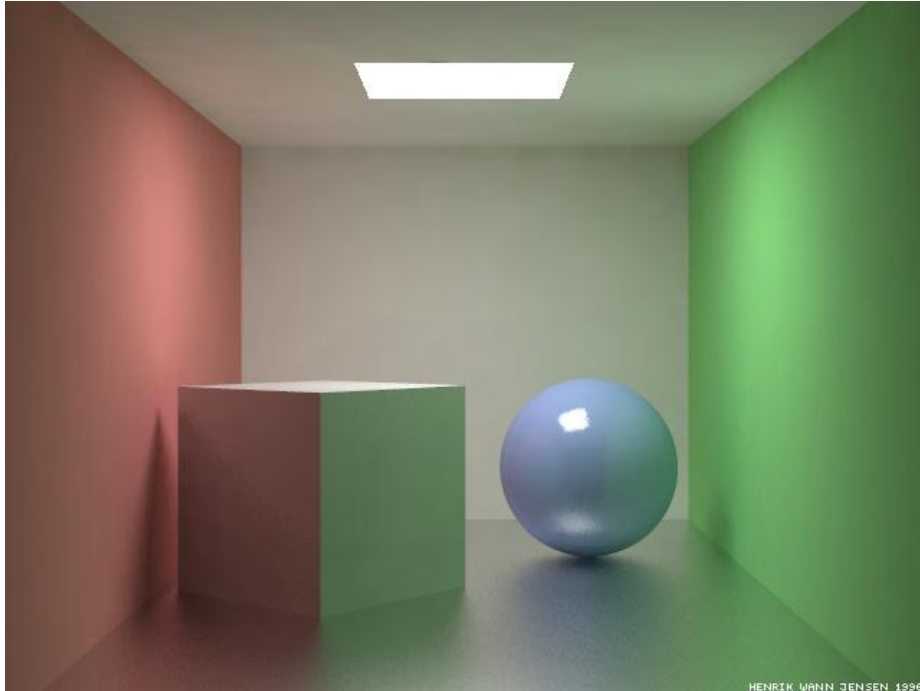


- Balancing the kd-tree makes a big difference
- Careful memory use, details about design for bit usage & total size
- Russian Roulette - random chance based on intensity to continue
  - Is it an appropriate & intuitive name?
- Various 'magic' constants, which (they say) are not difficult to set.
- Expecting paper about physics.. got paper about data structures!
- Well written, intuitive, easy to read, clear references to prior work
- Acknowledges limitations with the methods
- One column v. two column: specified by the publisher  
(journal/conference), e.g. this was in a book with pages < 8.5"x11"

# Reading for Today

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- “Global Illumination using Photon Maps”,  
Henrik Wann Jensen, *Rendering Techniques* 1996.



- Offloaded explanation of existing techniques to other papers (necessary to read other papers to understand this paper)
- Full details of hardware used & running times, direct comparison to other techniques/implementations
- Tradeoff between # of photons, memory, rendering time, & final quality
- Separate photon maps (with different resolutions) for different (matte v. specular) materials
- Are we finished? Have we correctly modeled/rendered all aspects of the physics of light – just limited by computational power?
- Take shortcuts (skip caustic photon map), when appropriate, for the scene & materials
- Important of random numbers. What random number generator is used? Are there artifacts if a bad number generator is used?

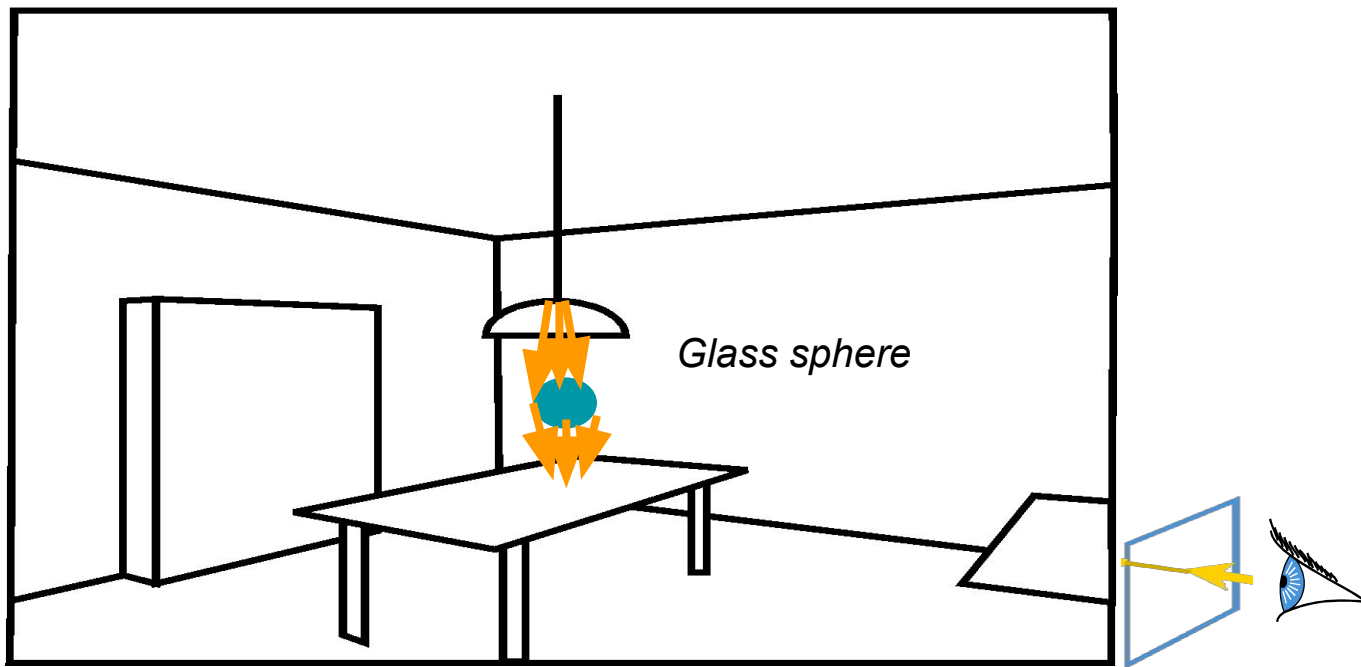


<https://www.housepaintingtutorials.com/painting-tone-on-tone-wall-stri>

# Photon Mapping - Caustics

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- Special photon map for specular reflection and refraction



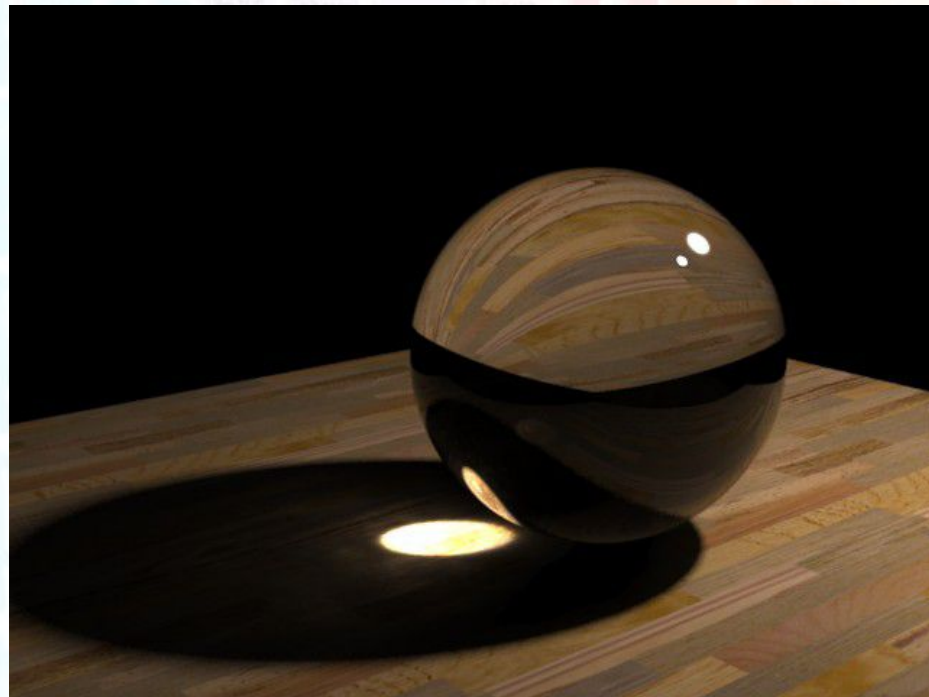


# Path Tracing & Photon Mapping Comparison

Path Tracing w/ 1000 paths/pixel



Photon mapping

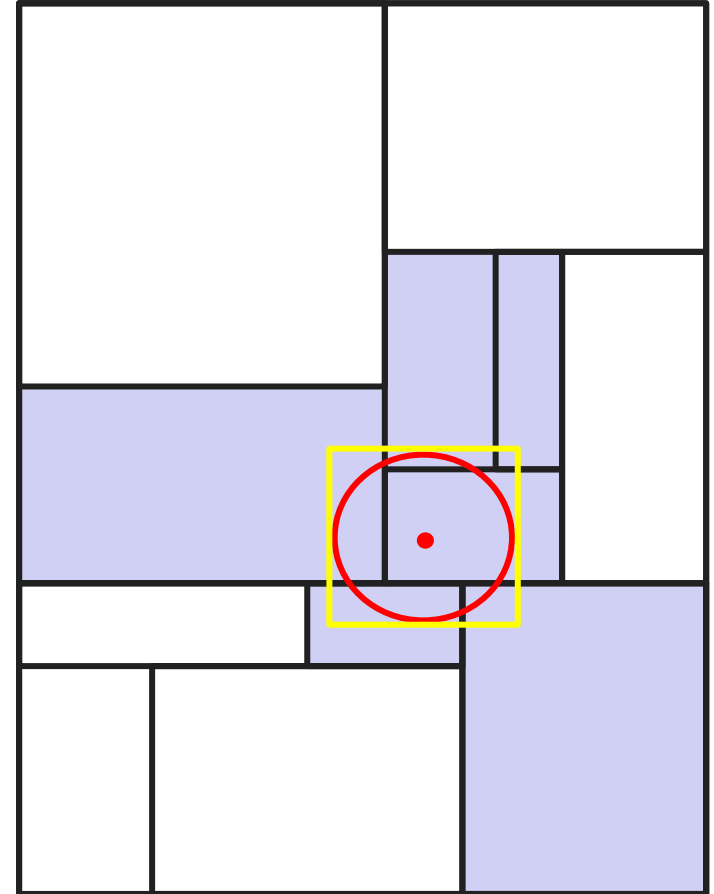


*(similar rendering time)*

Henrik Wann Jensen

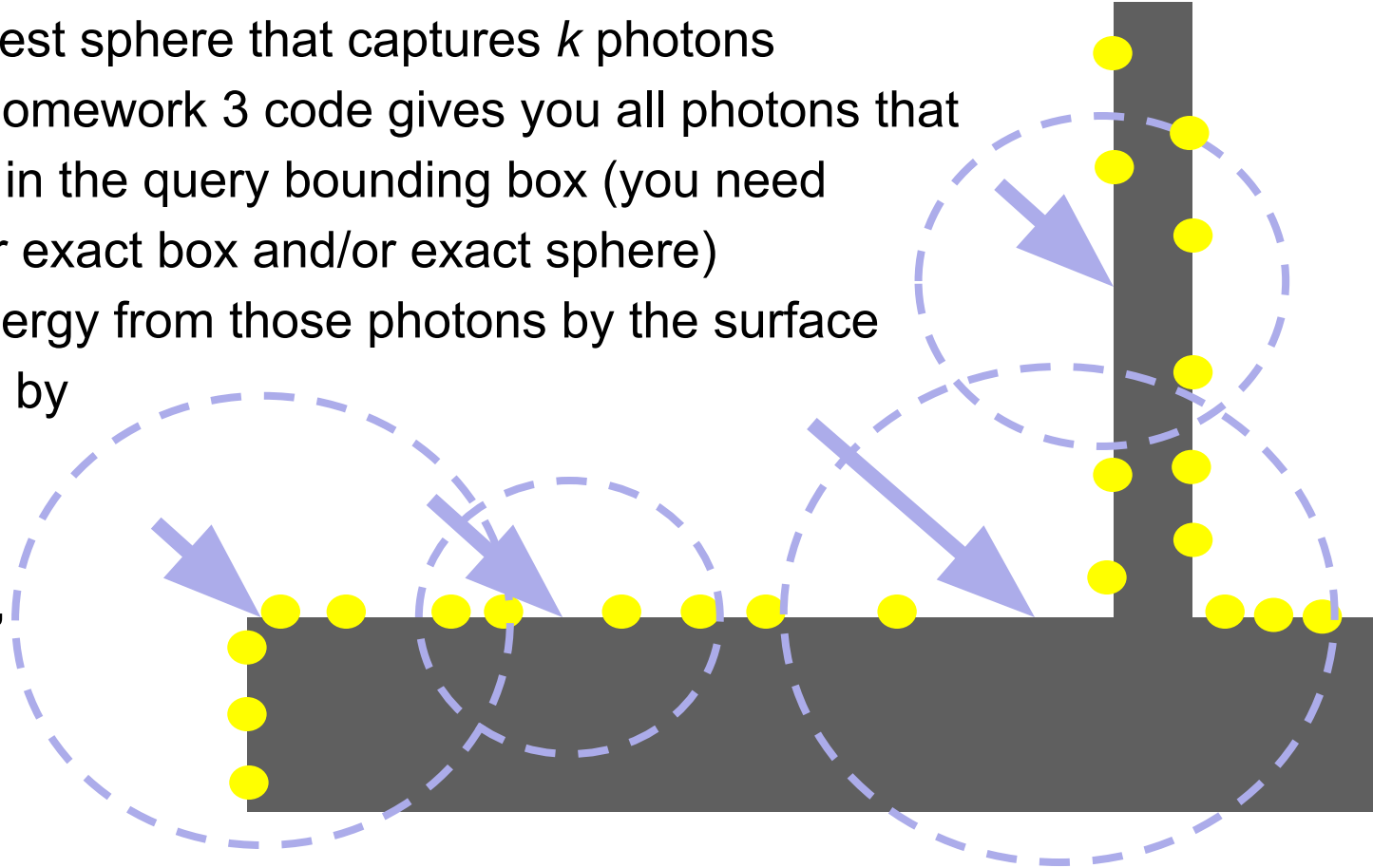
# Homework 3: Photons in the $k$ -D Tree

- You start with query point & radius (red)
- You input to the function `KDTree::CollectPhotonsInBox` a bounding box (yellow)
- The algorithm finds all  $k$ -d tree cells that overlap with bounding box (blue)
- The function returns all photons in those cells
- *You need to discard all photons not in your original query radius*



# Closest Photon Details

- Find the tightest sphere that captures  $k$  photons
  - NOTE: Homework 3 code gives you all photons that *might* be in the query bounding box (you need to test for exact box and/or exact sphere)
- Divide the energy from those photons by the surface area covered by that sphere
- What about thin surfaces, concave or convex corners?



# Today

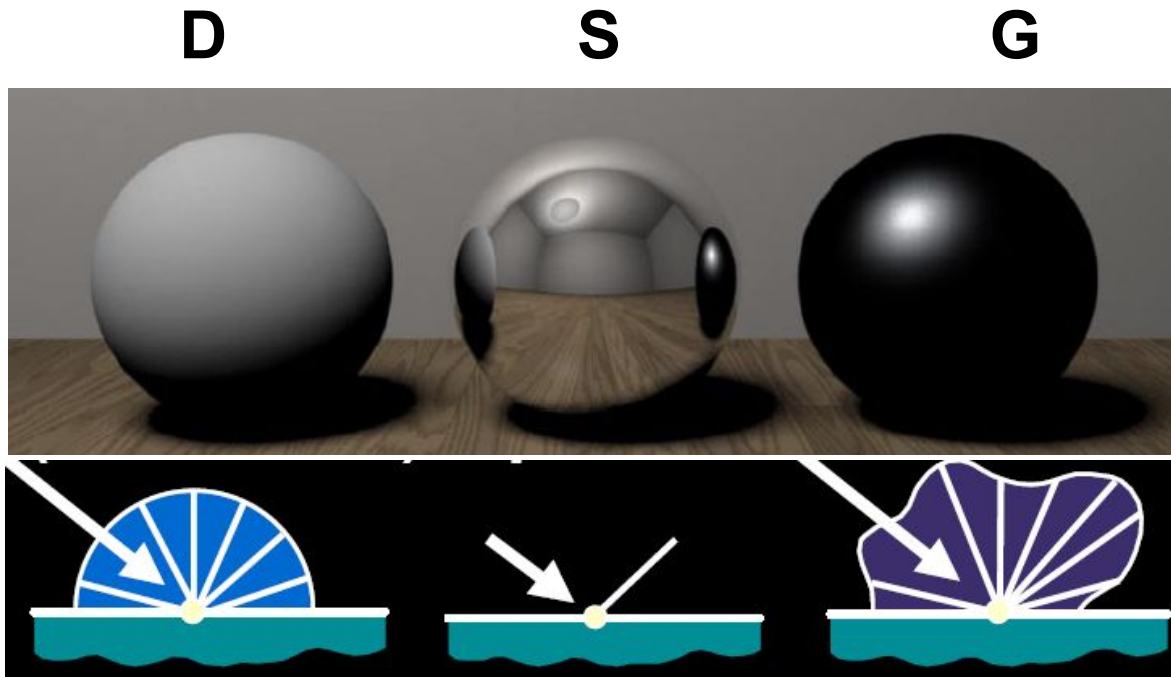
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# Ray Grammar

Classify local interaction:

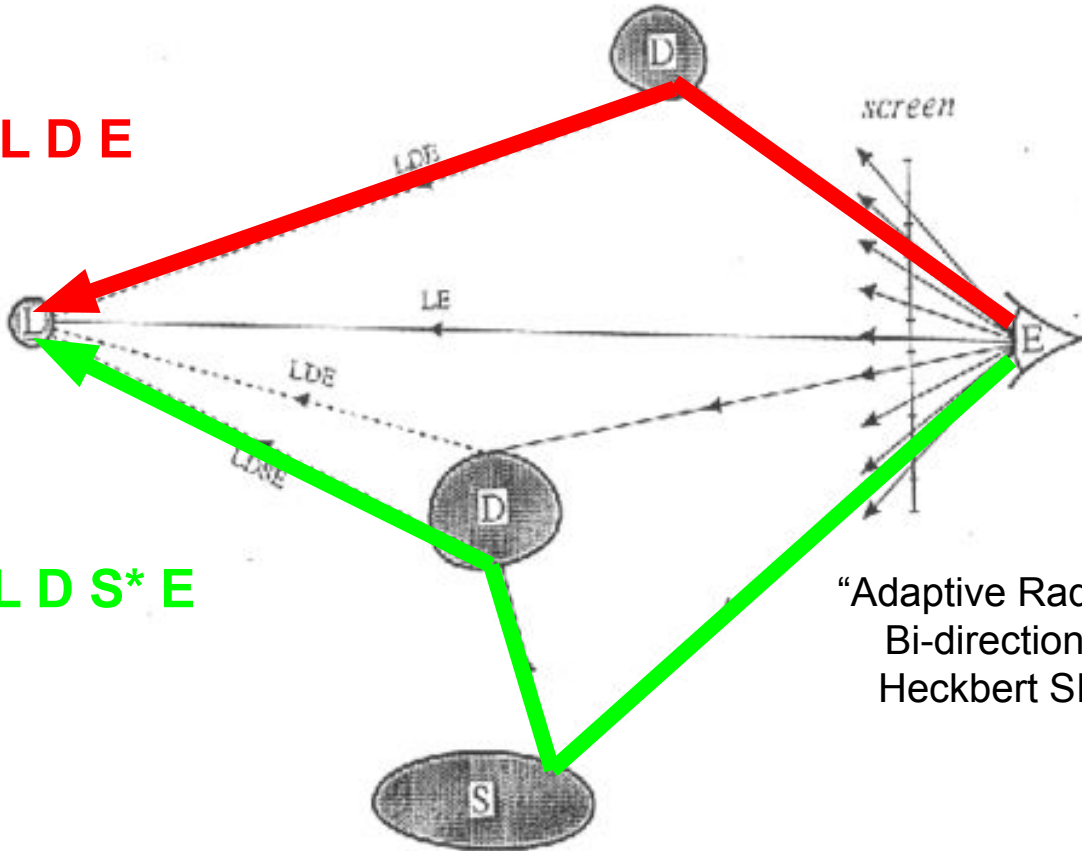
- E = eye
- L = light
- S = perfect specular (reflection or refraction)
- G = glossy scattering
- D = diffuse scattering



*From Dutre et al.'s slides*

# Classic Ray Casting/Tracing

Ray casting: L D E



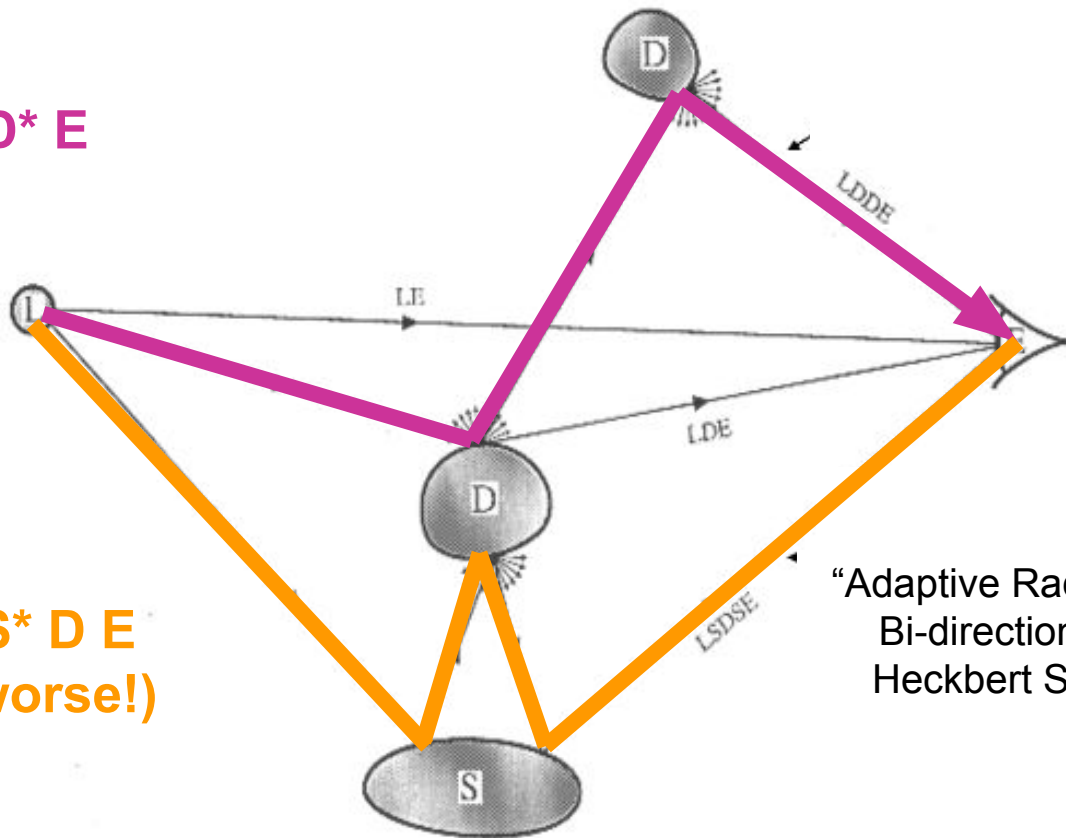
Ray tracing: L D S\* E

“Adaptive Radiosity Textures for  
Bi-directional Ray Tracing”  
Heckbert SIGGRAPH 1990

# Photon Tracing

Radiosity:  $L D^* E$

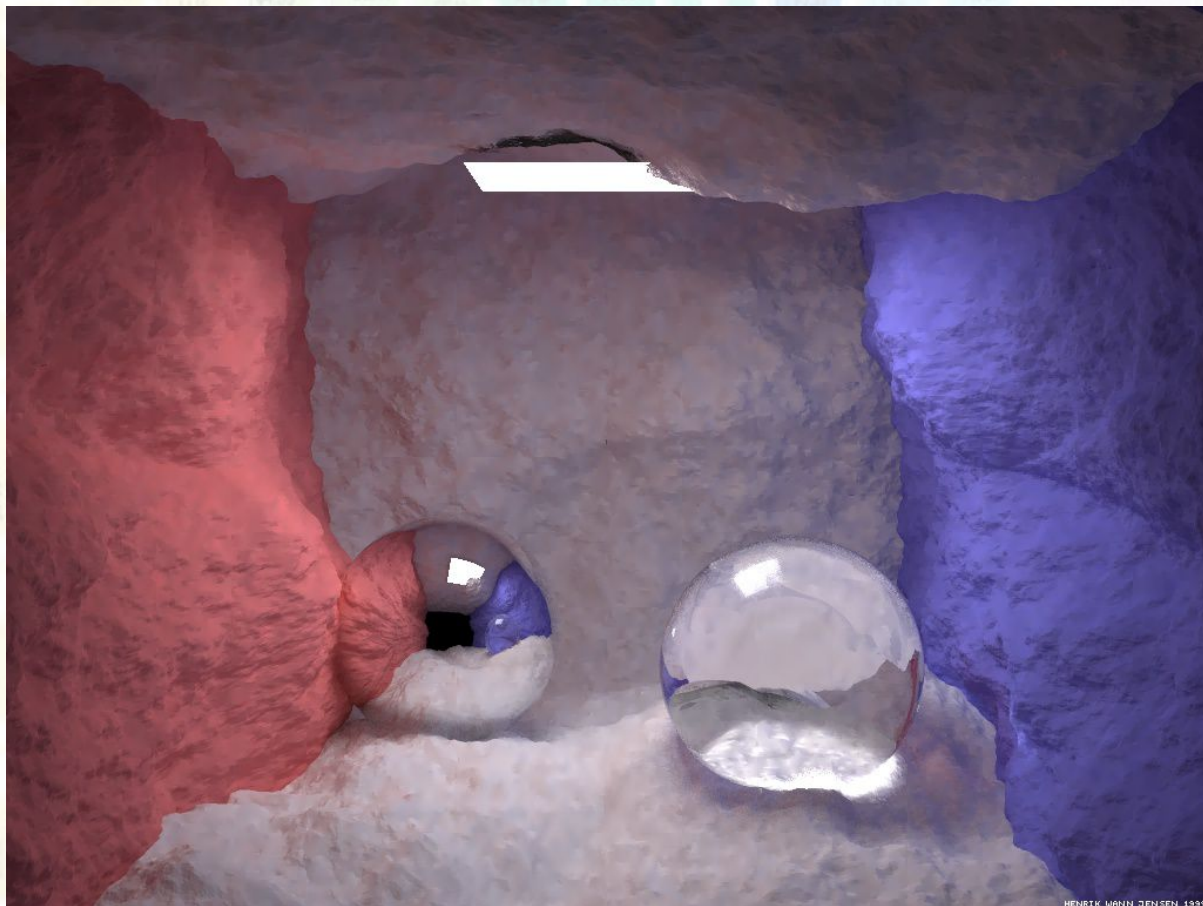
Caustics:  $L S^* D E$   
(or worse!)



“Adaptive Radiosity Textures for  
Bi-directional Ray Tracing”  
Heckbert SIGGRAPH 1990



# Questions?



Henrik Wann Jensen

# Today

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- Worksheet: Progressive Radiosity
- The Rendering Equation
- Ray Casting vs. Ray Tracing vs. Monte-Carlo Ray Tracing vs. Path Tracing
- Irradiance Caching
- Photon Mapping
- Papers for Today
- Ray Grammar
- Papers for Next Time

# Readings for Tuesday After Break: (*pick one*)

- “Correlated Multi-Jittered Sampling”,  
Andrew Kensler, Pixar Technical Memo, 2013

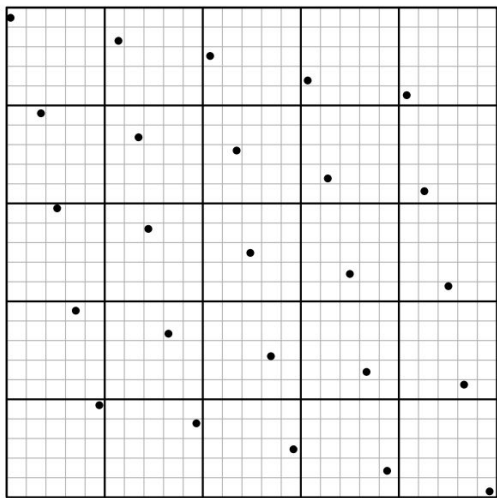


Figure 1: The canonical arrangement. Heavy lines show the boundaries of the 2D jitter cells. Light lines show the horizontal and vertical substrata of N-rooks sampling. Samples are jittered within the subcells.

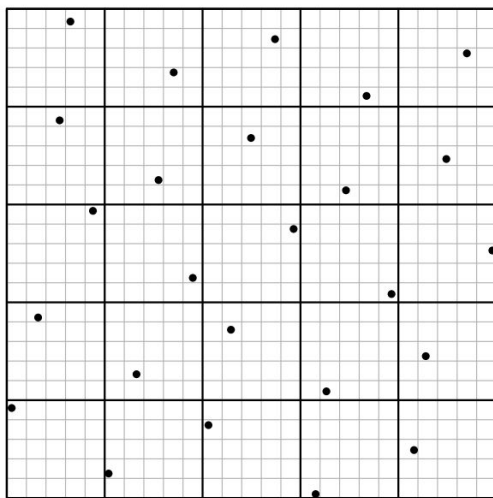


Figure 3: With correlated shuffling.

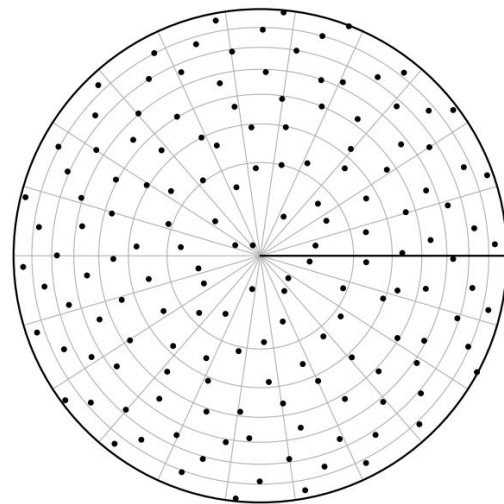


Figure 9: Polar warp with  $m = 22$ ,  $n = 7$ .

<sup>9</sup>G. J. Ward and P. S. Heckbert. Irradiance gradients. In *Third Eurographics Rendering Workshop*, pages 85–98, May 1992.

# Readings for Tuesday After Break: *(pick one)*

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- “Implicit Visibility and Antiradiance for Interactive Global Illumination”, Dachsbacher, Stamminger, Drettakis, and Durand Siggraph 2007





# Readings for Tuesday After Break: *(pick one)*

- “Recognition of Surface Reflectance Properties from a Single Image under Unknown Real-World Illumination”, Dror, Adelson, & Willsky, 2001.

Figure 1. The task addressed by our classifier. Using images of several surface materials under various illuminations as a training set, we wish to classify novel objects under novel illumination according to their surface material.

