CSCI 4530/6530 Advanced Computer Graphics

`https://www.cs.rpi.edu/~cutler/classes/advancedgraphics/S25/`

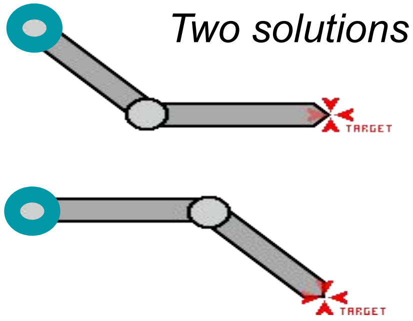
Lecture 11: Ray Tracing

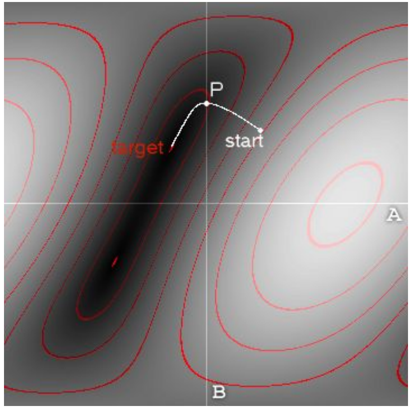
https://i.imgur.com/i7Aohc0.jpg

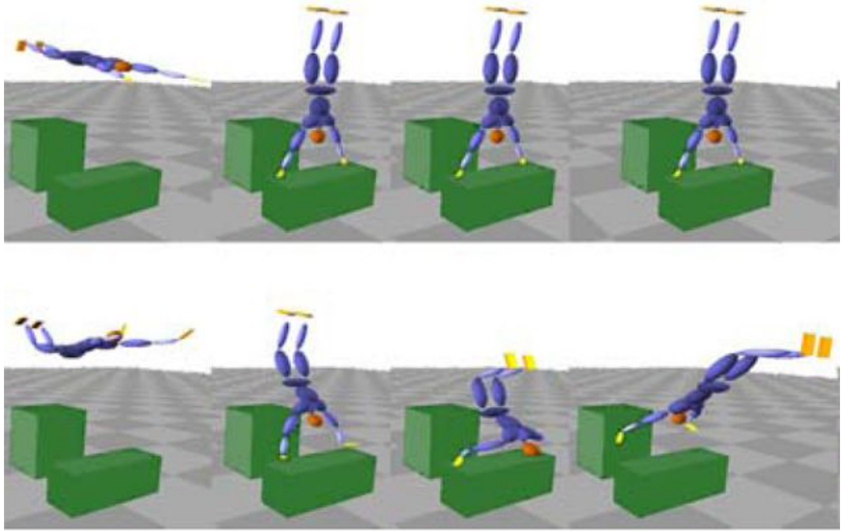
Fiat Lux, Debevec, 1999

Last Time?

- Keyframing
- Procedural Animation
- Physically-Based Animation
- Forward and Inverse Kinematics
- Motion Capture







Announcement: Quiz 1

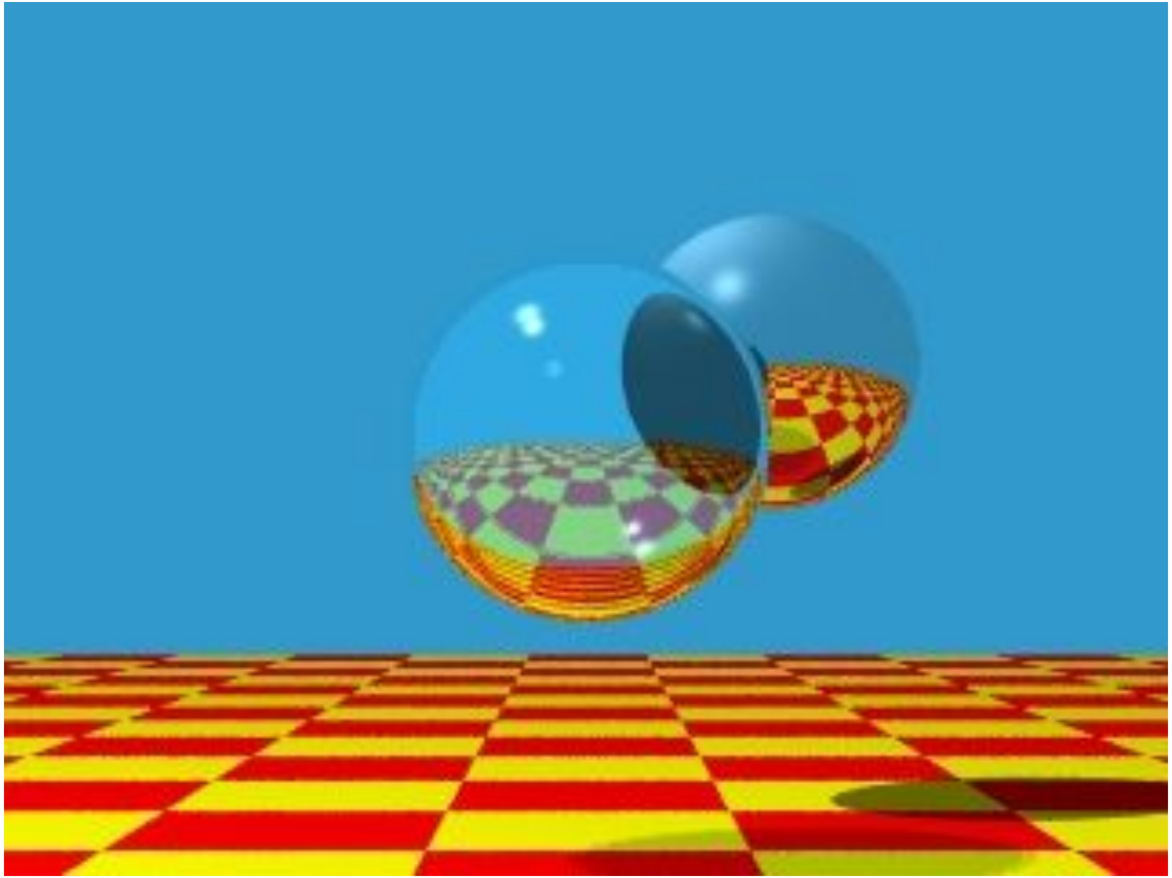
- Friday February 14th, during class (2:00-3:50pm)
  - Students w/ extra time accommodations may stay late as needed (please email Barb to request accommodations)
- One double-sided 8.5”x11” sheet of notes allowed
- Practice Problems (from 2014 & 2017) on the course calendar
- Coverage:
  - Lecture and assigned readings thru Lecture 10
  - When there was a choice of papers: you are responsible for having read one paper per lecture
  - Worksheets thru Lecture 10
  - Homeworks 0, 1, & 2

Today

- Reading for Today
- Ray Casting
- Ray Tracing
- Recursive Ray Tracing
- Distributed Ray Tracing
- Readings for Next Week

Reading for Tuesday

- "An improved illumination model for shaded display" Turner Whitted, Communications of the ACM, 1980.



- Title does not do justice to paper and its impact on field of graphics
- It’s just basic physics… but it’s backwards!
- Use of recursion makes real-time use impractical without significant optimizations/shortcuts/future hardware improvements
- Cannot cull faces behind the camera or on backside of objects (because they might be visible to various recursive rays)
- True diffuse reflection is overwhelmingly expensive
- 75-95% of cost is computing intersections
- Phong/Blinn-Phong reflection model still used today
- Diagrams were well done and very useful
- Detailed breakdown of running time by steps of algorithm

How to read a research paper?

-

How to read a research paper?

(especially an advanced paper in a new area)

- Multiple readings are often necessary
- Don't necessarily read from front to back
- Lookup important terms
- Target application & claimed contributions
- Experimental procedure
- How well results & examples support the claims
- Scalability of the technique (Big O Notation)
- Limitations of technique, places for future research
- Possibilities for hybrid systems with other work

Components of a well-written research paper?

-

Components of a well-written research paper?

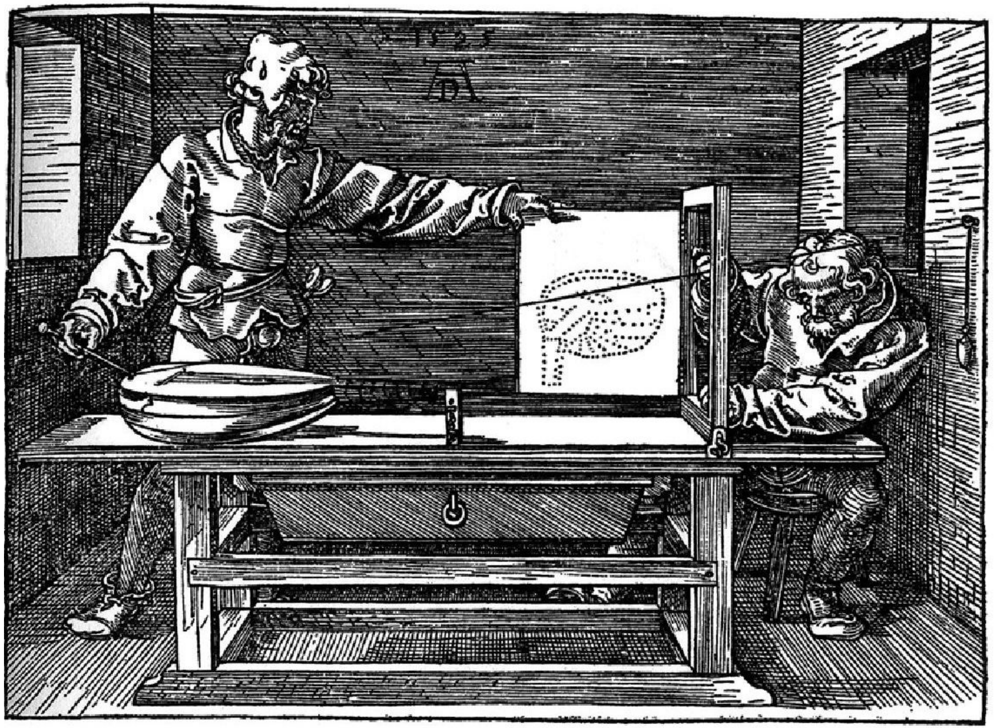
- Motivation/context/related work
- Contributions of this work
- Clear description of algorithm
  - Sufficiently-detailed to allow work to be reproduced
  - Work is theoretically sound (hacks/arbitrary constants discouraged)
- Results
  - well chosen examples
  - clear tables/illustrations/visualizations
- Conclusions
  - limitations of the method are clearly stated

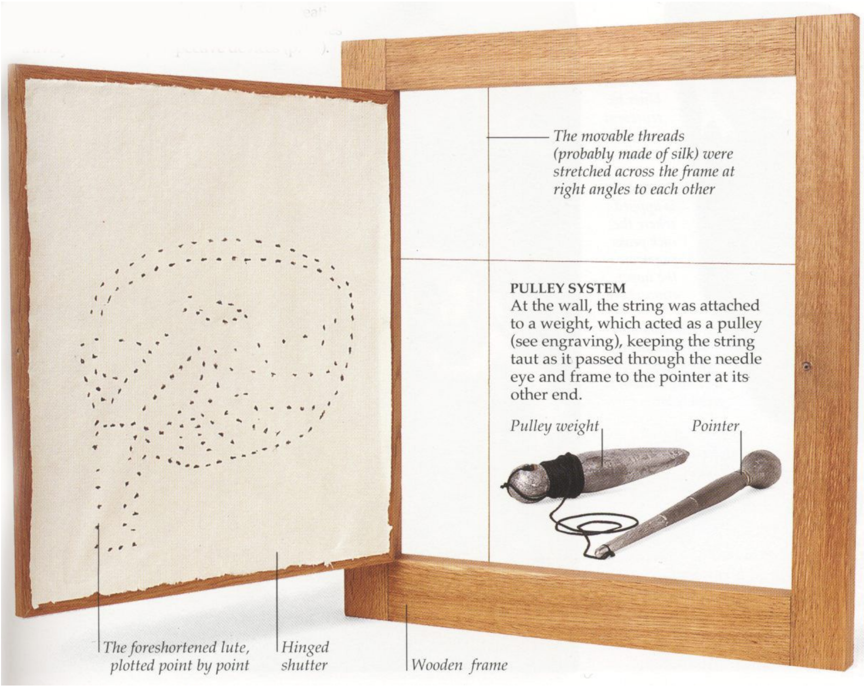
Today

- Reading for Today
- Ray Casting
  - Ray-Plane Intersection
  - Ray-Sphere Intersection
  - Point in Polygon
- Ray Tracing
- Recursive Ray Tracing
- Distributed Ray Tracing
- Readings for Next Week

Durer's Ray Casting Machine

- Albrecht Durer, 16th century

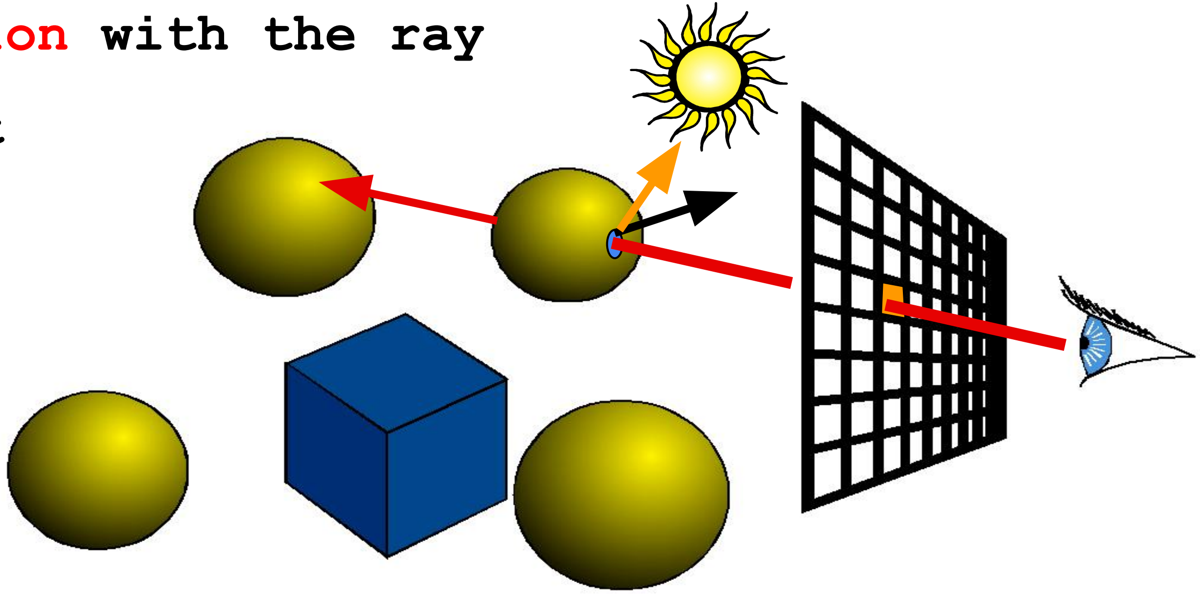




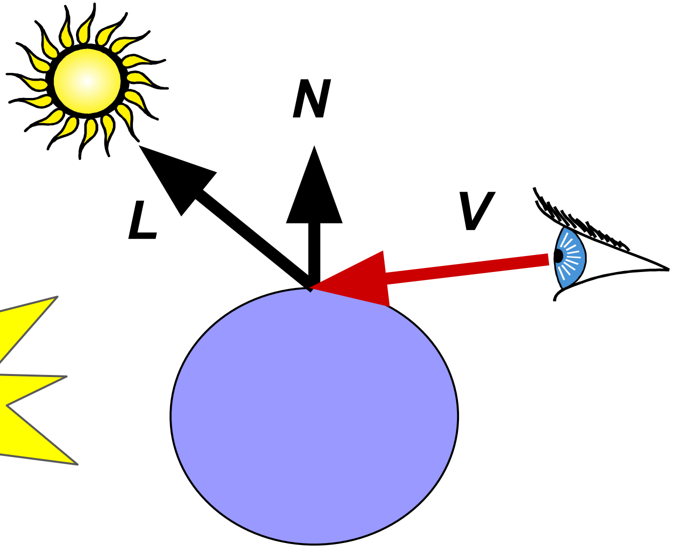
Ray Casting

```
For every pixel
   Construct a ray from the eye
   For every object in the scene
      Find intersection with the ray
      Keep if closest
   Shade depending
   on light and
   normal vector
```

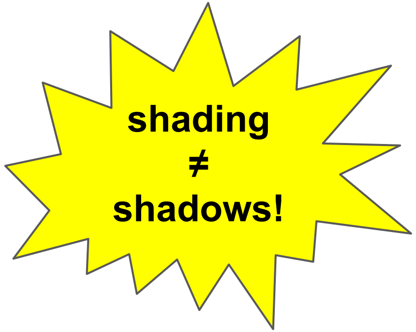
Finding the intersection and normal is the central part of ray casting

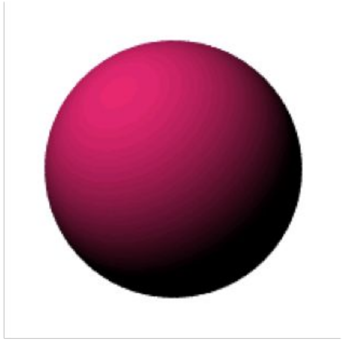


What is “*Local Shading*” ?

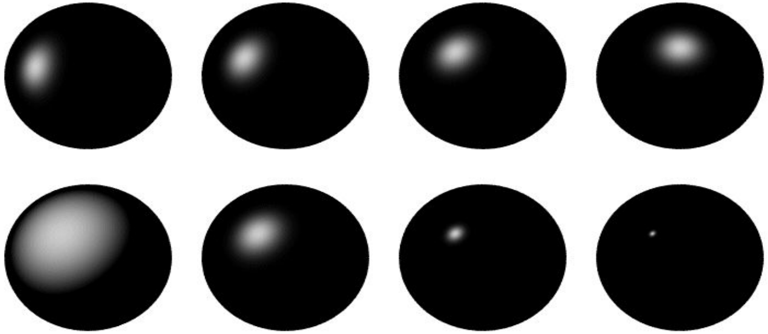


- Surface/Scene Characteristics:
  - surface normal, ***N***
  - direction to light, ***L***
  - viewpoint, ***V***
- Material Properties
  - color/texture
  - diffuse (matte)
  - specular (shiny)
  - …
- *More about material models later!*





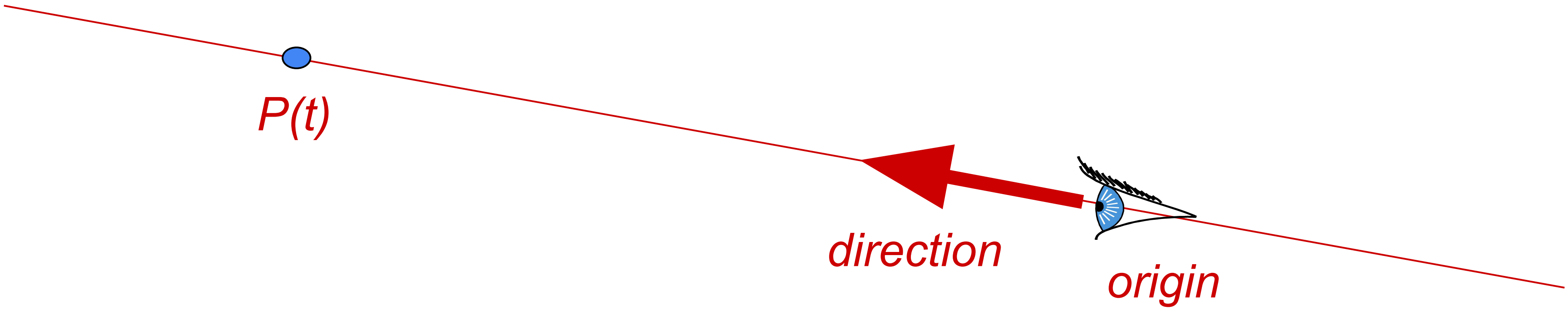
Diffuse sphere



Specular spheres

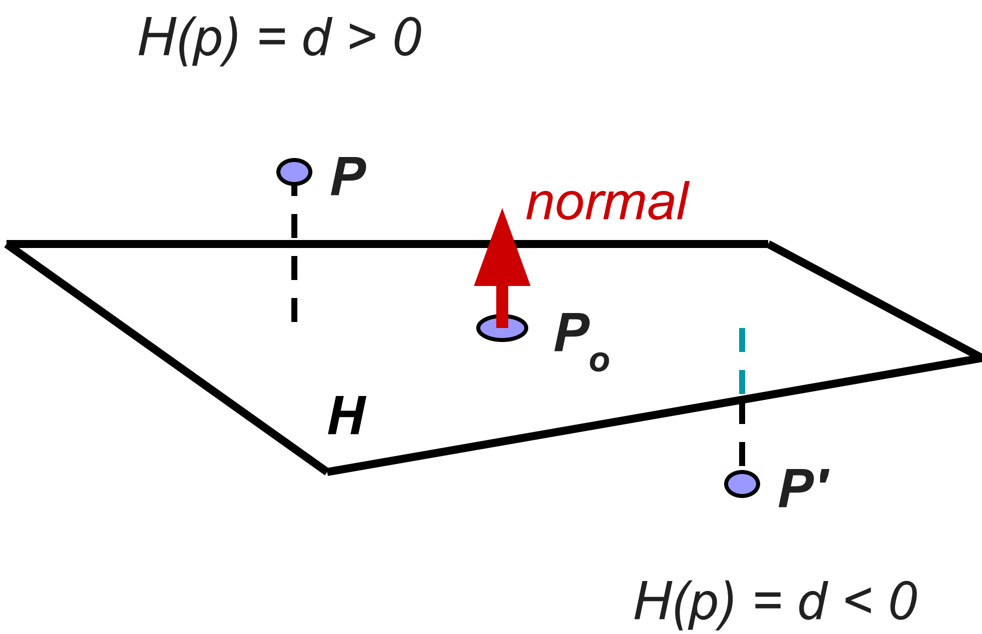
Ray Representation?

- Two vectors:
  - Origin
  - Direction (normalized is better)
- Parametric line (*explicit* representation)
  - *P(t) = origin + t * direction*



3D Plane Representation?

- Plane defined by
  - $P_o = (x,y,z)$
  - $n = (A,B,C)$
- *Implicit* plane equation
  - $H(P) = Ax+By+Cz+D = 0$
    $= n \cdot P + D = 0$
- Point-Plane distance?
  - If n is normalized,
    distance to plane, d = H(P)
  - d is the *signed distance*!



Explicit vs. Implicit?

- Ray equation is explicit $P(t) = R_o + t * R_d$
  - Parametric
  - Generates points
  - Harder to verify that a point is on the ray

- Plane equation is implicit $H(P) = n \cdot P + D = 0$
  - Solution of an equation
  - Does not generate points
  - Verifies that a point is on the plane

Ray-Plane Intersection

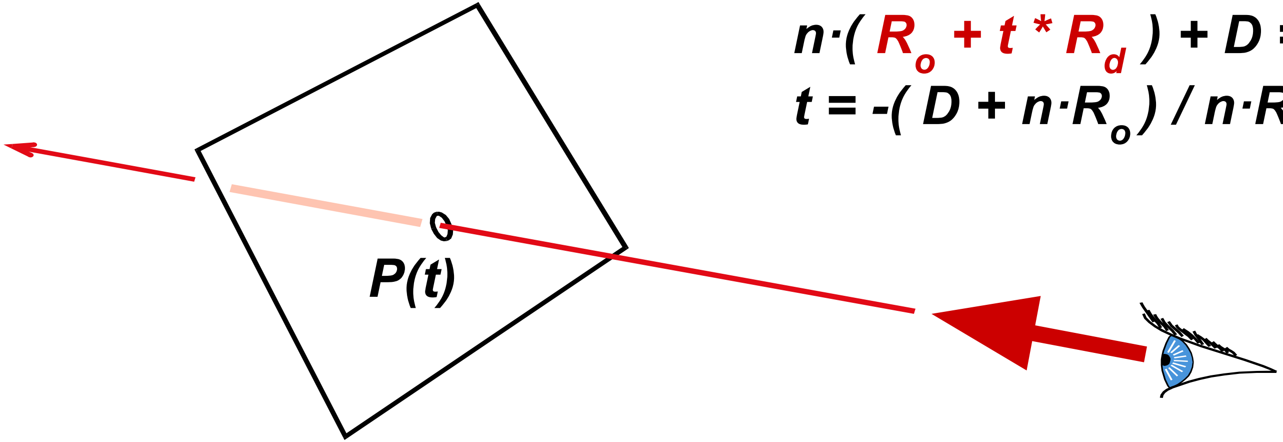
- Intersection means both are satisfied
- So, insert explicit equation of ray into implicit equation of plane
- Then solve for ***t***

$$P(t) = R_o + t * R_d$$

$$H(P) = n \cdot P + D = 0$$

$$n \cdot (R_o + t * R_d) + D = 0$$

$$t = -(D + n \cdot R_o) / n \cdot R_d$$



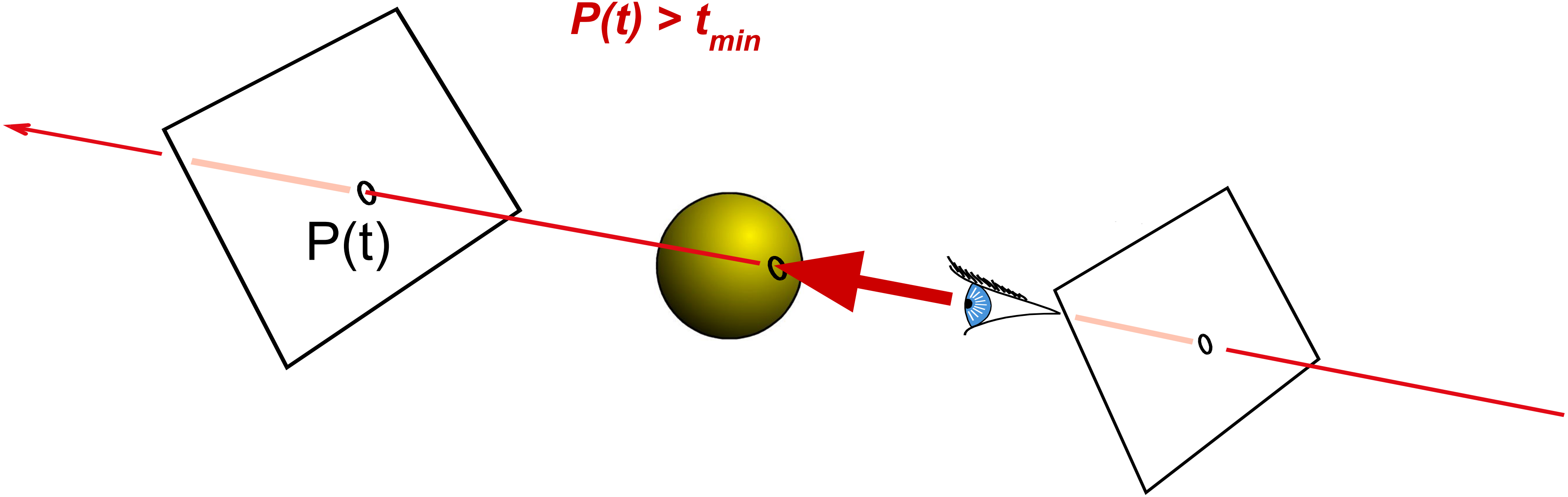
Additional Housekeeping

- Verify that intersection is closer than previous

$$P(t) < t_{current}$$

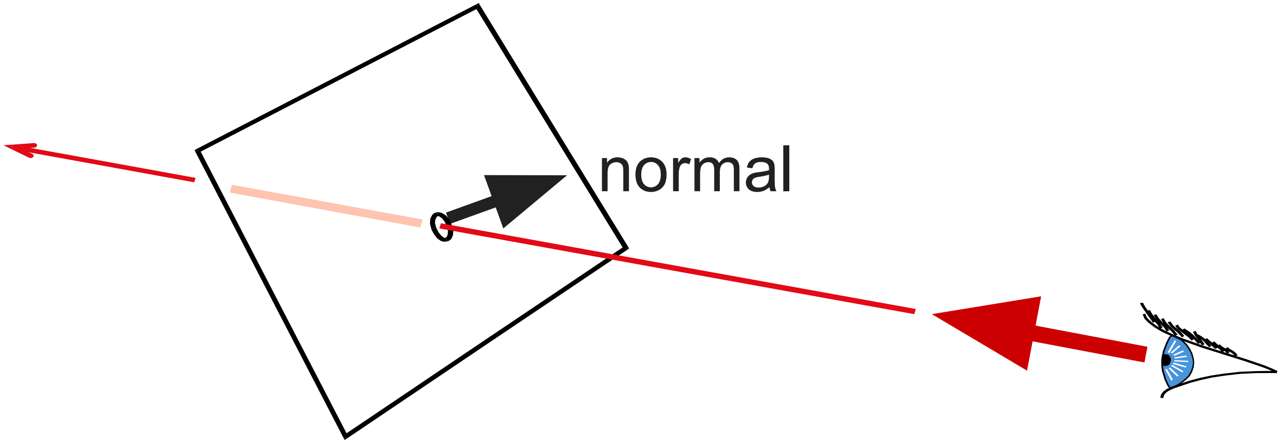
- Verify that it is not out of range (behind eye)

$$P(t) > t_{min}$$



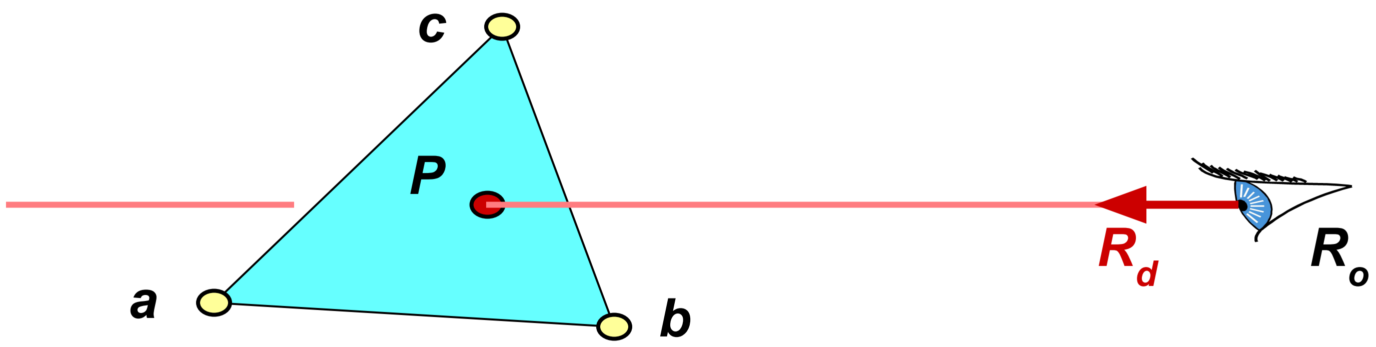
Normal at Surface Intersection

- Needed for shading
  - E.g., diffuse material: dot product between light and normal
- Normal of a plane is constant!



Ray-Triangle Intersection

- Intersect with the plane…
- Then use barycentric coordinates:
  - $\boldsymbol{P(\alpha, \beta, \gamma) = \alpha a + \beta b + \gamma c}$
    with $\alpha + \beta + \gamma = 1$
  - If $0 < \alpha < 1$ & $0 < \beta < 1$ & $0 < \gamma < 1$
    then the point is inside the triangle!



Barycentric Coordinates Definition

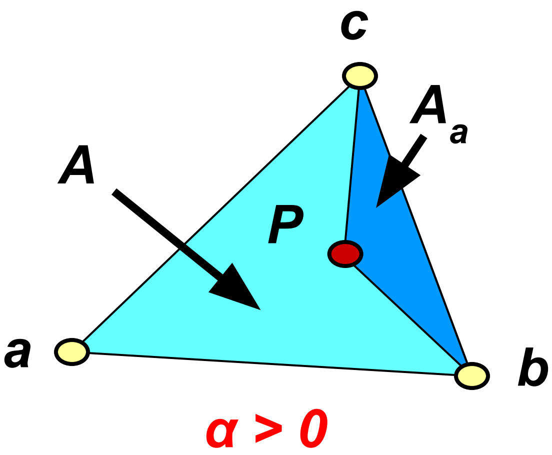
- Ratio of opposite sub-triangle area to total area

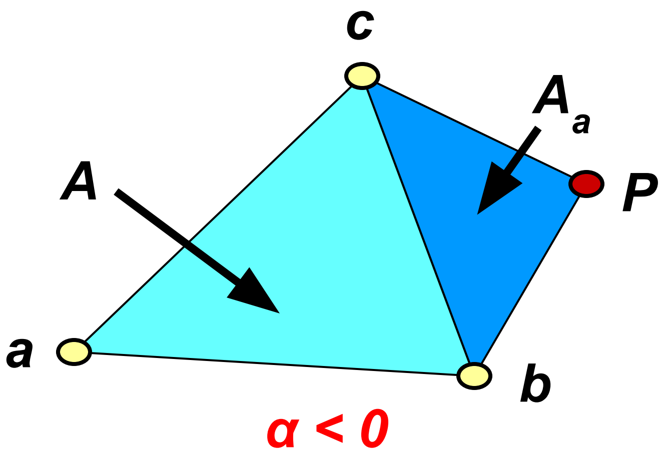
$$\alpha = A_a / A \qquad \beta = A_b / A \qquad \gamma = A_c / A$$

- Use signed areas for points outside the triangle
- α is approximately 1.0 when ***P*** is close to ***a***
- α is approximately 0.0 when ***P*** is near line **bc**

But how do I know if the point is outside the triangle?

That's what I was trying to determine!





Barycentric Coordinates using Cramer's Rule

- Used to solve for one variable at a time in system of equations

$$\beta = \frac{\begin{vmatrix} a_x - R_{ox} & a_x - c_x & R_{dx} \\ a_y - R_{oy} & a_y - c_y & R_{dy} \\ a_z - R_{oz} & a_z - c_z & R_{dz} \end{vmatrix}}{|A|} \qquad \gamma = \frac{\begin{vmatrix} a_x - b_x & a_x - R_{ox} & R_{dx} \\ a_y - b_y & a_y - R_{oy} & R_{dy} \\ a_z - b_z & a_z - R_{oz} & R_{dz} \end{vmatrix}}{|A|}$$

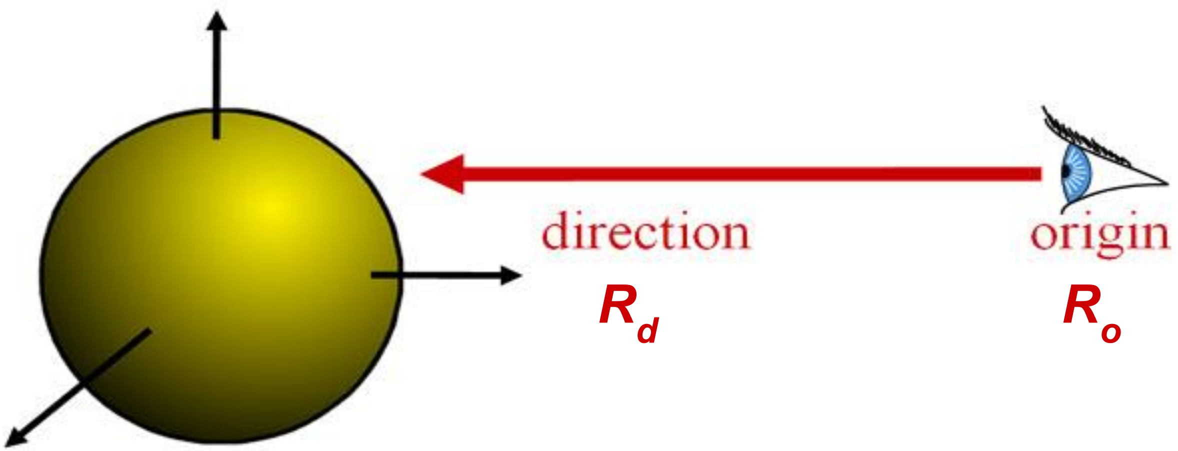
$$t = \frac{\begin{vmatrix} a_x - b_x & a_x - c_x & a_x - R_{ox} \\ a_y - b_y & a_y - c_y & a_y - R_{oy} \\ a_z - b_z & a_z - c_z & a_z - R_{oz} \end{vmatrix}}{|A|}$$

| | denotes the determinant

Can be copied mechanically into code

Sphere Representation?

- Implicit sphere equation
  - Assume centered at origin (easy to translate)
  - $H(P) = P \cdot P - r^2 = 0$



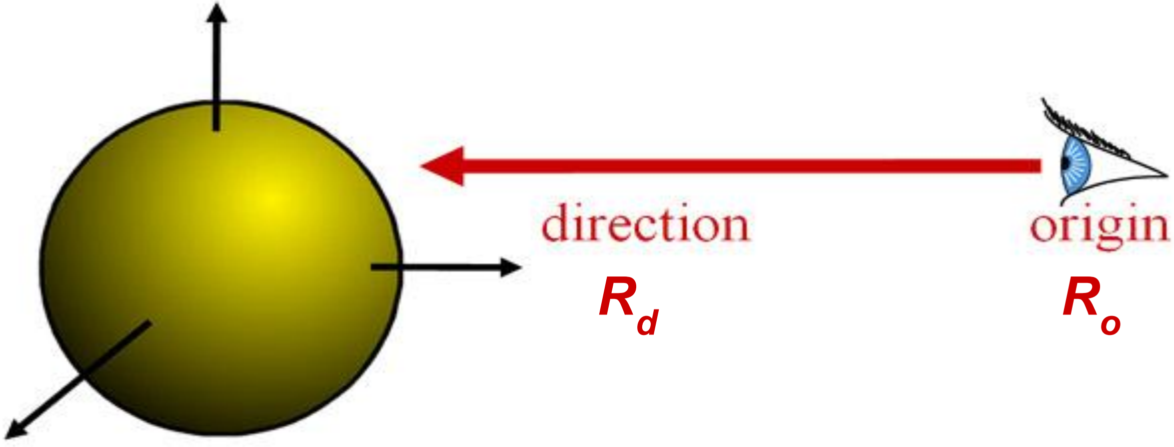
Ray-Sphere Intersection

- Insert explicit equation of ray into implicit equation of sphere & solve for t

$$P(t) = R_o + t^*R_d \qquad H(P) = P \cdot P - r^2 = 0$$

$$(R_o + tR_d) \cdot (R_o + tR_d) - r^2 = 0$$

$$R_d \cdot R_d t^2 + 2R_d \cdot R_o t + R_o \cdot R_o - r^2 = 0$$



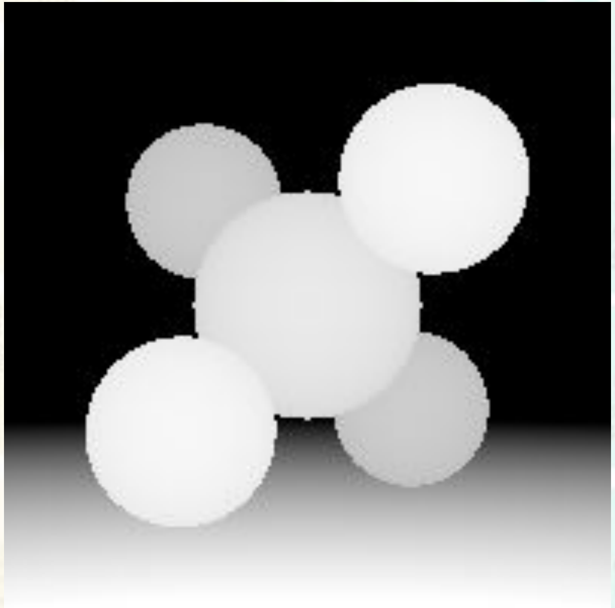
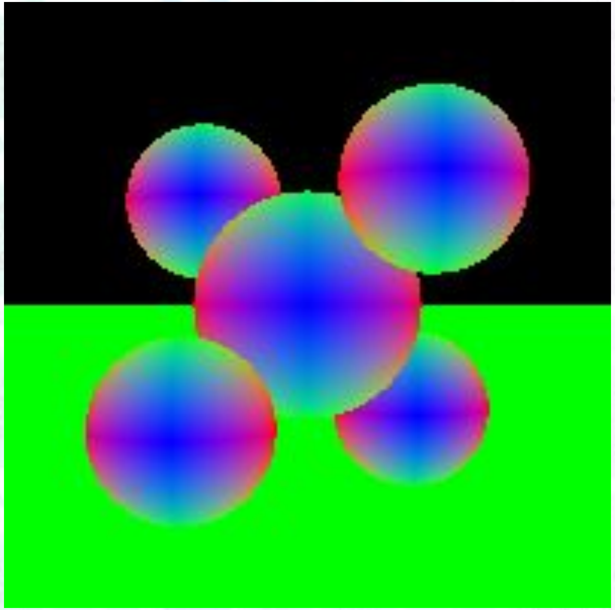
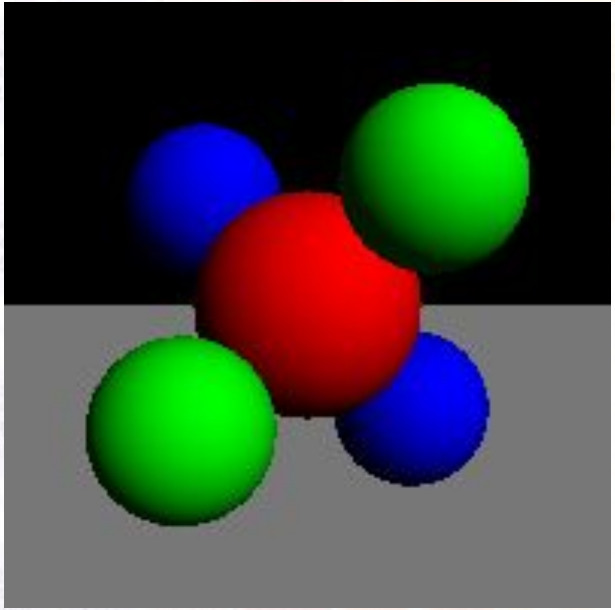
Ray-Sphere Intersection

- $\boldsymbol{R_d \cdot R_d t^2 + 2R_d \cdot R_o t + R_o \cdot R_o - r^2 = 0}$
- Solve using quadratic formula: $\mathbf{at^2 + bt + c = 0}$
  - **a** = 1 (remember, $||\mathbf{R_d}||$ = 1, a.k.a. *normalized*)
  - $\mathbf{b = 2R_d \cdot R_o}$
  - $\mathbf{c = R_o \cdot R_o - r^2}$

- with discriminant $d = \sqrt{b^2 - 4ac}$
- and solutions $t_\pm = \dfrac{-b \pm d}{2a}$

- *What does it mean if there are no solutions, 1 solution, or 2 solutions?*
- *Don't forget to account for t < 0…*

Questions?

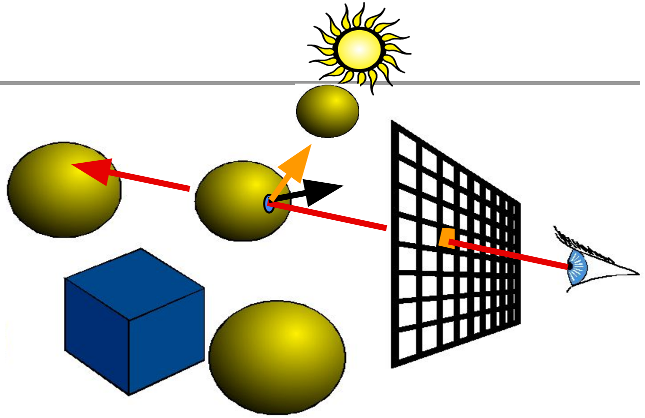




depth *normals* *local shading*

Today

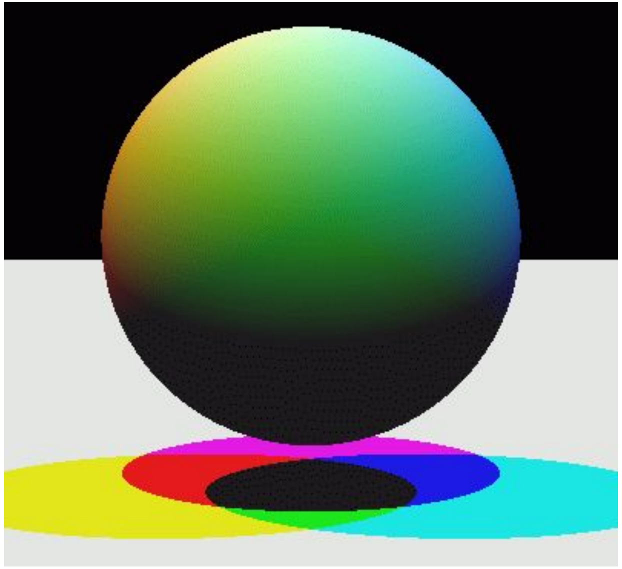
- Reading for Today
- Ray Casting
- Ray Tracing
  - Shadows
  - Reflection
  - Refraction
- Recursive Ray Tracing
- Distributed Ray Tracing
- Readings for Next Week

How to Add Ray Traced Shadows



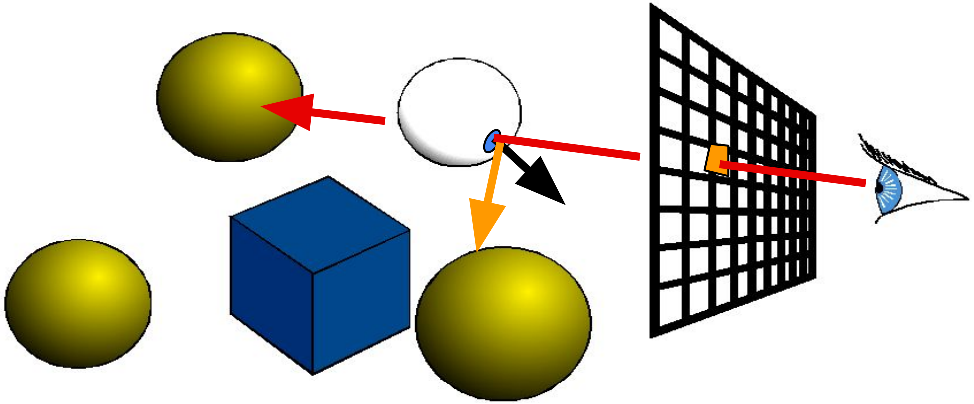
```
Find point to be shaded
For every light
   Construct ray from point to light
   For every object
      Intersect light ray with object
   If no objects between point
      and light, then add
      contribution from light
```

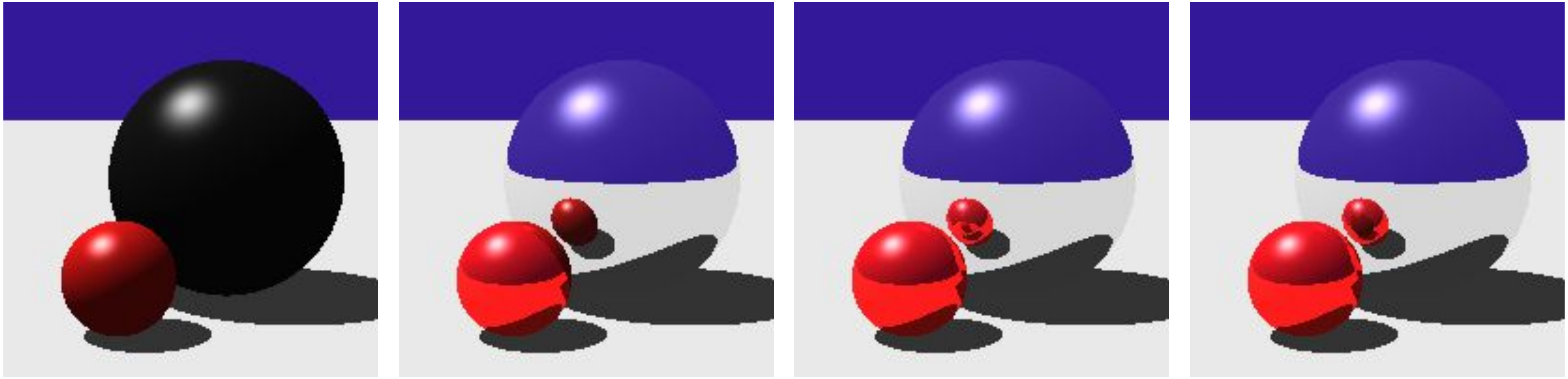
How many lights in this scene?
Where are they positioned?
What color are they?



How to Add Mirror Reflections

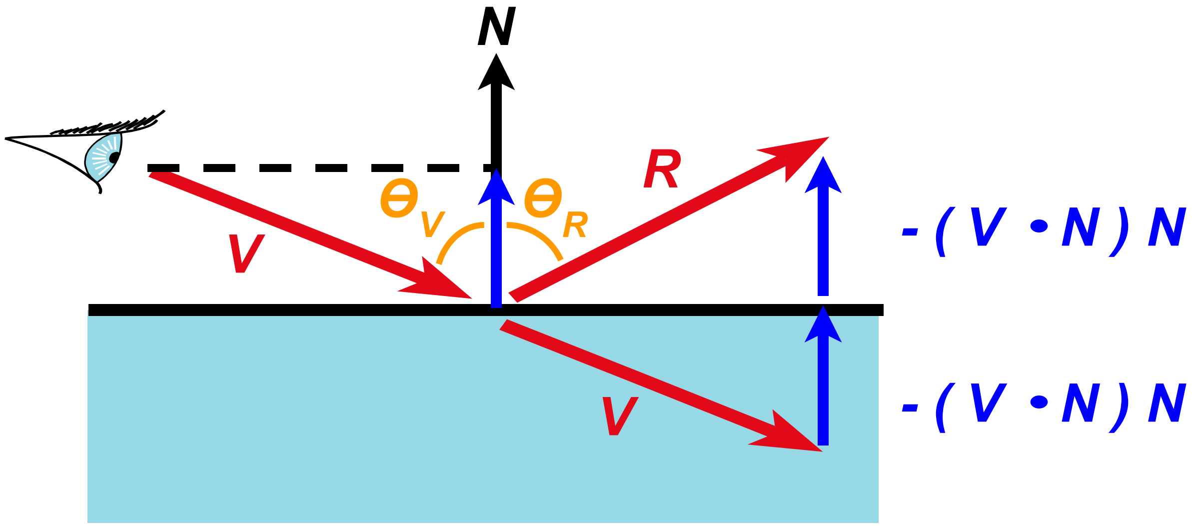
- Cast ray symmetric with respect to the normal
- Multiply by reflection coefficient (color)





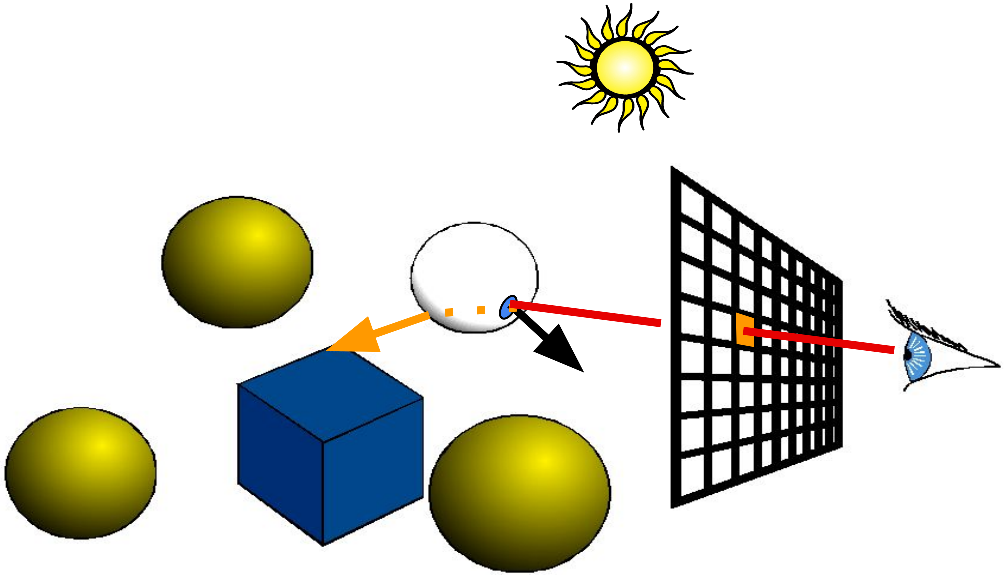
How to Construct Reflected Ray?

- Reflection angle $\boldsymbol{\theta}_V$ = view angle $\boldsymbol{\theta}_R$
- $\mathbf{R} = \mathbf{V} - 2\,(\mathbf{V} \cdot \mathbf{N})\,\mathbf{N}$

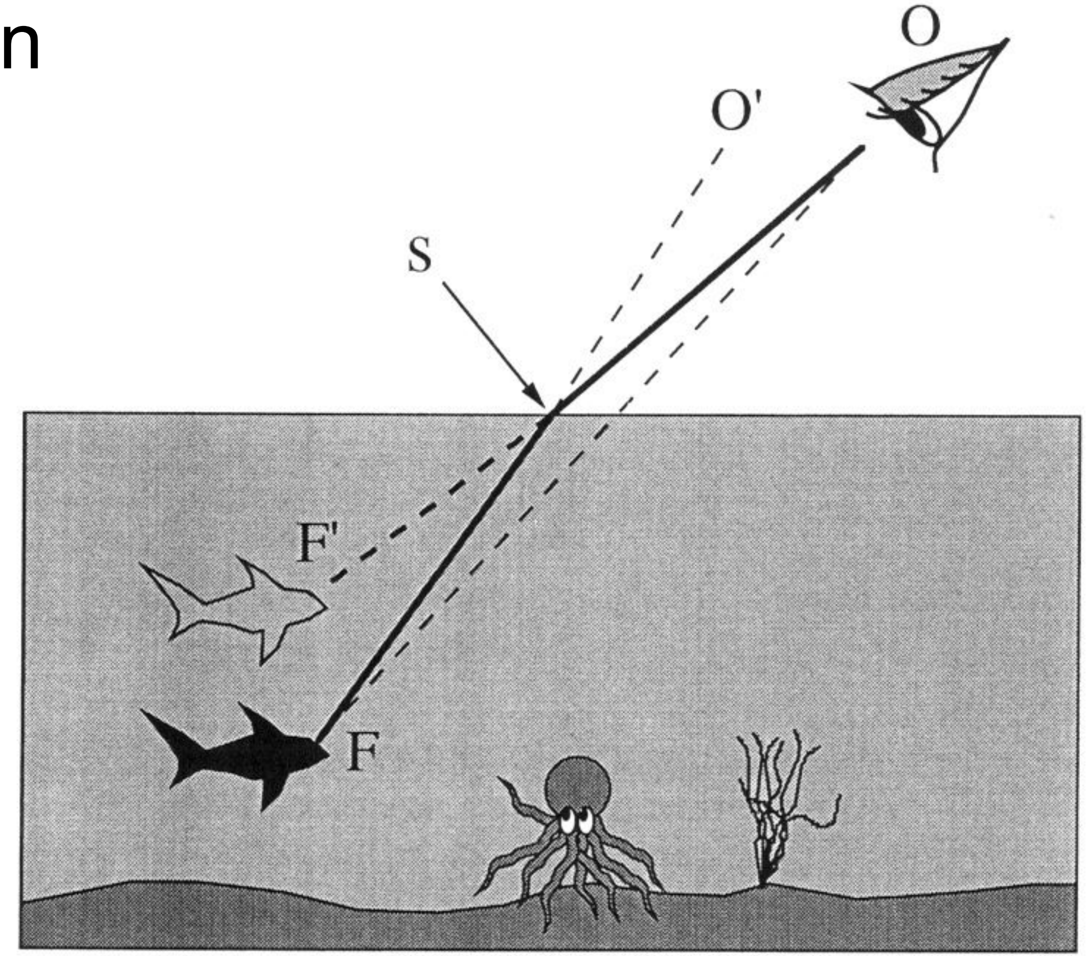


Transparency

- Cast ray in refracted direction
- Multiply by transparency coefficient (color)

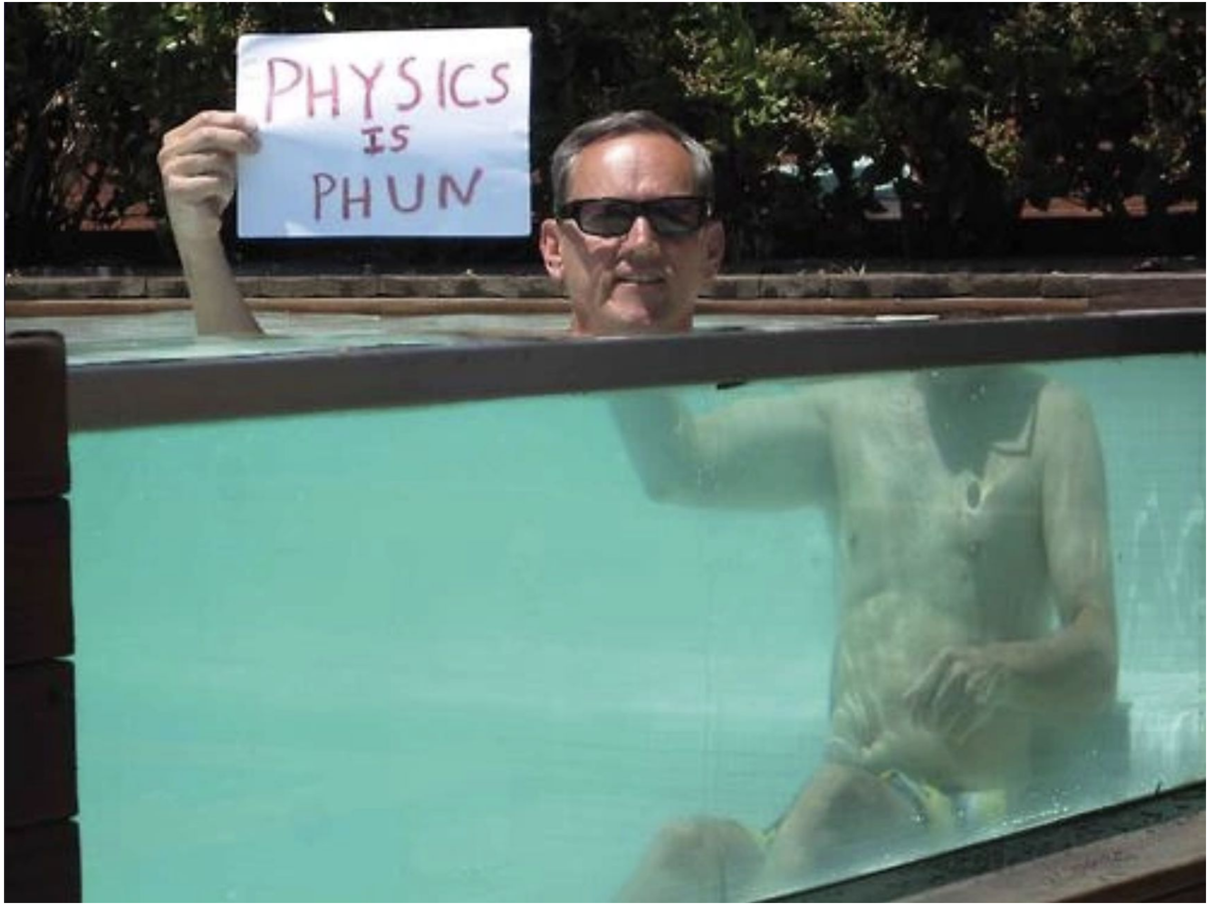


Qualitative Refraction



From "Color and Light in Nature" by Lynch and Livingston

Qualitative Refraction



Refraction

Note: The math works the same tracing the ray either "forwards" or "backwards", but it's really easy to get confused and have a sign error in the direction.

$$\mathbf{I} = \mathbf{N}\cos\Theta_i - \mathbf{M}\sin\Theta_i$$

$$\mathbf{M} = (\mathbf{N}\cos\Theta_i - \mathbf{I}) / \sin\Theta_i$$

$$\mathbf{T} = -\mathbf{N}\cos\Theta_T + \mathbf{M}\sin\Theta_T$$

$$= -\mathbf{N}\cos\Theta_T + (\mathbf{N}\cos\Theta_i - \mathbf{I})\sin\Theta_T / \sin\Theta_i$$

$$= -\mathbf{N}\cos\Theta_T + (\mathbf{N}\cos\Theta_i - \mathbf{I})\,\eta_r$$

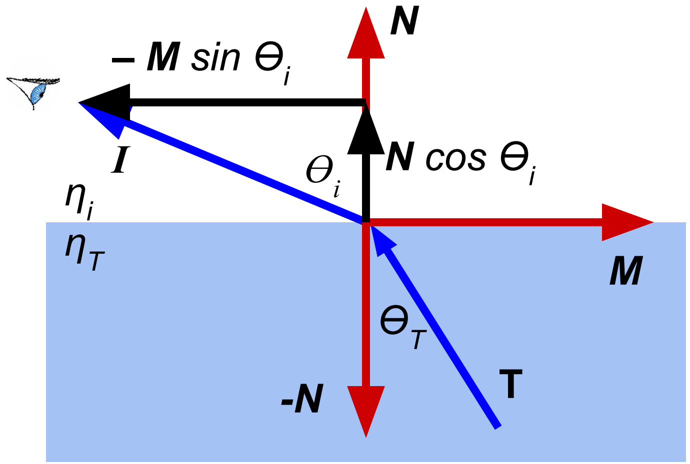
$$= [\,\eta_r\cos\Theta_i - \cos\Theta_T]\,\mathbf{N} - \eta_r\,\mathbf{I}$$

$$= [\,\eta_r\cos\Theta_i - \sqrt{(1 - \sin^2\Theta_T)}]\,\mathbf{N} - \eta_r\,\mathbf{I}$$

$$= [\,\eta_r\cos\Theta_i - \sqrt{(1 - \eta_r^2\sin^2\Theta_i)}]\,\mathbf{N} - \eta_r\,\mathbf{I}$$

$$= [\,\eta_r\cos\Theta_i - \sqrt{(1 - \eta_r^2(1 - \cos^2\Theta_i))}]\,\mathbf{N} - \eta_r\,\mathbf{I}$$

$$= [\,\eta_r(\mathbf{N}\cdot\mathbf{I}) - \sqrt{(1 - \eta_r^2(1 - (\mathbf{N}\cdot\mathbf{I})^2))}]\,\mathbf{N} - \eta_r\,\mathbf{I}$$



Snell-Descartes Law:

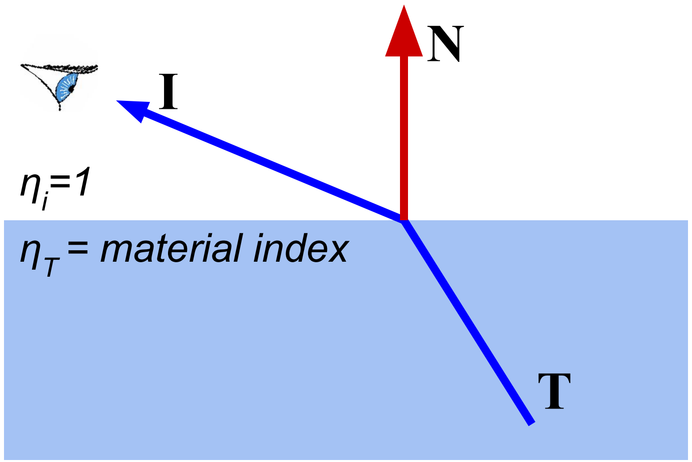
$$\eta_i \sin\Theta_i = \eta_T \sin\Theta_T$$

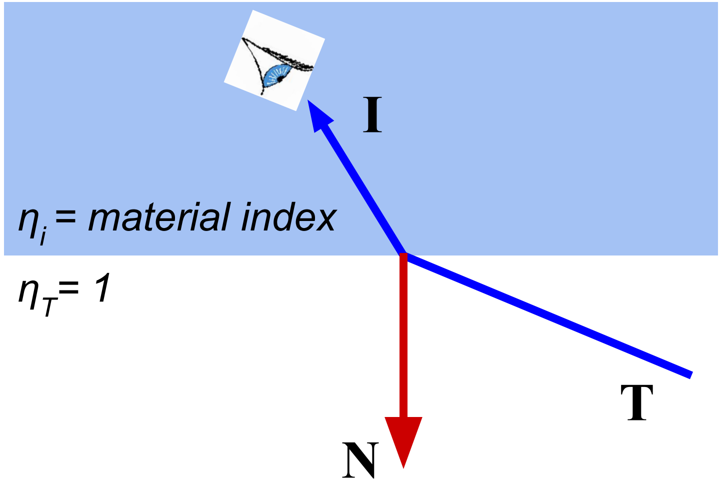
$$\frac{\sin\Theta_T}{\sin\Theta_i} = \frac{\eta_i}{\eta_T} = \eta_r$$

- *Total internal reflection when the square root is imaginary*
- *Don't forget to normalize!*

Refraction & the Sidedness of Objects

- Make sure you know whether you are entering or leaving the transmissive material:





Light bends towards the surface normal when entering a denser material.
It bends away from the normal when leaving the denser material.

Refraction & the Sidedness of Objects

What about intersecting transparent objects?

Image by Henrik Wann Jensen

Total Internal Reflection

From "Color and Light in Nature"
by Lynch and Livingston

Fig. 3.7A The optical manhole. From under water, the entire celestial hemisphere is compressed into a circle only 97.2° across. The dark boundary defining the edges of the manhole is not sharp due to surface waves. The rays are analogous to the crepuscular type seen in hazy air, Section 1.9. (Photo by D. Granger)

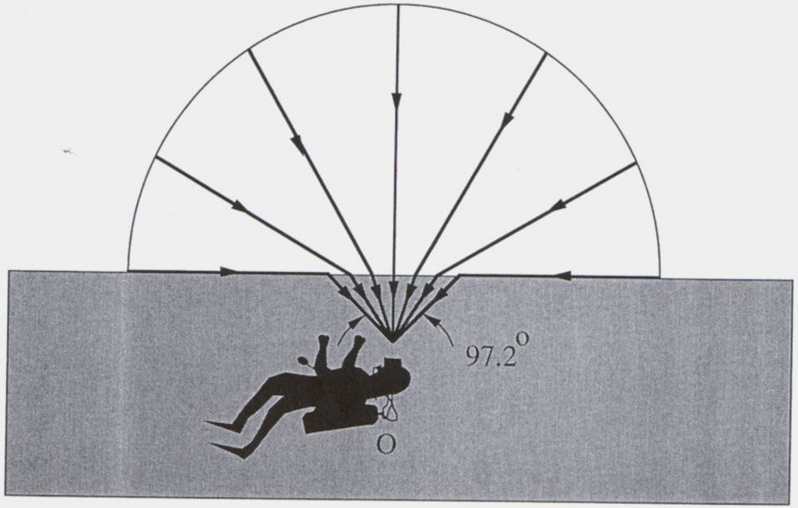


Fig. 3.7B The optical manhole. Light from the horizon (angle of incidence = 90°) is refracted downward at an angle of 48.6°. This compresses the sky into a circle with a diameter of 97.2° instead of its usual 180°.

Today

- Reading for Today
- Ray Casting
- Ray Tracing
- Recursive Ray Tracing
- Distributed Ray Tracing
- Readings for Next Week

Ray Tracing

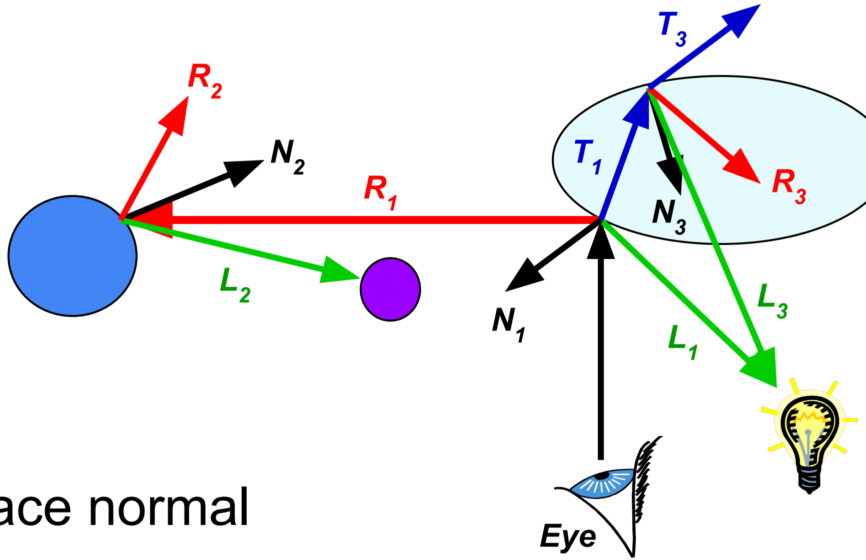
Does it ever end?

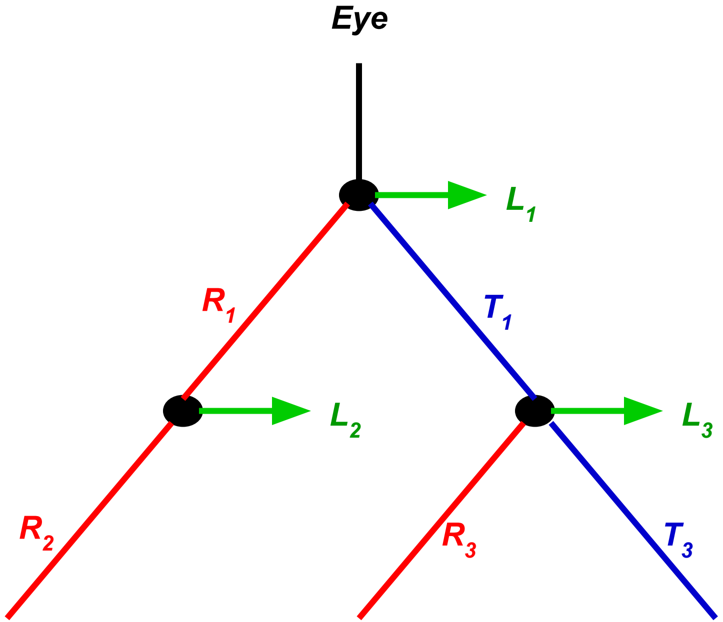
```
trace ray
    Intersect all objects
    color = ambient term
    For every light
        cast shadow ray
        color += local shading term
    If mirror
        color += color_refl * trace reflected ray

    If transparent
        color += color_trans * trace transmitted ray
```

Stopping criteria:

- Recursion depth: Stop after a number of bounces
- Ray contribution: Stop if reflected / transmitted contribution becomes too small

The Ray Tree





$\boldsymbol{N_i}$ surface normal

$\boldsymbol{R_i}$ reflected ray

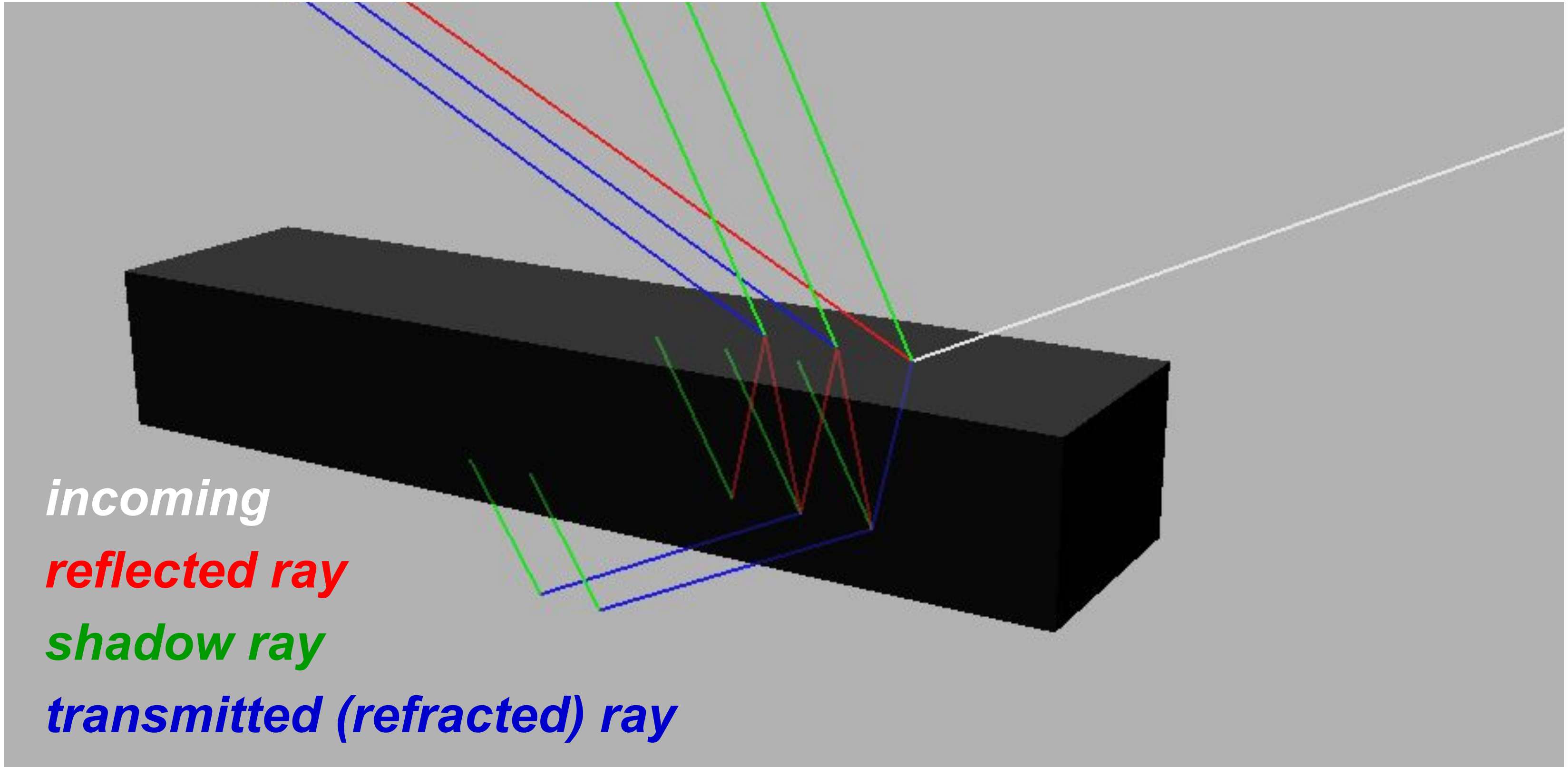
$\boldsymbol{L_i}$ shadow ray

$\boldsymbol{T_i}$ transmitted (refracted) ray

Big O Notation Complexity?

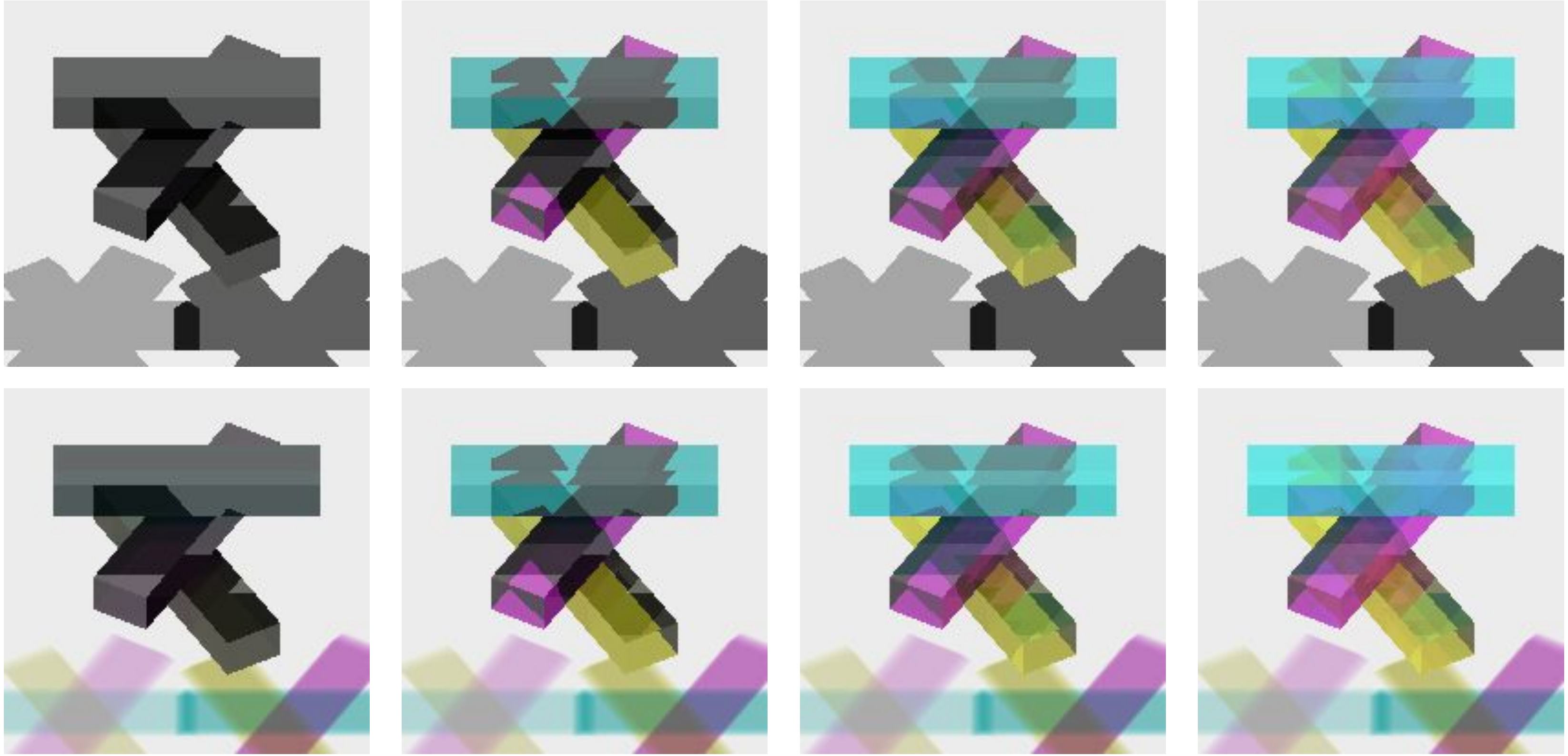
Ray Debugging

- Visualize the ray tree for single image pixel



Shadows of Transparent Objects

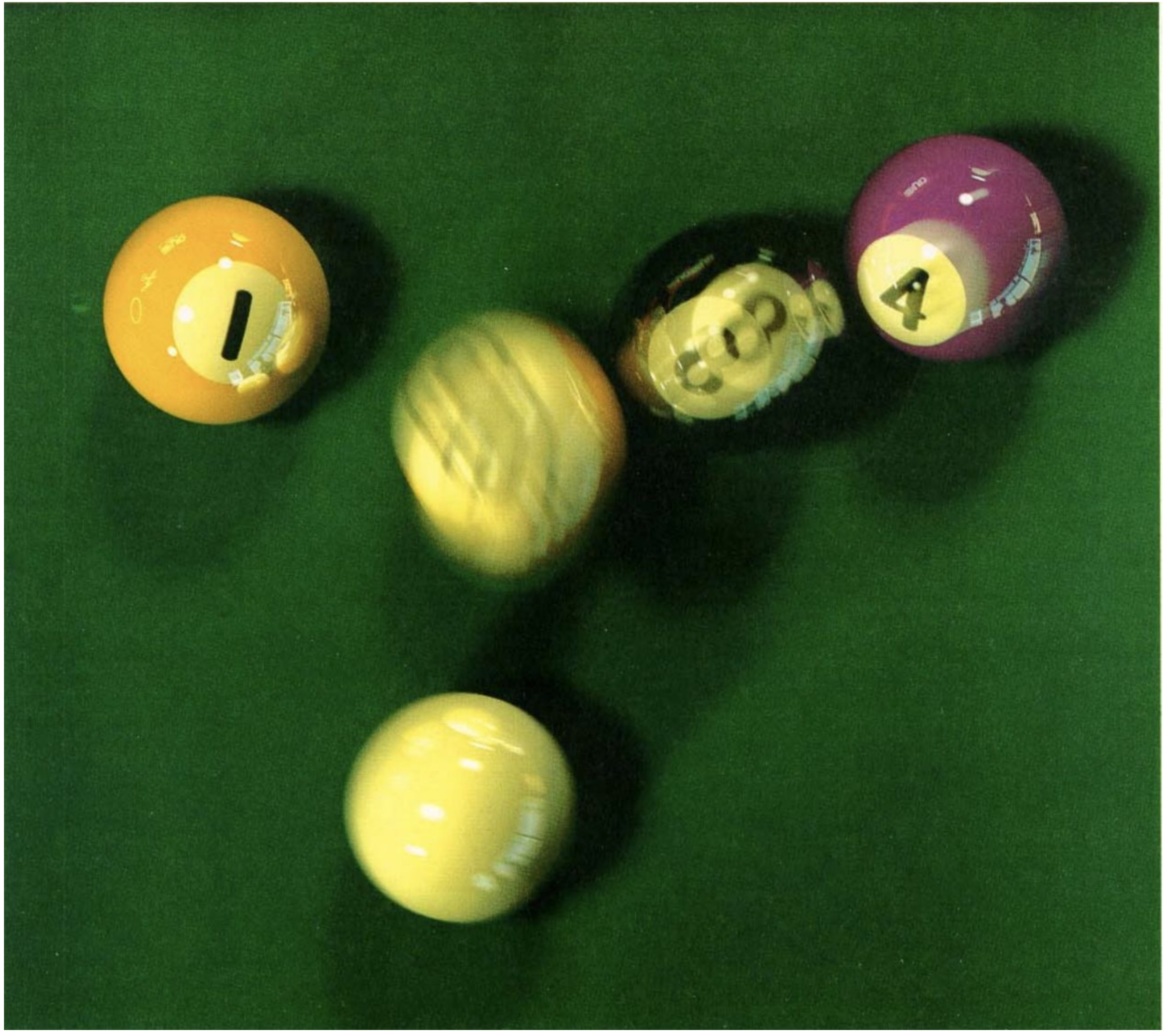
- Is this physically accurate?



Today

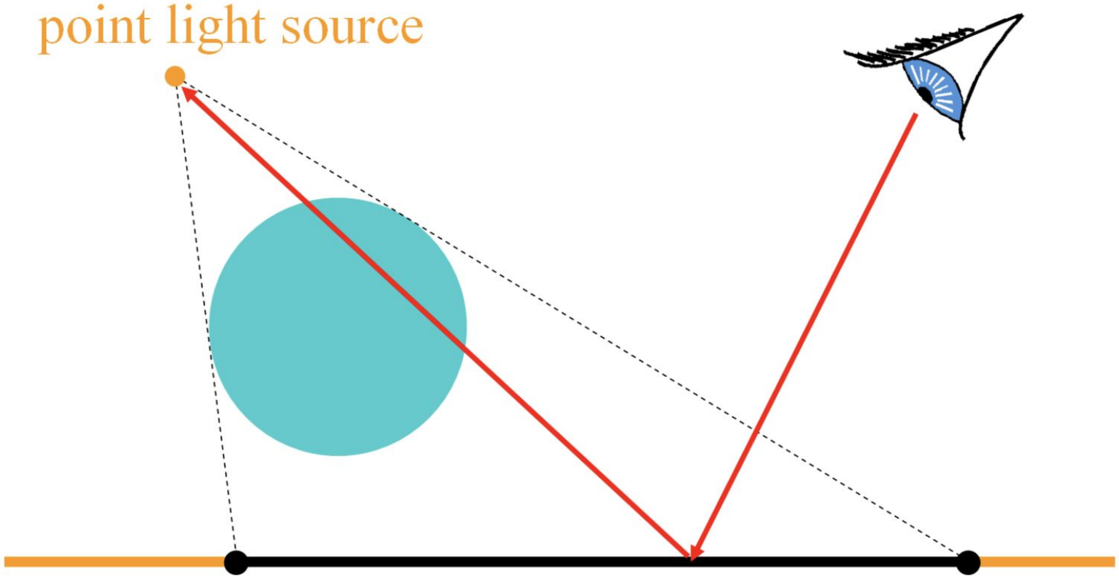
- Reading for Today
- Ray Casting
- Ray Tracing
- Recursive Ray Tracing
- Distributed Ray Tracing
  - Soft shadows
  - Antialiasing (getting rid of jaggies)
  - Glossy reflection
  - Motion blur
  - Depth of field (focus)
- Readings for Next Week

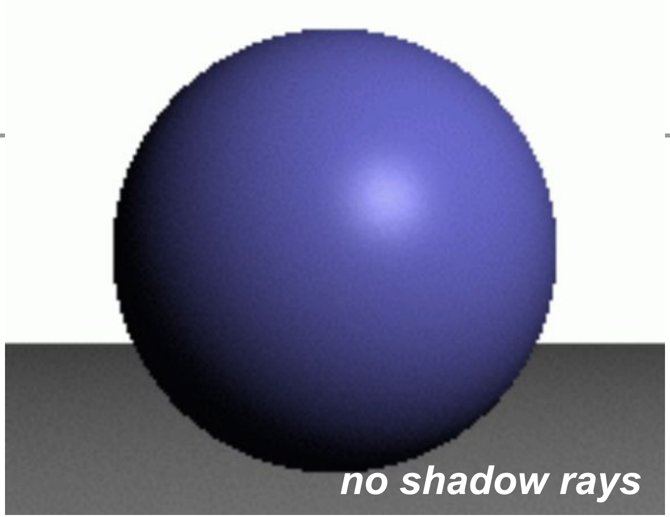
"Distributed Ray Tracing", Cook, Porter, & Carpenter, SIGGRAPH 1984.

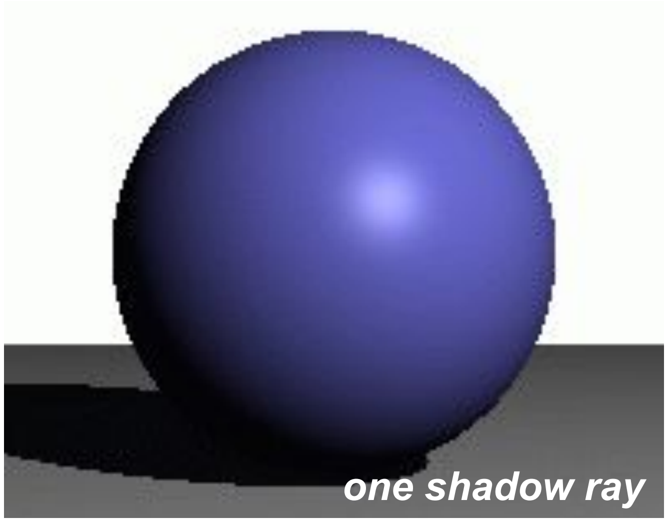


Ray Tracing Shadows

- One shadow ray
  per intersection
  per point light source

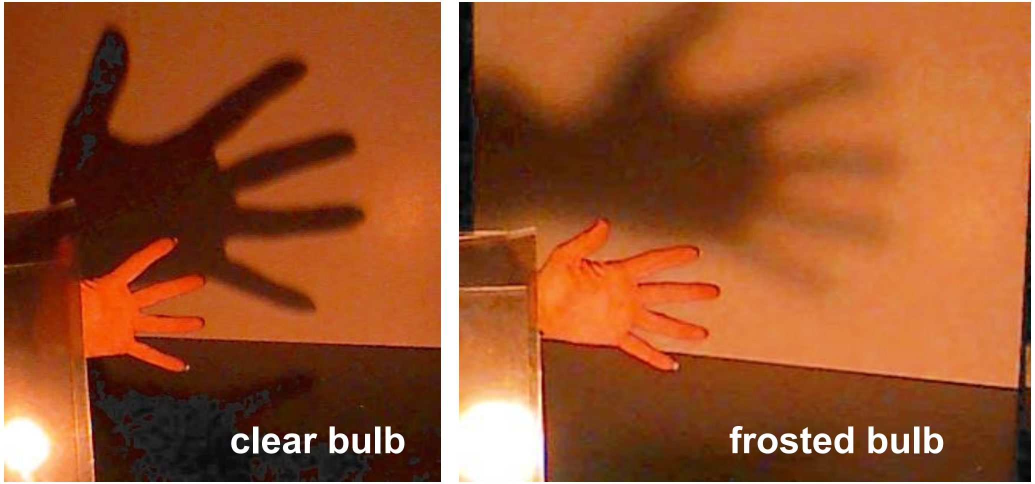






Shadows & Light Sources

http://www.davidfay.com/index.php

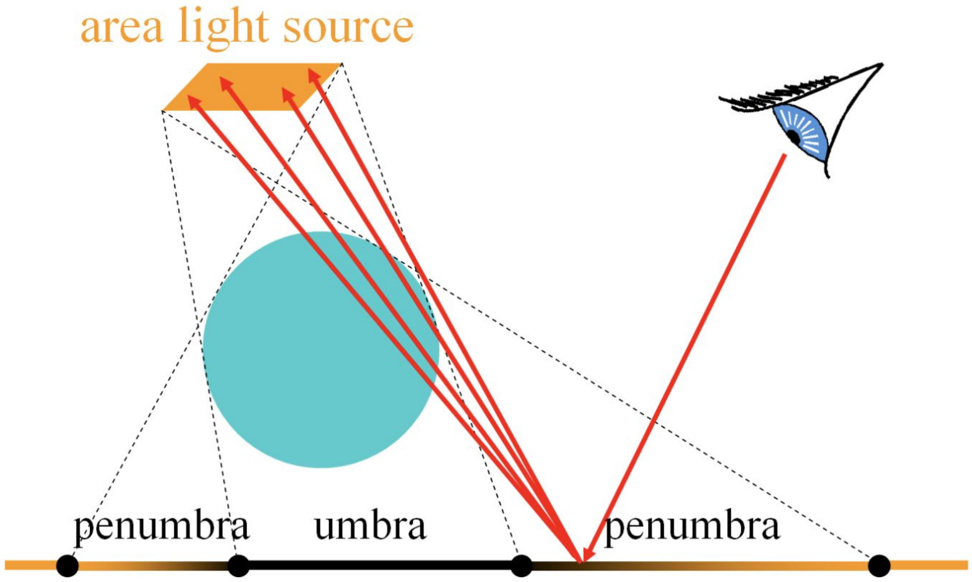


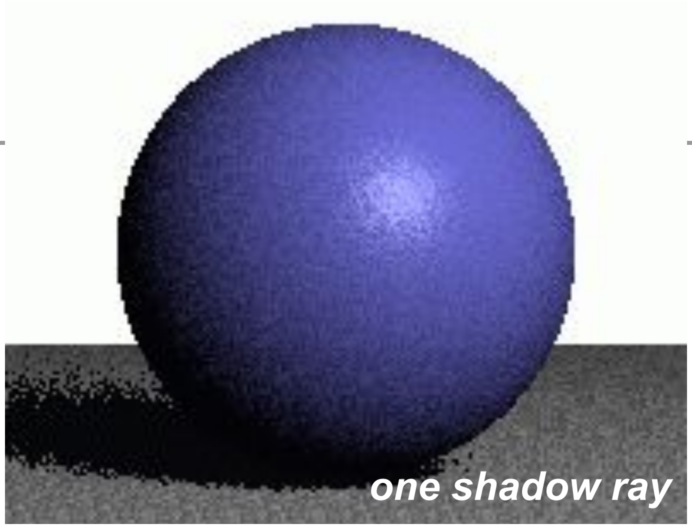
http://3media.initialized.org/photos/2000-10-18/index_gall.htm

http://www.pa.uky.edu/~sciworks/light/preview/bulb2.htm

Ray Tracing Soft Shadows

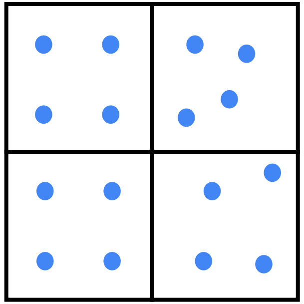
- multiple shadow rays to sample area light source

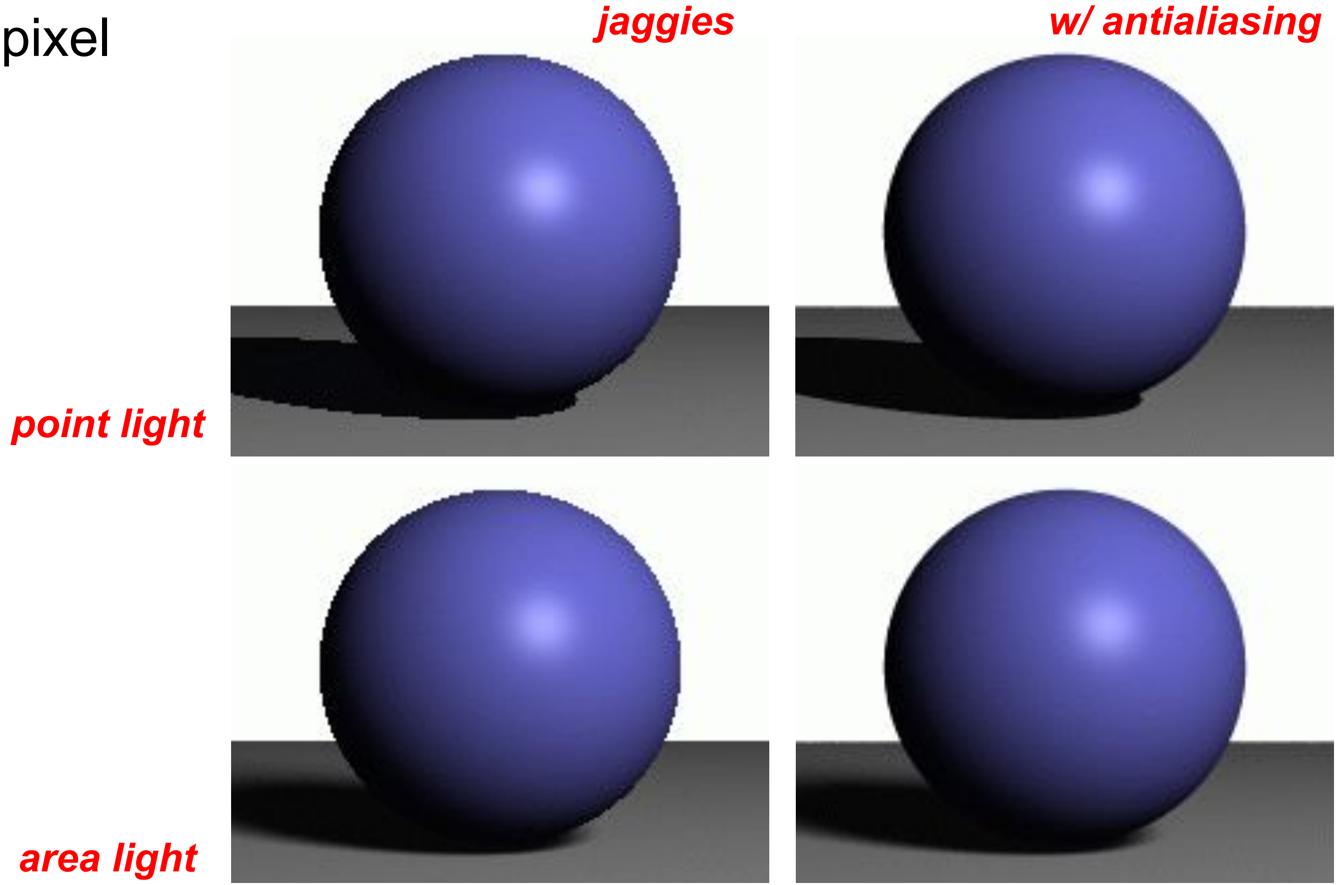




Antialiasing – Supersampling

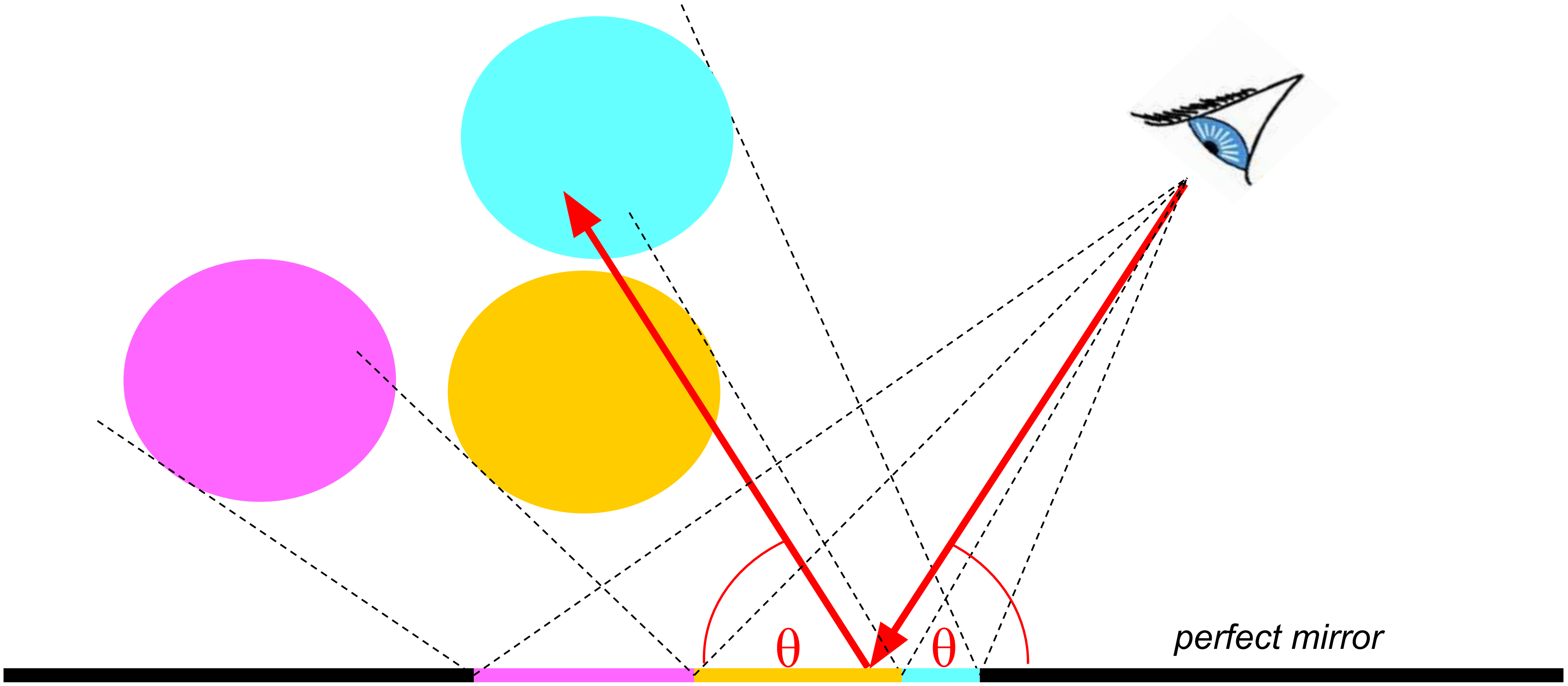
- multiple rays per pixel





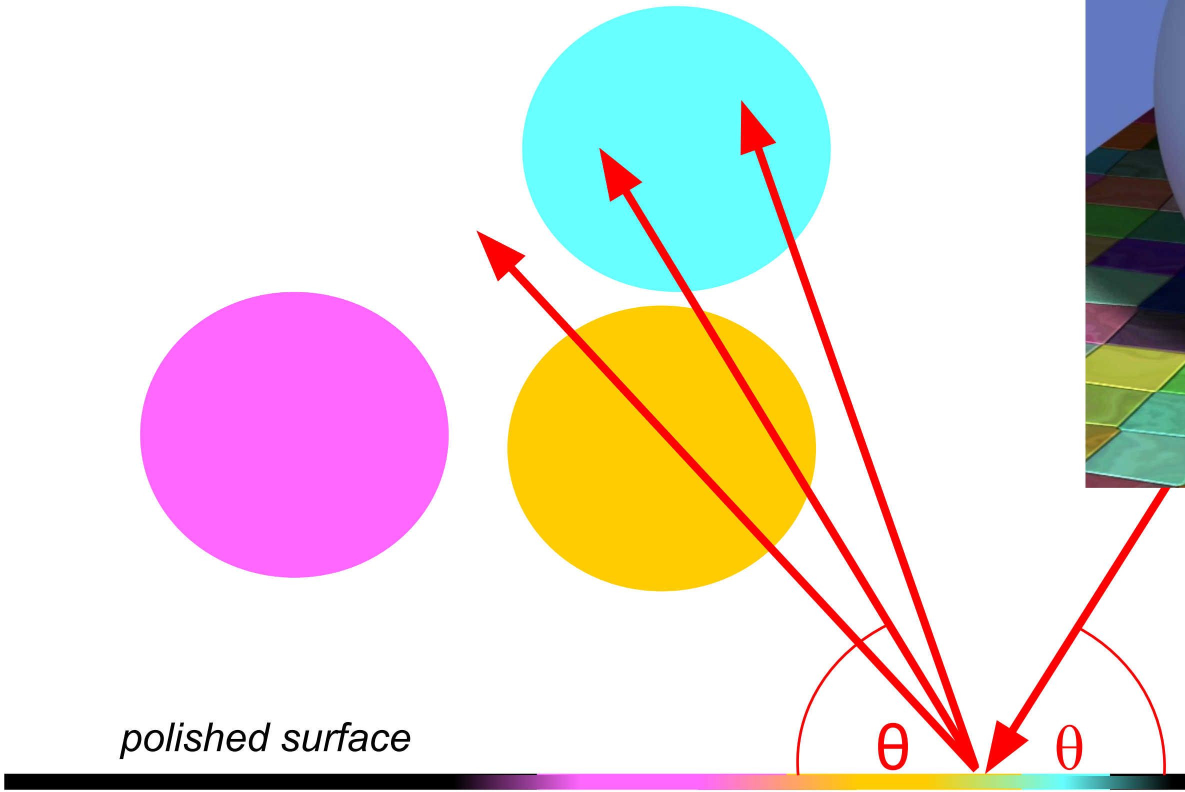
Ray Tracing Perfect Mirror Reflection

- one reflection ray per intersection



Ray Tracing Glossy Reflection

- multiple reflection rays



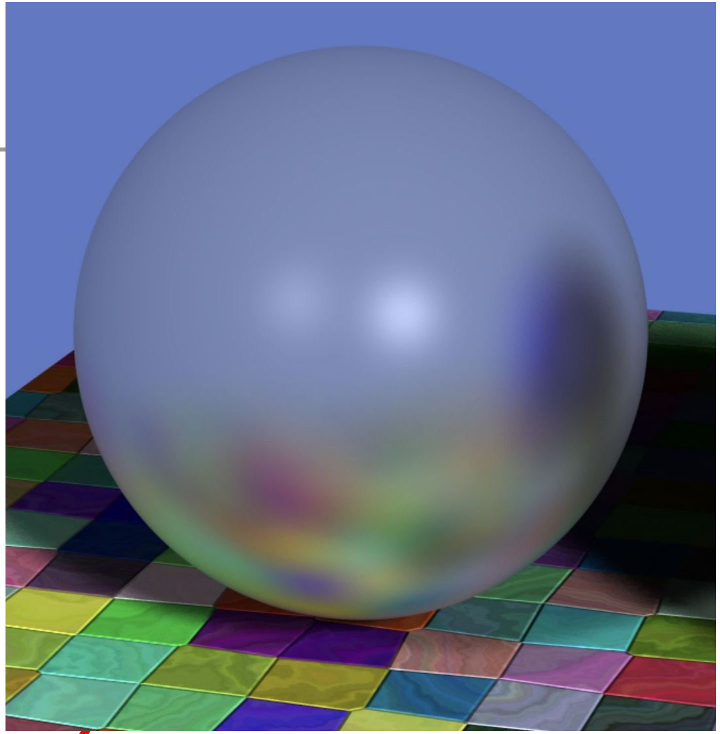
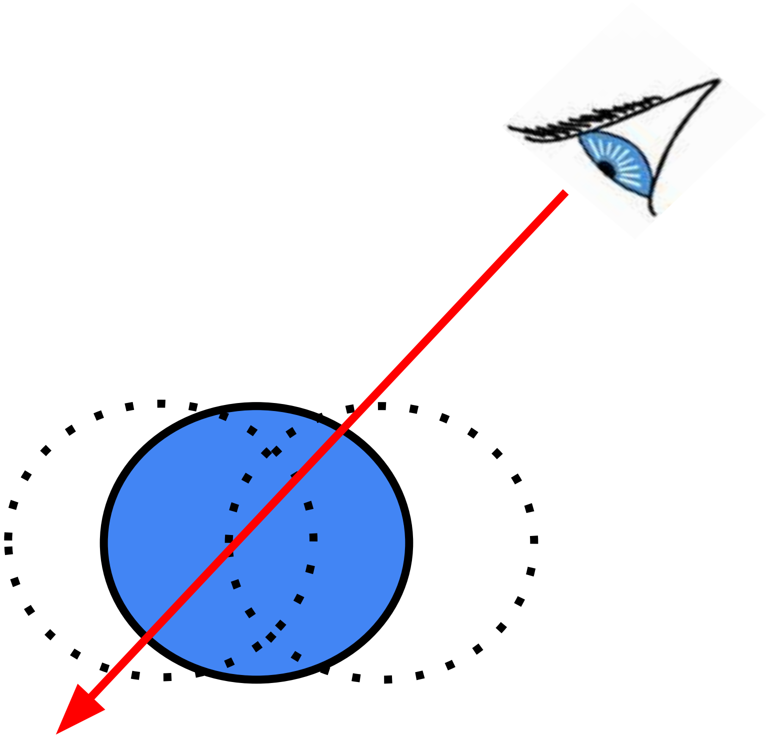


Image by Justin Legakis

Ray Tracing Motion Blur

- Sample objects temporally



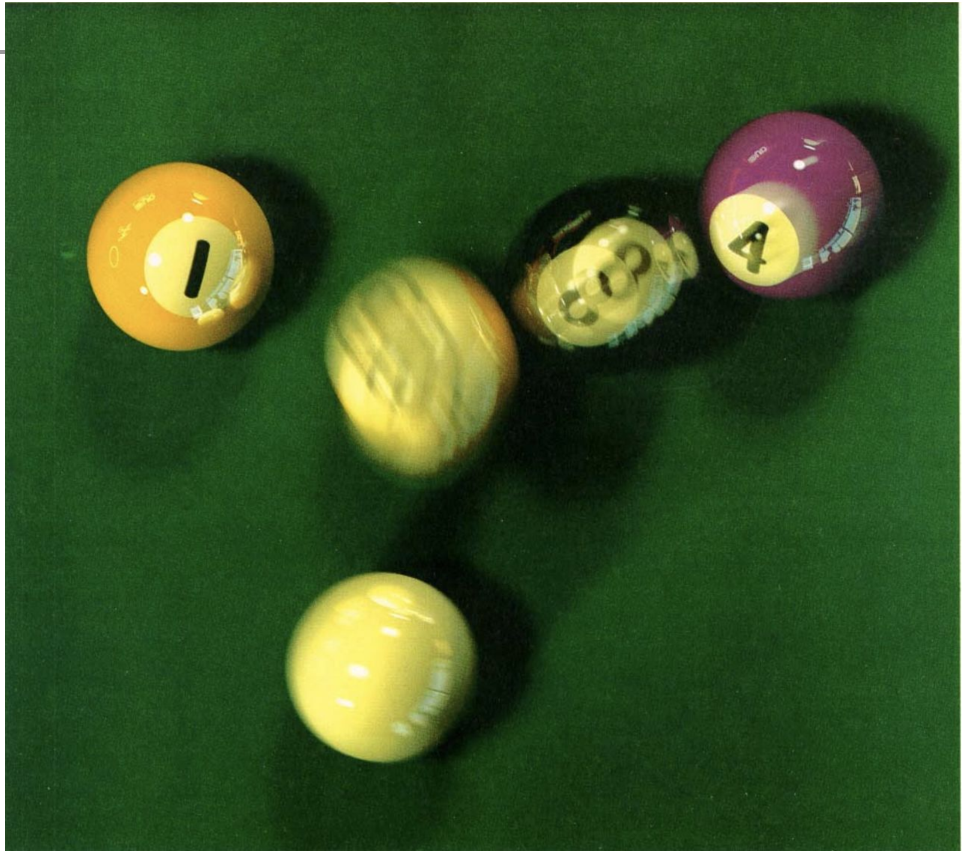
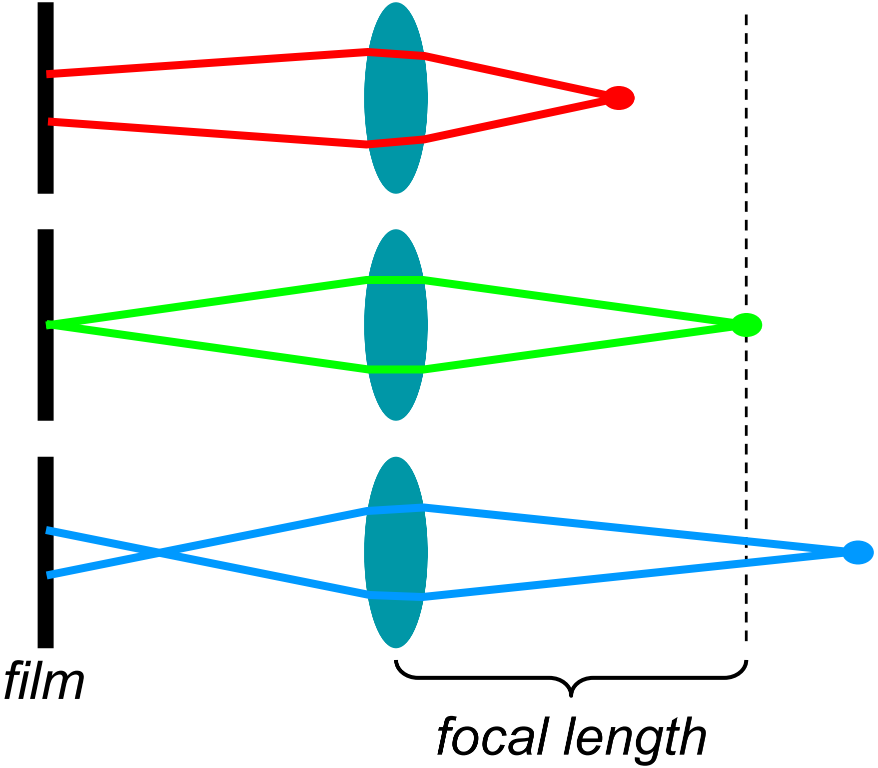


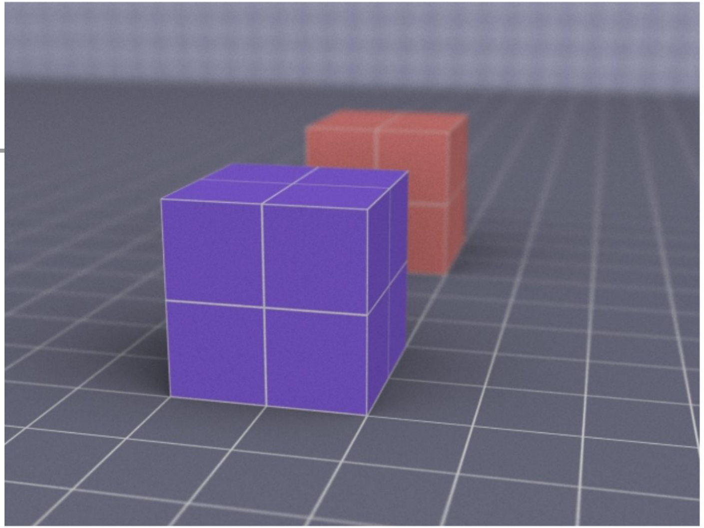
Image by Rob Cook

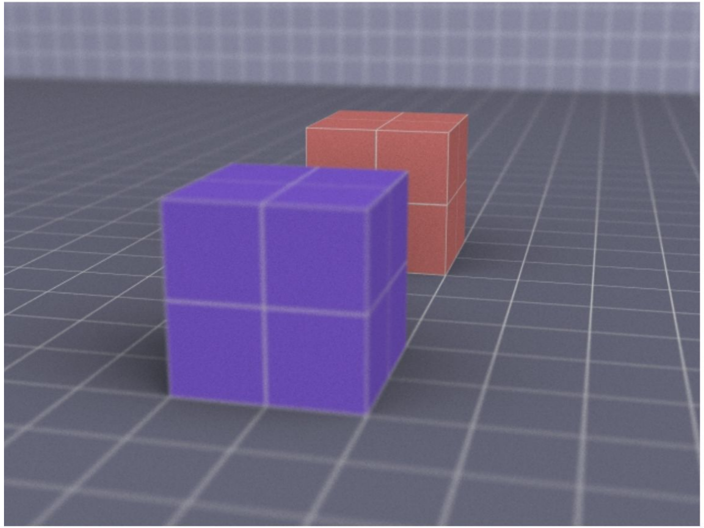
Depth of Field

Images by Justin Legakis

- multiple rays per pixel







Ray Tracing Algorithm Analysis

- Ray casting
- Lots of primitives
- Recursive
- Distributed Ray Tracing Effects
  - Soft shadows
  - Anti-aliasing
  - Glossy reflection
  - Motion blur
  - Depth of field

cost ≈ height * width *
num primitives *
intersection cost *
size of recursive ray tree *
num shadow rays *
num supersamples *
num glossy rays *
num temporal samples *
num focal samples *
. . .

can we reduce this?

these can serve double duty

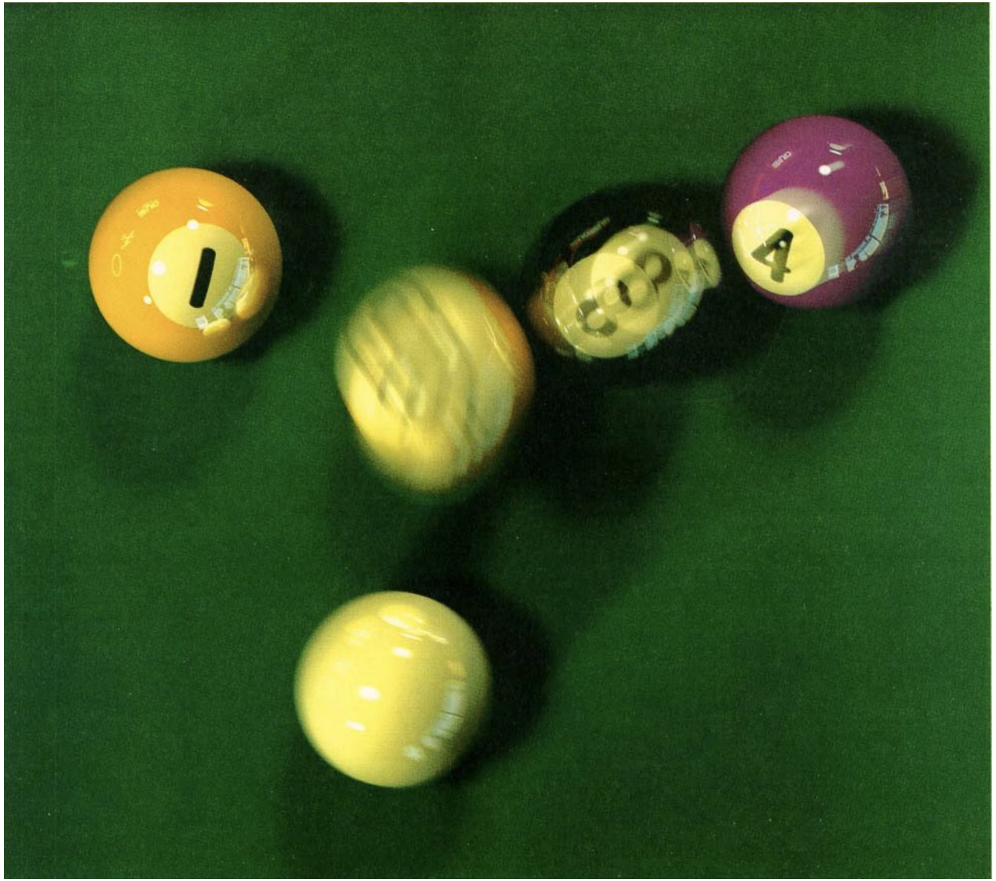
Today

- Reading for Today
- Ray Casting
- Ray Tracing
- Recursive Ray Tracing
- Distributed Ray Tracing
- Readings for Next Week

Reading for Next Time

Everyone should read this paper for HW3

"Distributed Ray Tracing", Cook, Porter, & Carpenter, SIGGRAPH 1984.



Reading for Next Time *(optional)*

"Measuring and Modeling Anisotropic Reflection",
Greg Ward,
SIGGRAPH 1992