## CSCI 4530/6530 Advanced Computer Graphics

`https://www.cs.rpi.edu/~cutler/classes/advancedgraphics/S25/`

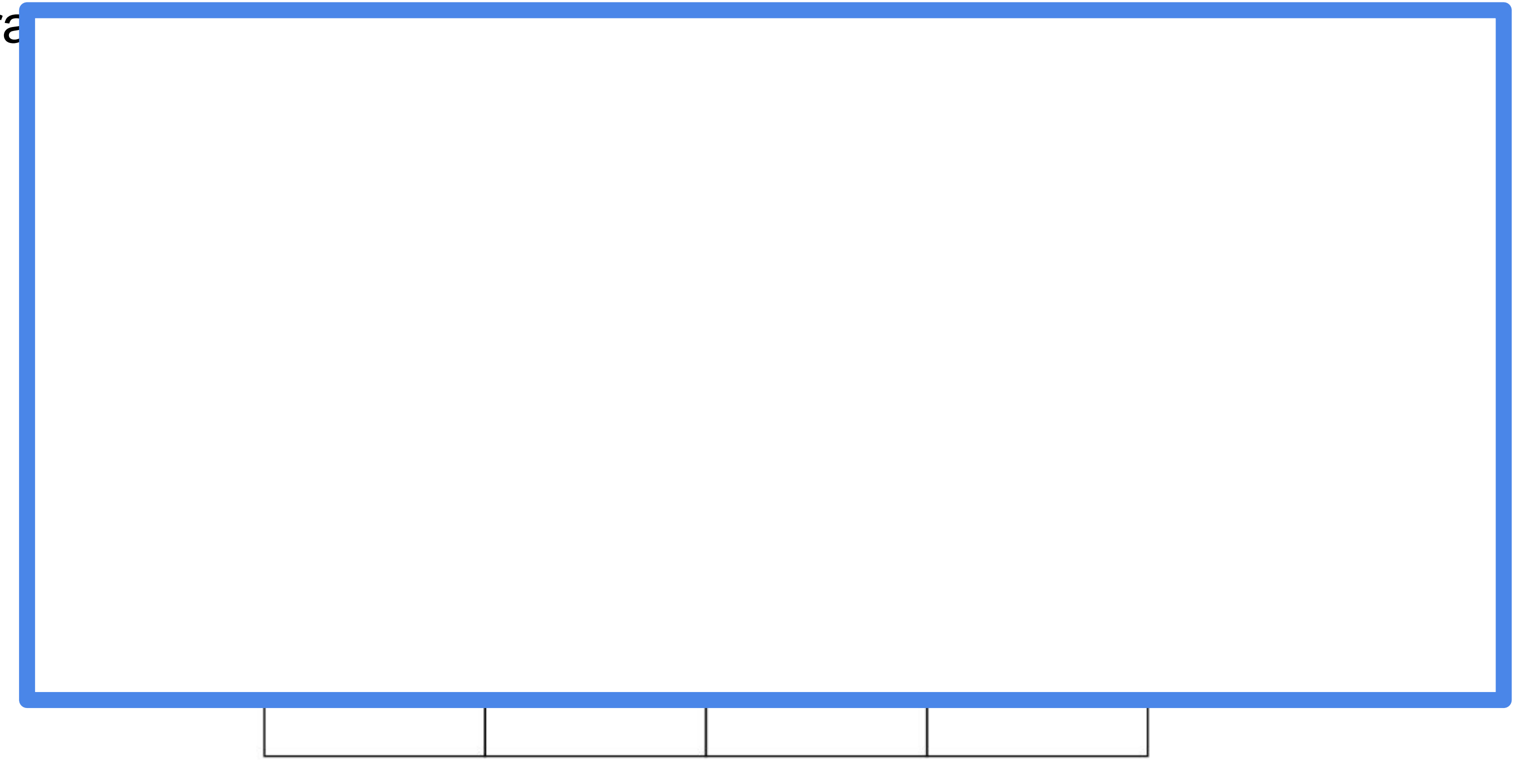
# Lecture 9: Fracture & Tetrahedral Models

# Worksheet: Fluid Velocity Interpolation

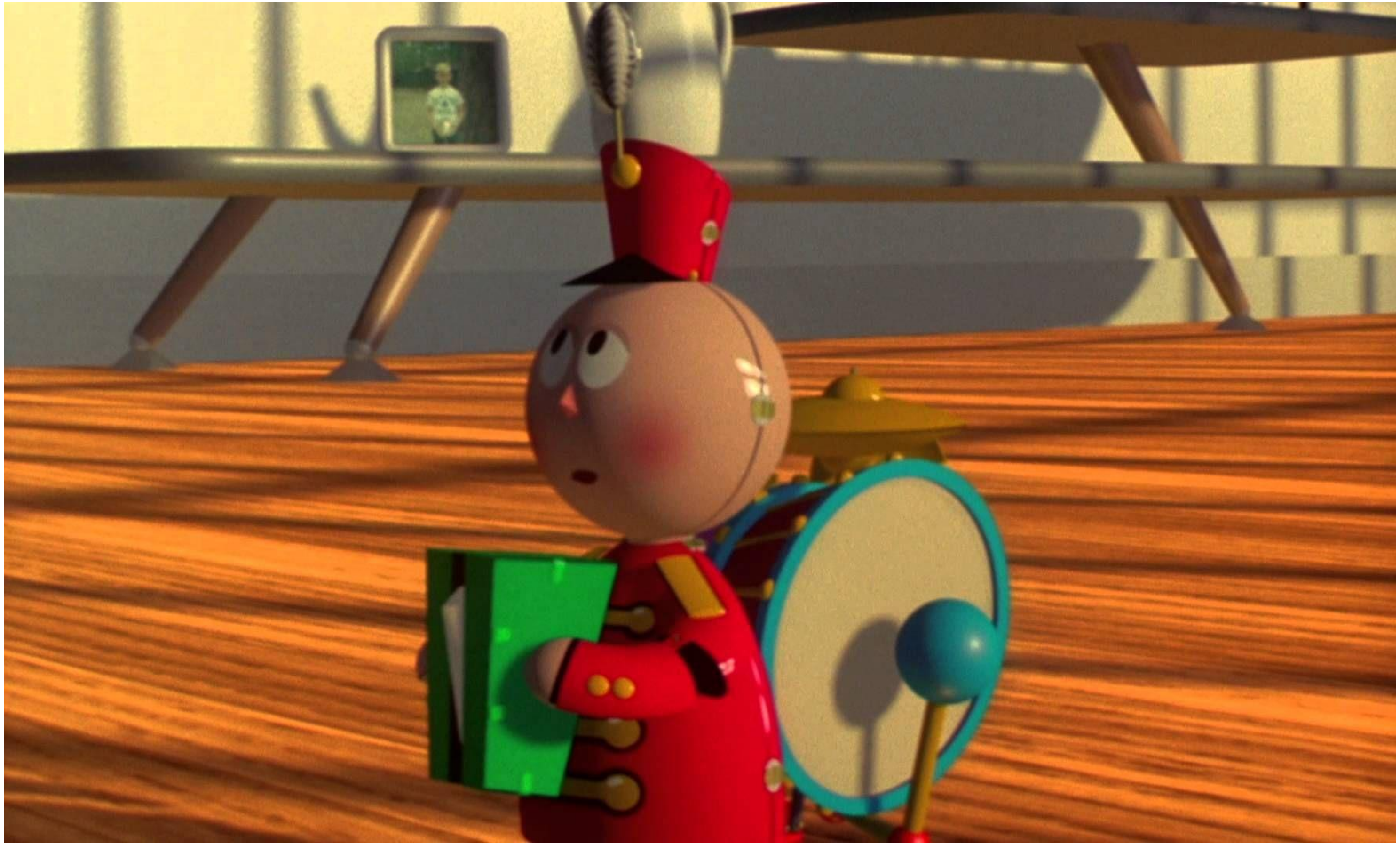
- Determine the inte 2D v (x = y = at th whit in th

# Worksheet: Fluid Incompressibility

- What are the horizontal and face velocities after 1, 2, and many itera



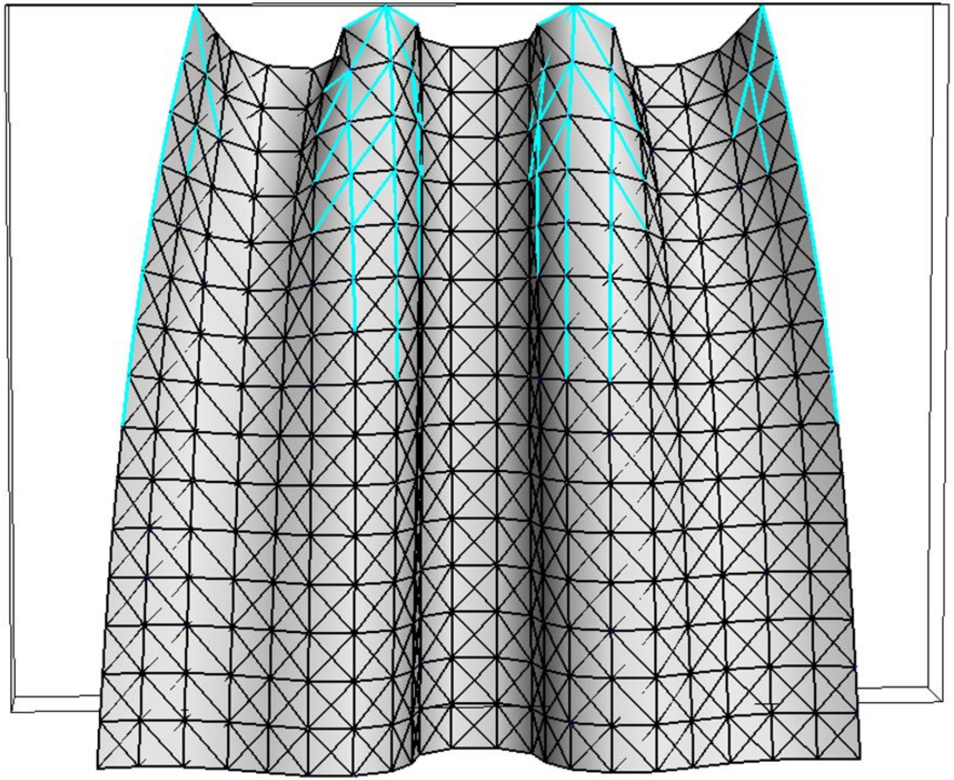
# Tin Toy, Pixar Animation Studios, 1988

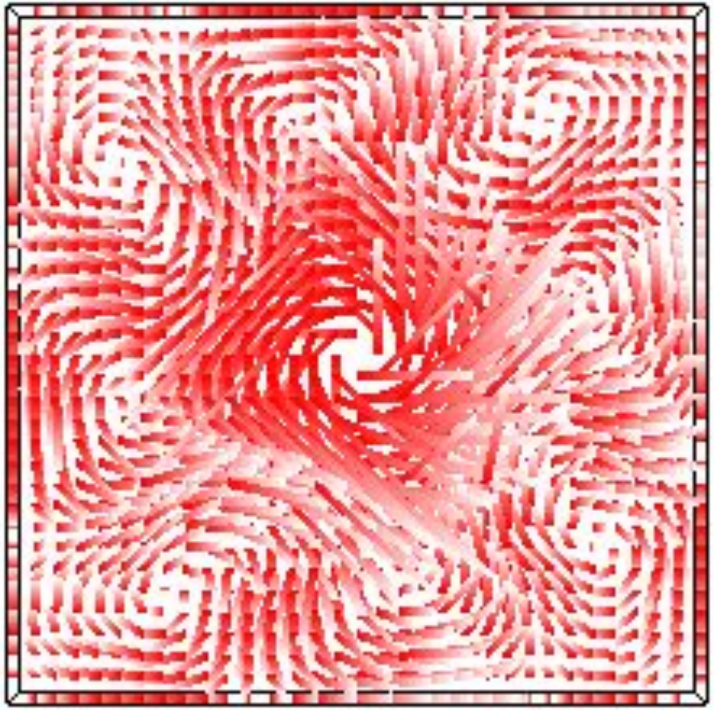


# Acura Bullet, The Mill, SIGGRAPH 2009

# HW2: Cloth & Fluid Simulation

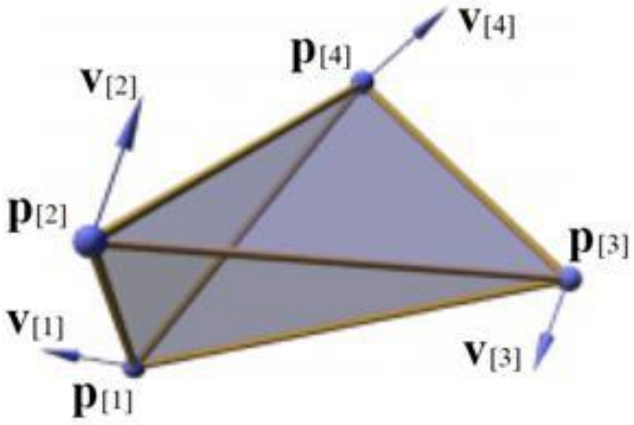
*HW2 deadline*
*Thursday @ 11:59pm*

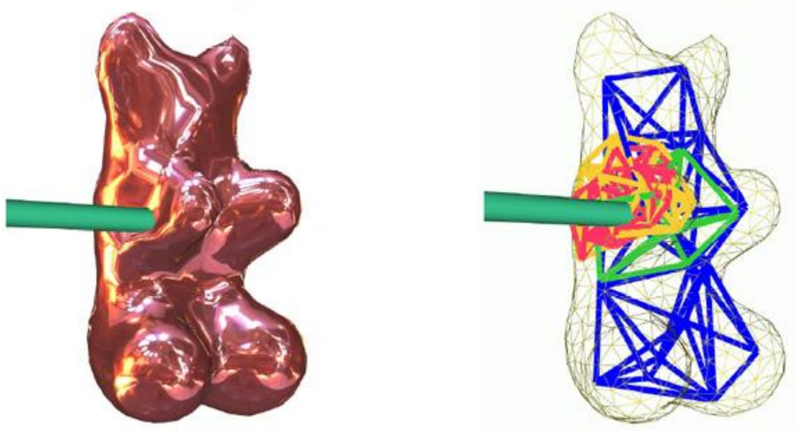


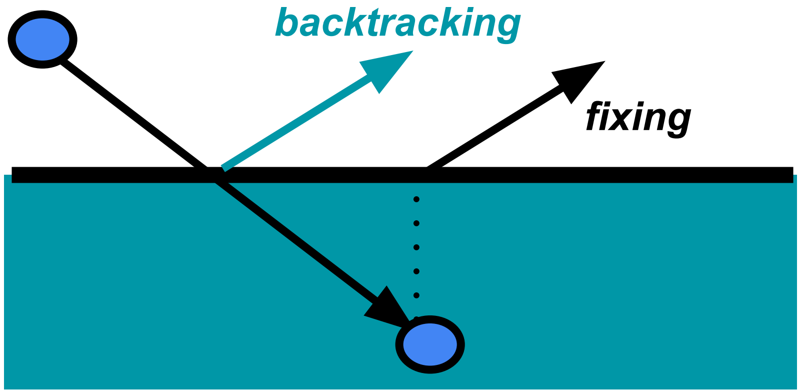


# Last Time?

- Rigid Body
- Collision Response
- Finite Element Method
  - Stress/Strain
- Deformation
  - Level of Detail





# Today

- Worksheet: Fluid Velocity & Incompressibility
- Continuing from Last Time…
  - Rigid Body Dynamics
  - Collision Response
  - Non-Rigid, Deformable Objects
  - Finite Element Method
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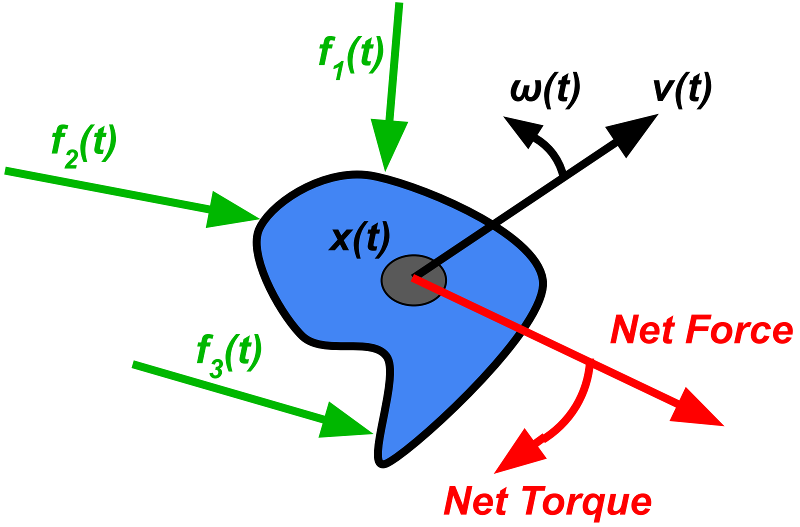
- Tetrahedral Element Quality
- Papers for Next Time

# Rigid Body Dynamics

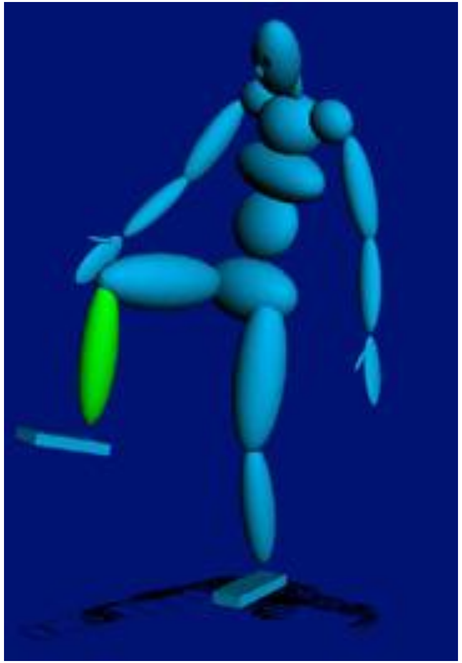
- How do we simulate this object's motion over time?
- We could discretize the object into many particles…
  - But a rigid body does *not* deform
  - Only a few *degrees of freedom*
- Instead, we use only one particle at the center of mass
  - Body has velocity ***v(t)*** *and* angular velocity ***ω(t)***
  - Compute net force & net torque



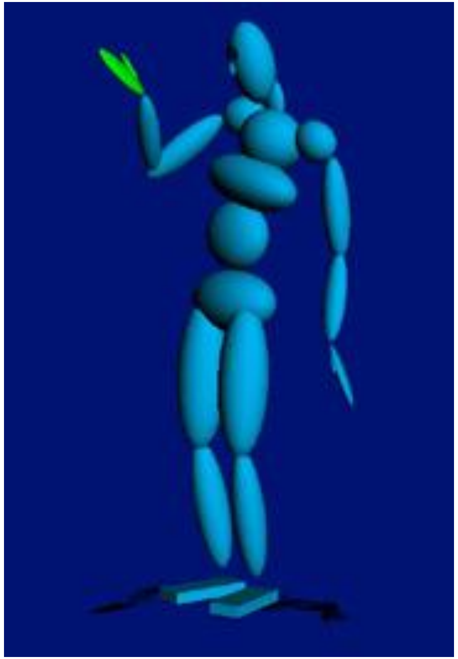

*Nice Reference Material: http://www.pixar.com/companyinfo/research/pbm2001/*

# Degree of Freedom (DOF)

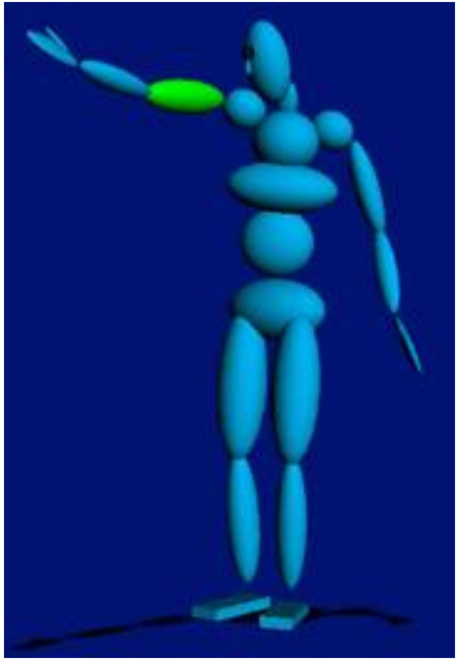
- Rotations:



1 DOF: knee



2 DOF: wrist

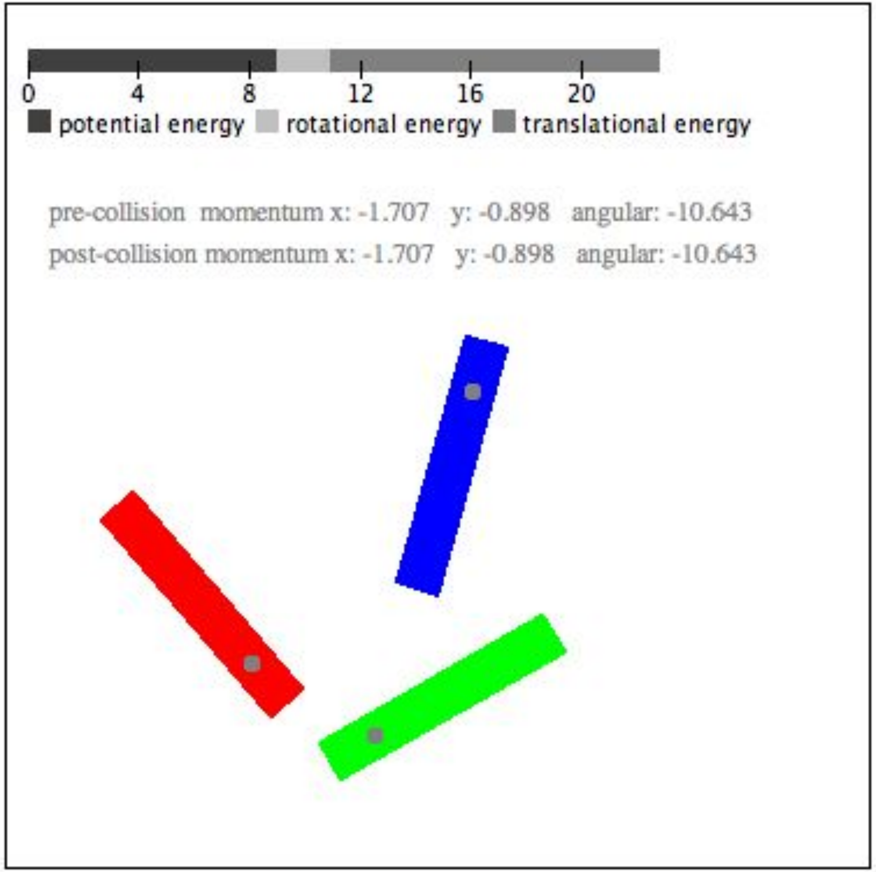


3 DOF: arm

- Translations count too… ➔ *6 Degrees of Freedom (DOF)*

# Energy & Rigid Body Collisions

- Total Energy = Kinetic Energy + Potential Energy + Rotational Energy
- Total Energy stays constant if there is no damping and no friction
- Rotational Energy is constant between collisions




*http://www.myphysicslab.com/collision.html*

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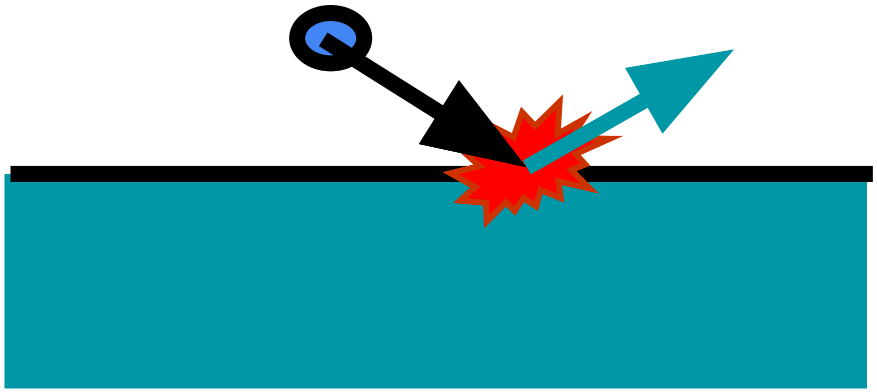
# Collisions

- Detection
- Response
- Overshooting problem
  (when we enter the solid)

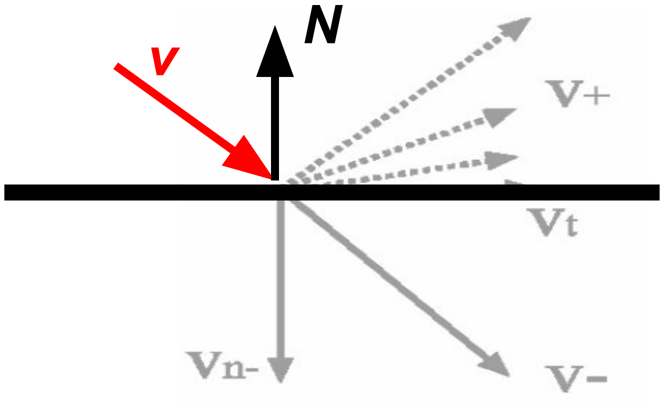


# Collision Response

- tangential velocity $\boldsymbol{v_t}$ unchanged
- normal velocity $\boldsymbol{v_n}$ reflects:

$$v = v_t + v_n$$

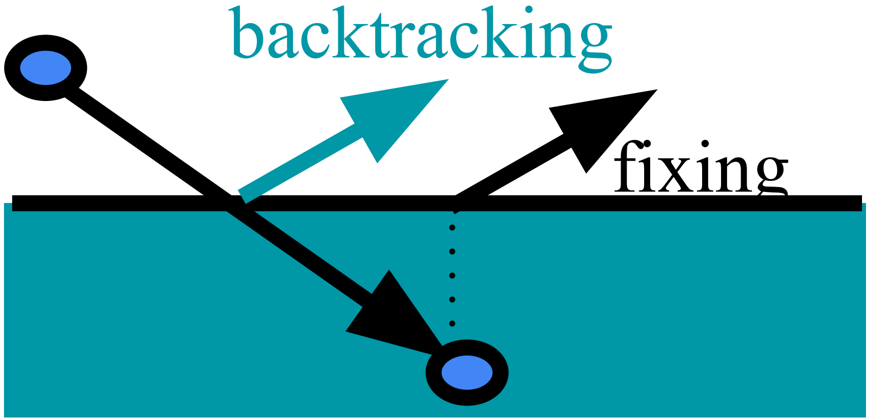
$$v \leftarrow v_t - \varepsilon v_n$$




- coefficient of restitution
  - 1 for elastic
  - 0 for plastic
- change of velocity = $-(1+\epsilon)v$
- change of momentum *Impulse* = $-m(1+\epsilon)v$

# Collisions - Overshooting

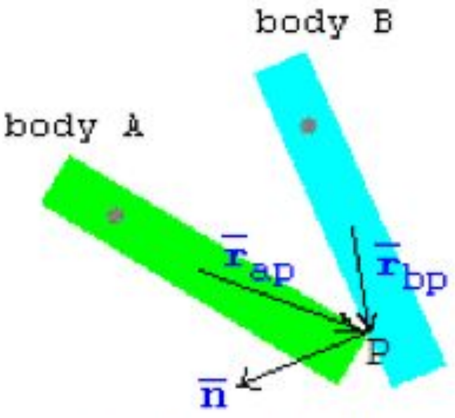
- Usually, we detect collision when it’s too late: we’re already inside
- Solutions: back up
  - Compute intersection point
  - Compute response there
  - Advance for remaining fractional time step
- Other solution:
  Quick and dirty fixup
  - Just project back to object closest point



# Collision Between Two Objects

- Suppose a vertex on body A is colliding into an edge of body B at point P. Define the following variables:

$m_a, m_b$ = mass of bodies A, B
$\bar{\mathbf{r}}_{ap}$ = distance vector from center of mass of body A to point P
$\bar{\mathbf{r}}_{bp}$ = distance vector from center of mass of body B to point P
$\omega_{a1}, \omega_{b1}$ = initial pre-collision angular velocity of bodies A, B
$\omega_{a2}, \omega_{b2}$ = final post-collision angular velocity of bodies A, B
$\bar{\mathbf{v}}_{a1}, \bar{\mathbf{v}}_{b1}$ = initial pre-collision velocities of center of mass bodies A, B
$\bar{\mathbf{v}}_{a2}, \bar{\mathbf{v}}_{b2}$ = final post-collision velocities of center of mass bodies A, B
$\bar{\mathbf{v}}_{ap1}$ = initial pre-collision velocity of impact point on body A
$\bar{\mathbf{v}}_{bp1}$ = initial pre-collision velocity of impact point on body B
$\bar{\mathbf{n}}$ = normal (perpendicular) vector to edge of body B
$e$ = elasticity (0 = inelastic, 1 = perfectly elastic)

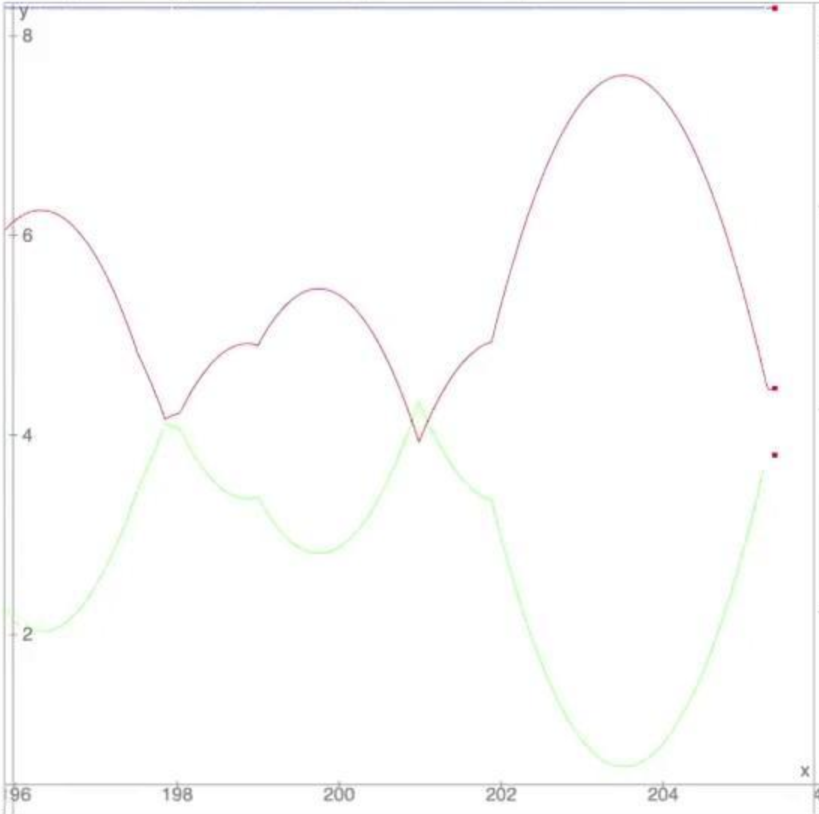


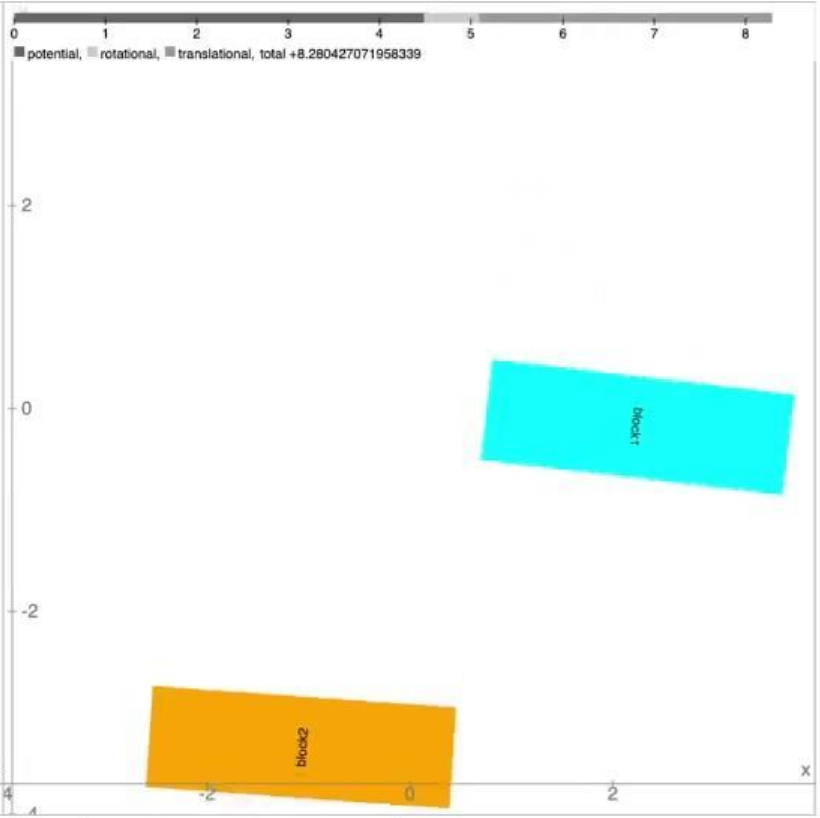
vectors involved in collision

*http://www.myphysicslab.com/collision.html*

Sim | Graph | Time Graph | Multi Graph

English | previous | next




green: kinetic energy

red: potential energy

blue: total energy

X: time

time window 10.0

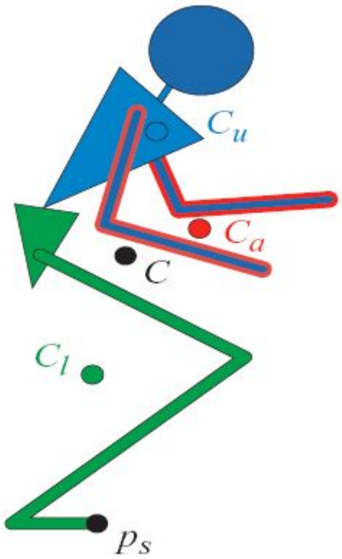
clear graph

pan-zoom

share

# Center of Mass & Moment of Inertia

- Center of Mass: mean location of all mass in the system
- Moment of Inertia: a measure of an object's resistance to changes to its rotation
- If a solid cylinder & a hollow tube have the same radius & the same mass, which will reach the bottom of the ramp first?

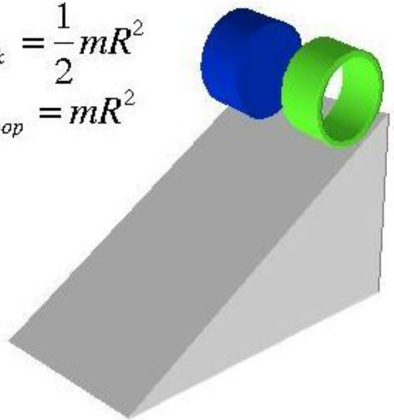



Liu & Popović

http://en.wikipedia.org/wiki/Fosbury_Flop

$$I_{disk} = \frac{1}{2} m R^2$$

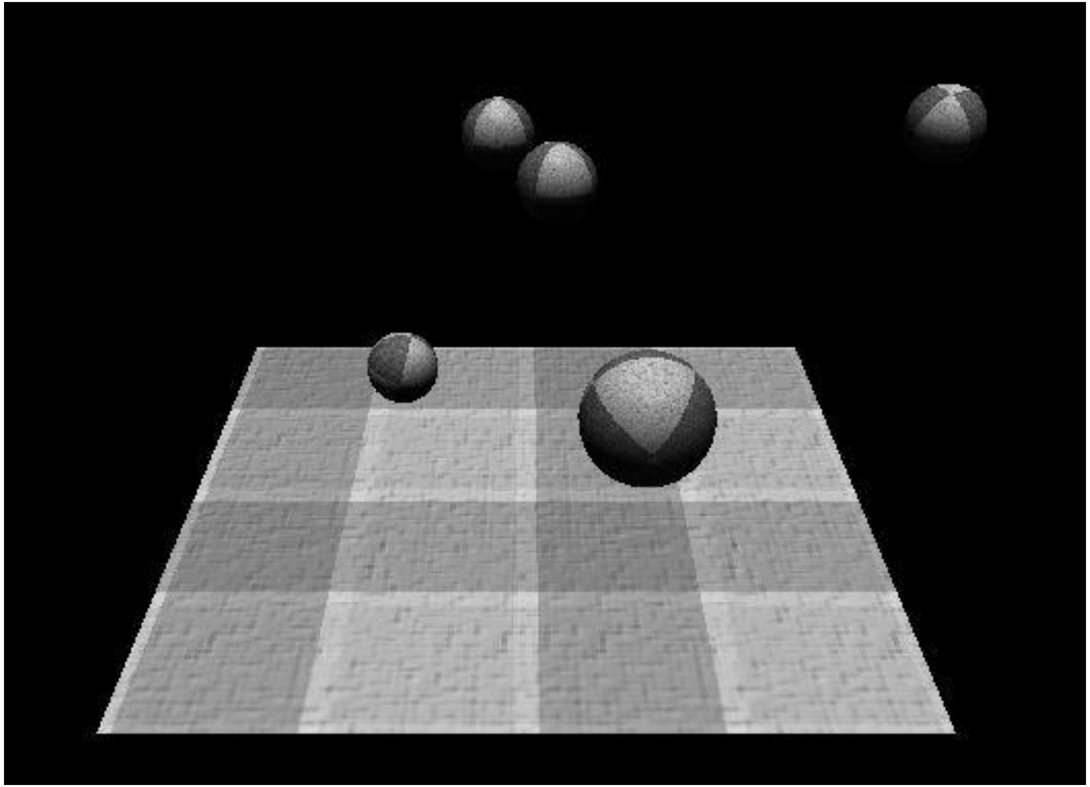
$$I_{hoop} = m R^2$$



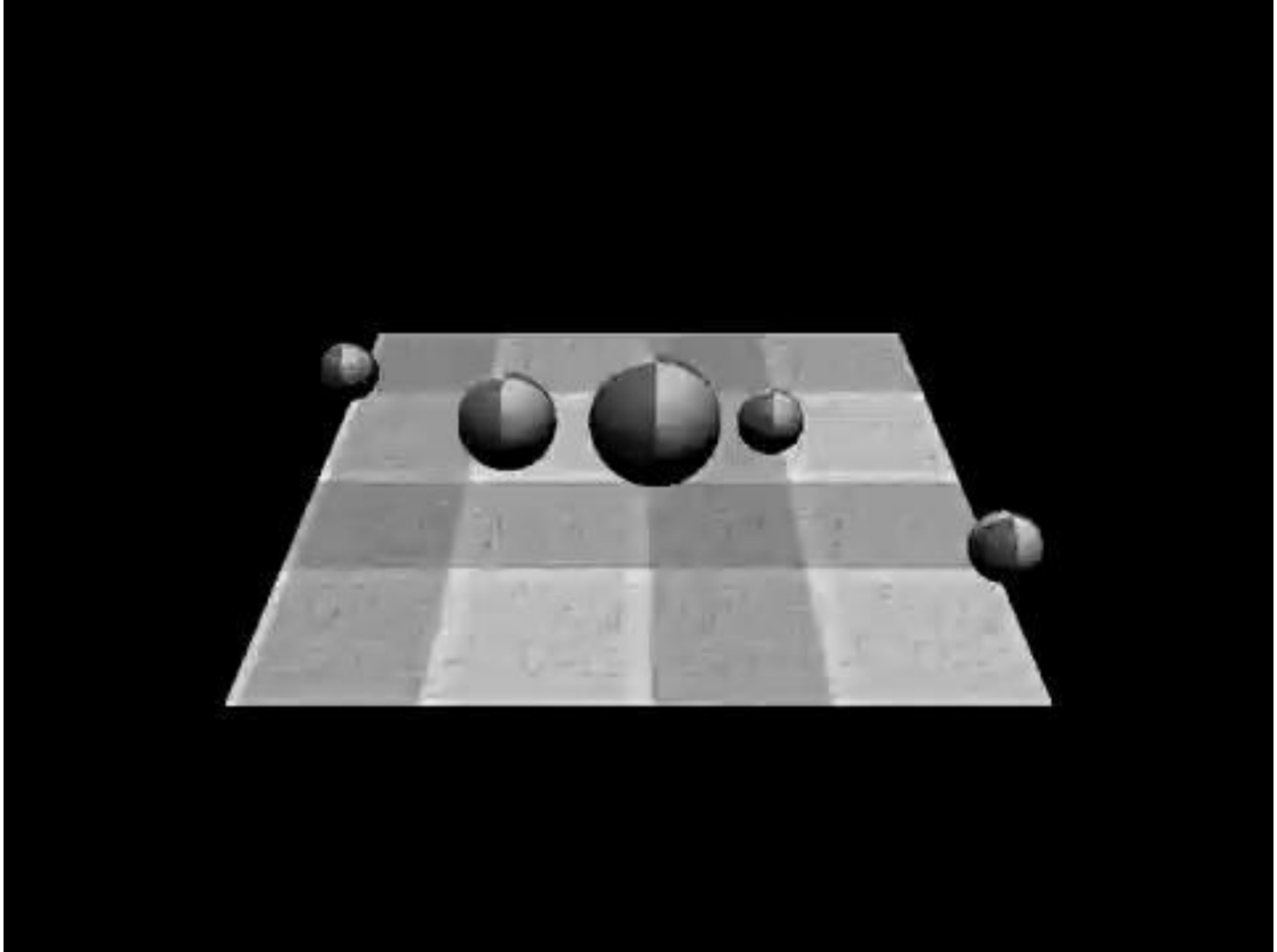
http://solomon.physics.sc.edu/~tedeschi/demo/demo12.html
http://hyperphysics.phy-astr.gsu.edu/hbase/hoocyl2.html

# Rigid Body Dynamics

- Physics
  - Velocity
  - Acceleration
  - Angular Momentum
- Collisions
- Friction

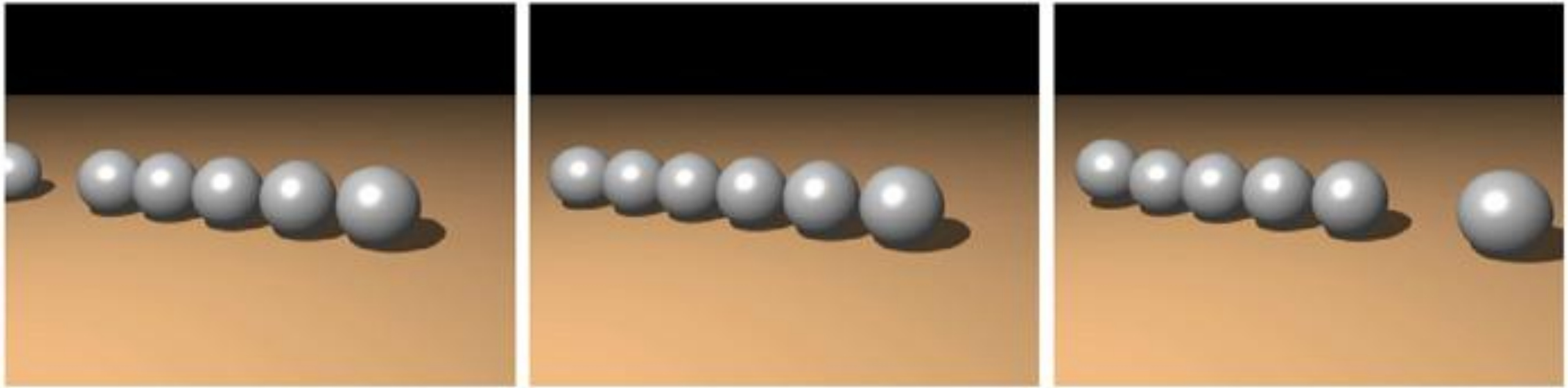


*from: Darren Lewis*
*http://www-cs-students.stanford.edu/~dalewis/cs448a/rigidbody.html*



# Advanced Collisions

- What about friction?
- What if the contact between two objects is not a single point?
- What if more than two objects collide simultaneously?

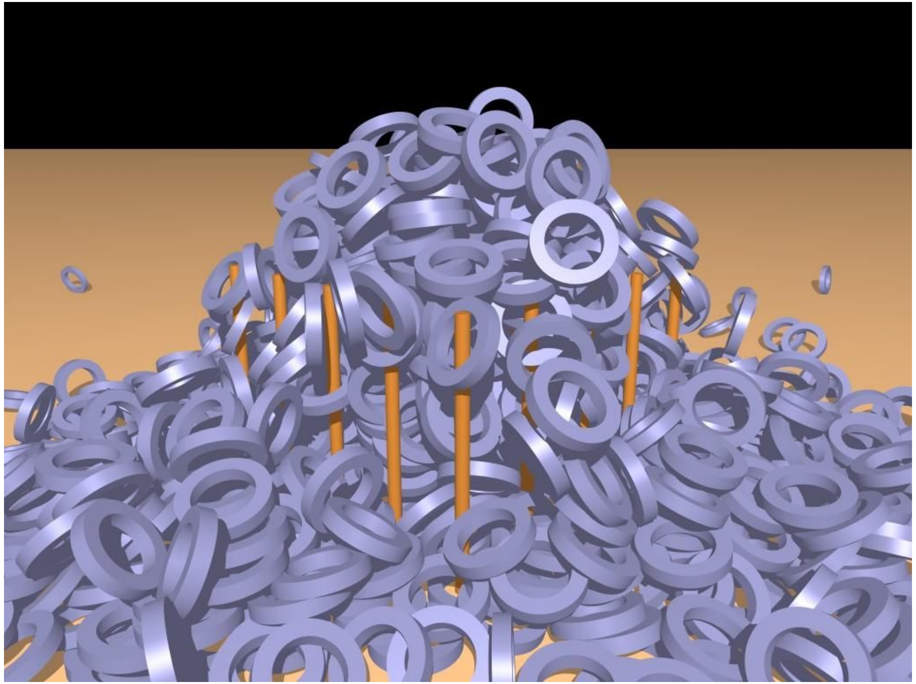


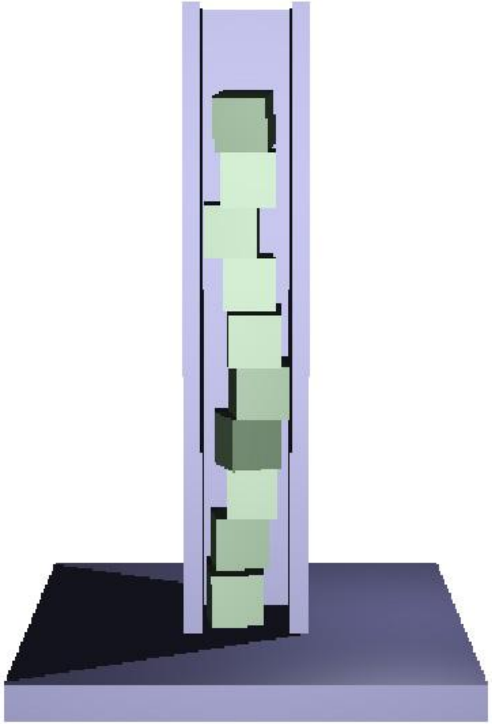
*Guendelman, Bridson & Fedkiw*
*Nonconvex Rigid Bodies with Stacking*
*SIGGRAPH 2003*

# Resting Collisions

*Victor J. Milenkovic & Harald Schmidl*
*Optimization-Based Animation*
*SIGGRAPH 2001*

- We know how to simulate bouncing really well
- But resting collisions are harder to manage





*Guendelman, Bridson & Fedkiw*
*Nonconvex Rigid Bodies with Stacking,*
*SIGGRAPH 2003*

# Today

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# Simulation of Non-Rigid Objects

- We modeled string & cloth using mass-spring systems.
  Can we do the same?

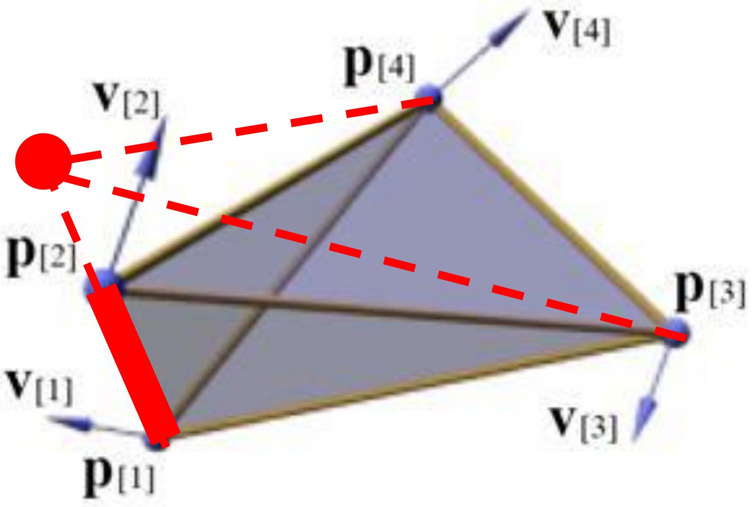
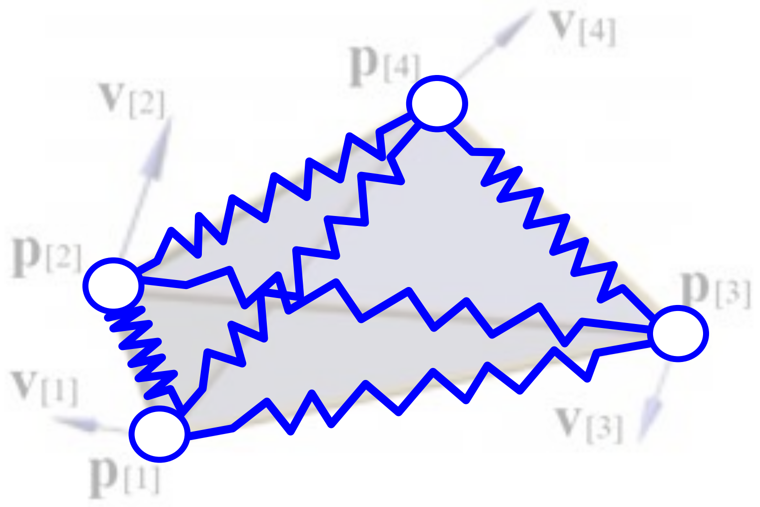


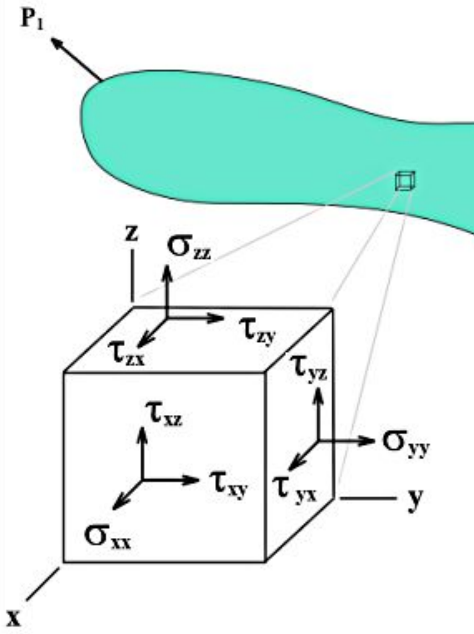

*Image from O'Brien et al. 1999*




- Yes… But a more physically accurate model uses *volumetric elements*

# Strain & Stress

- Stress
  - the internal distribution of forces within a body that balance and react to the loads applied to it
  - *normal stress* & *shear stress*
- Strain
  - material deformation caused by stress.
  - measured by the change in length of a line or by the change in angle between two lines



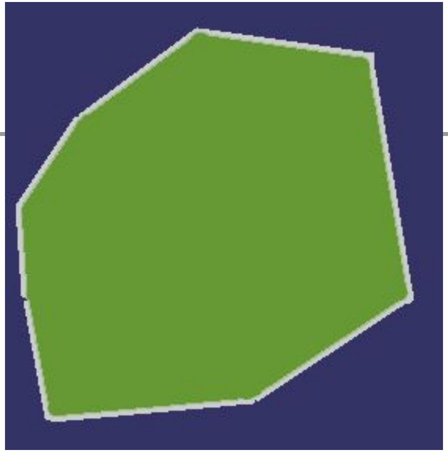

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

$$\varepsilon = \frac{\Delta_l}{l_0}$$

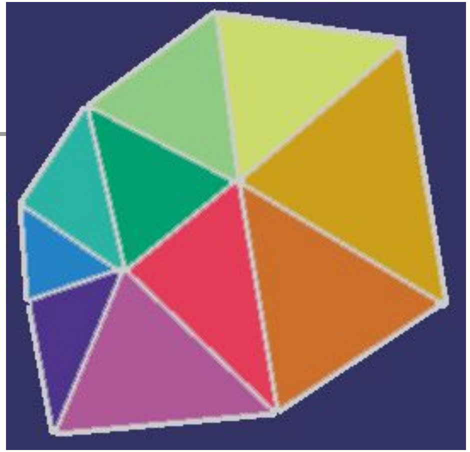
*http://en.wikipedia.org/wiki/Image:Stress_tensor.png*

# Finite Element Method

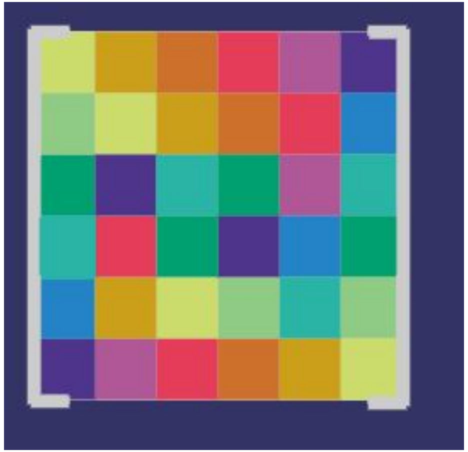
- To solve the continuous problem (deformation of all points of the object)
  - Discretize the problem
  - Express the interrelationship
  - Solve a big linear system
- More principled than Mass-Spring



*object*



*finite elements*



*large matricial system*

*Diagram from Debunne et al. 2001*

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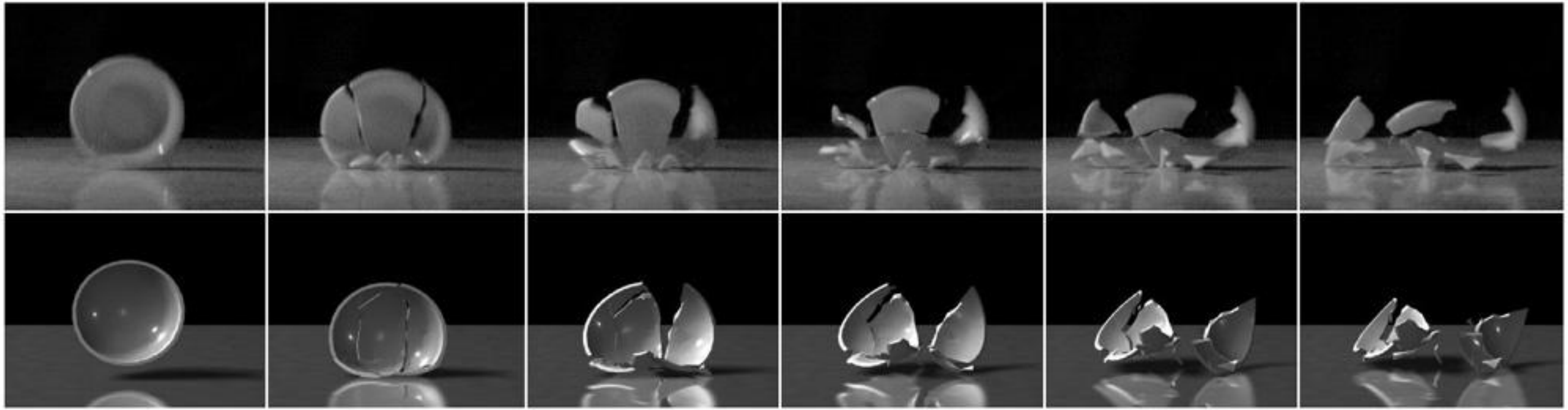
# Readings for Tuesday *(pick one)*

- James O'Brien & Jessica Hodgins "*Graphical Modeling and Animation of Brittle Fracture*" SIGGRAPH 1999.



- Fracture threshold
- Remeshing
  - need connectivity info!
- Material properties
- Parameter tuning

- Creating tetrahedron to fill interior of object
- Fracture along mesh or subdivide elements along fracture plane?
  - Dynamic remeshing
- Fracture propagation, cracks cannot(do not generally?) cross each other
- Lots of science & math, but maybe not enough implementation detail?
- Slow/offline runtime, but impressive & accurate results
- Comparison to high speed footage of object fracture is impressive
- Used an explicit solver, wouldn’t implicit give more accurate results?
- Noted limitations, but only very brief explanation/justification of choices
- Only works on homogenous (isotropic) materials

# Fracture Opening Modes

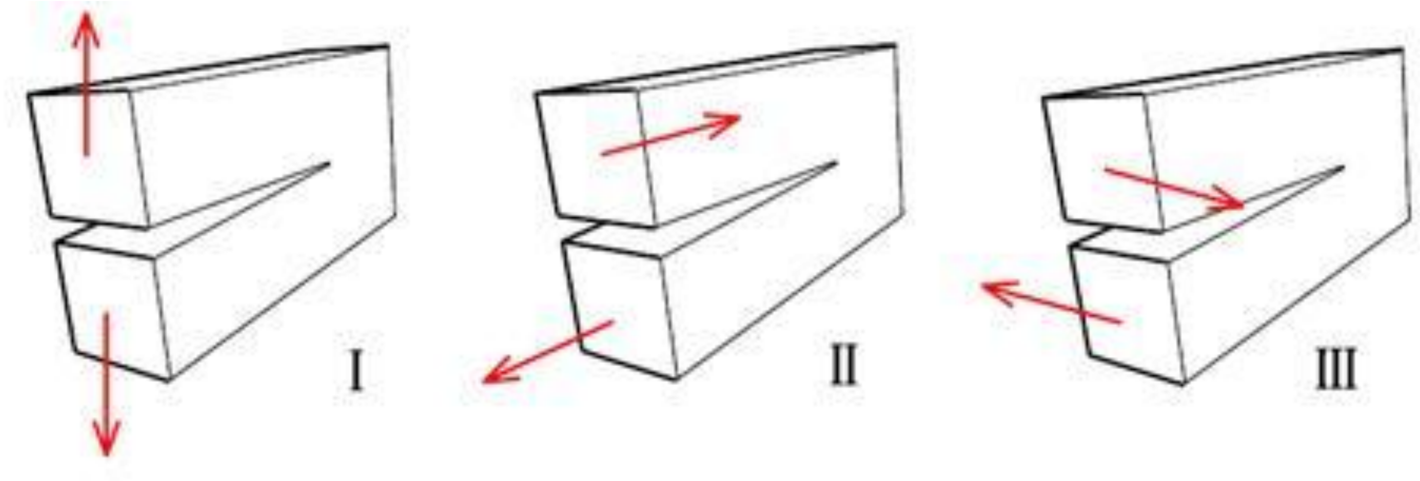
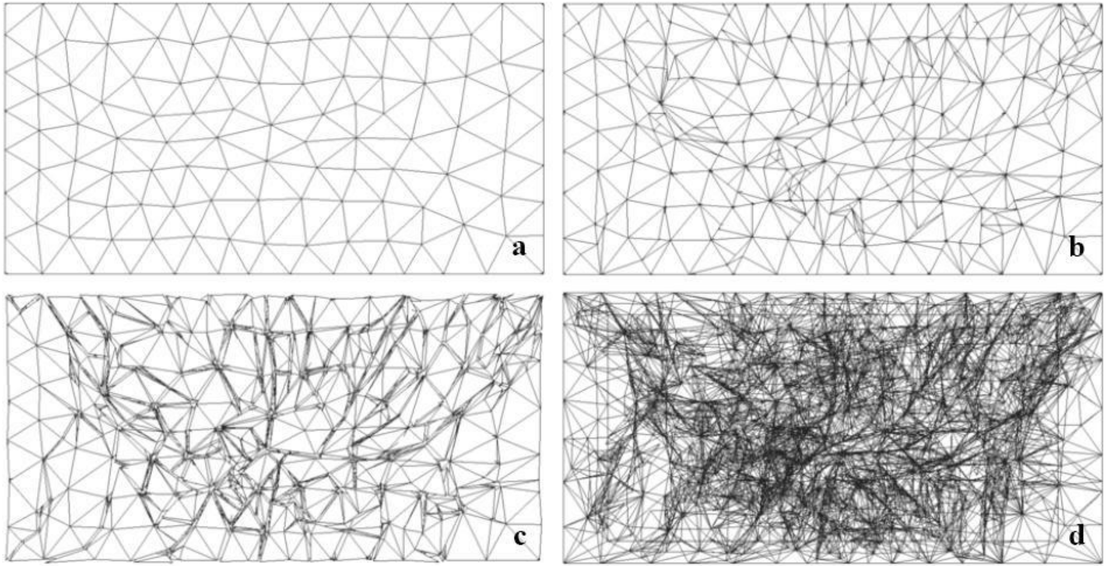
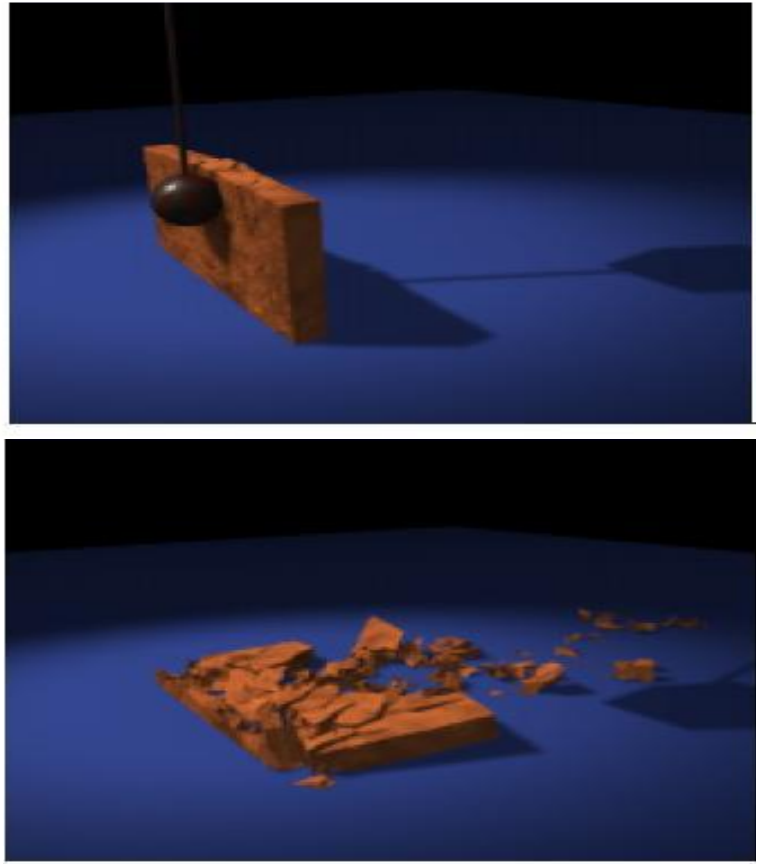



Figure 6: Three loading modes that can be experienced by a crack. Mode I: Opening, Mode II: In-Plane Shear, and Mode III: Out-of-Plane Shear. Adapted from Anderson [1].

*Image from O'Brien et al. 1999*

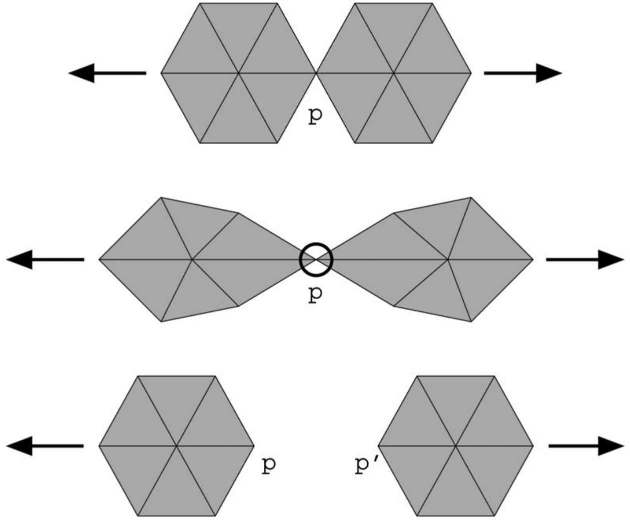
# Local Mesh Refinement

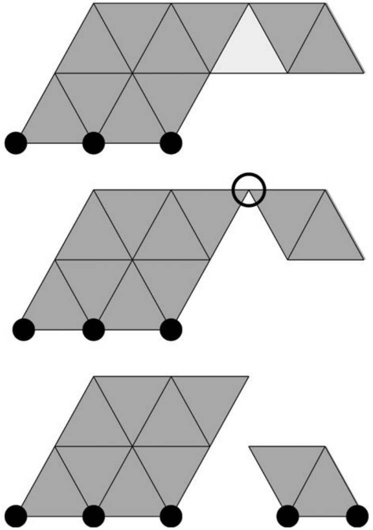


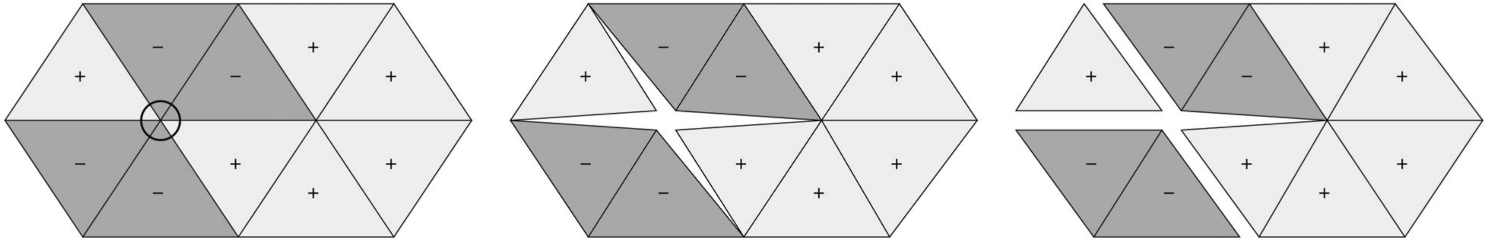



*Images from O'Brien et al. 1999*

# Managing Fracture Adjacency





*Cutler 2003*

# Fracture Propagation Difficulties

- Need to track direction of fracture propagation?

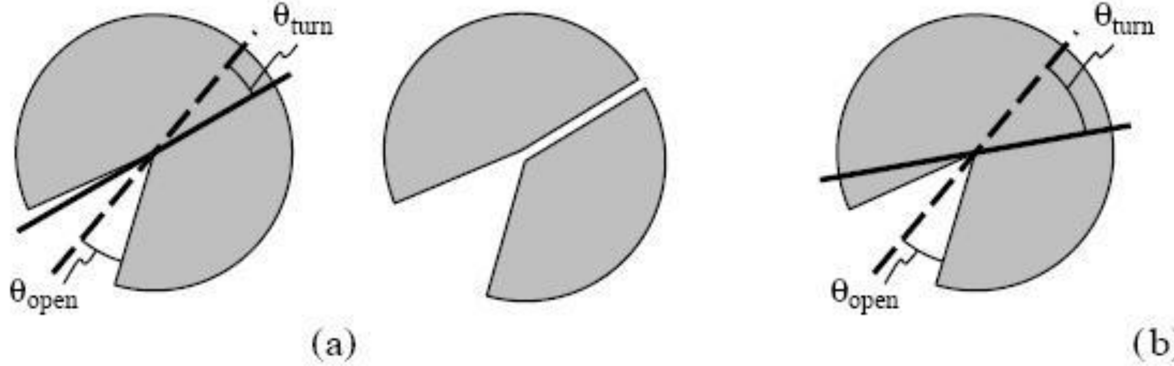
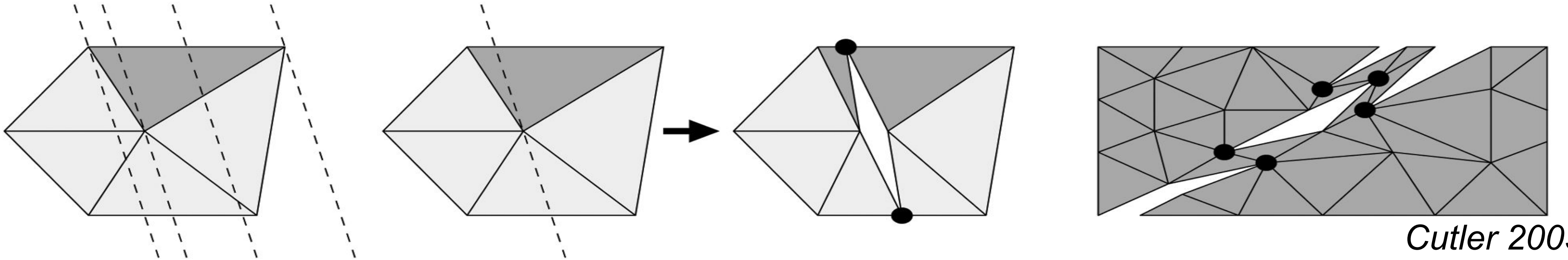



*Image from O'Brien et al. 1999*

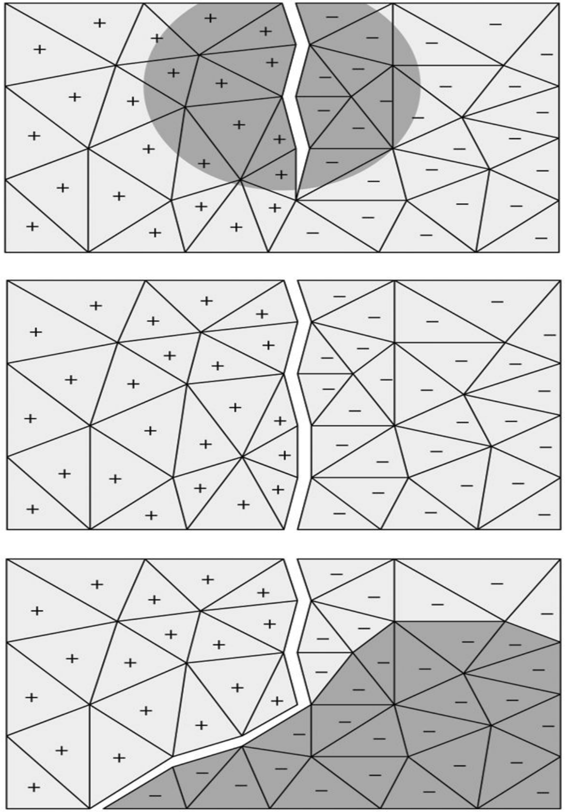
- Need to track crack tip?



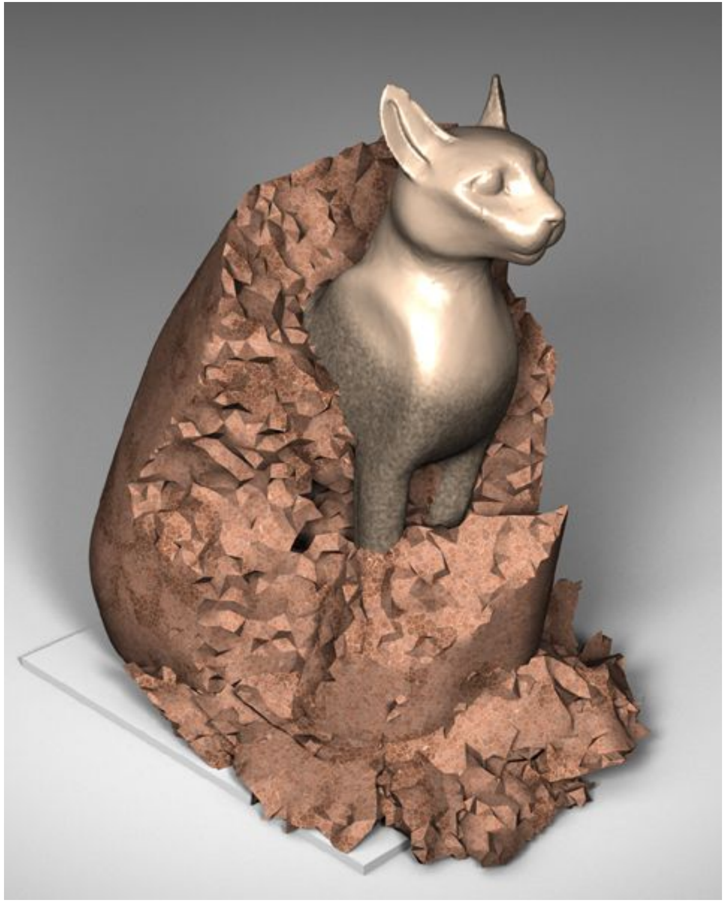
*Cutler 2003*

# Controlling Speed of Propagation

*Procedural Authoring of Solid Models, Cutler 2003*

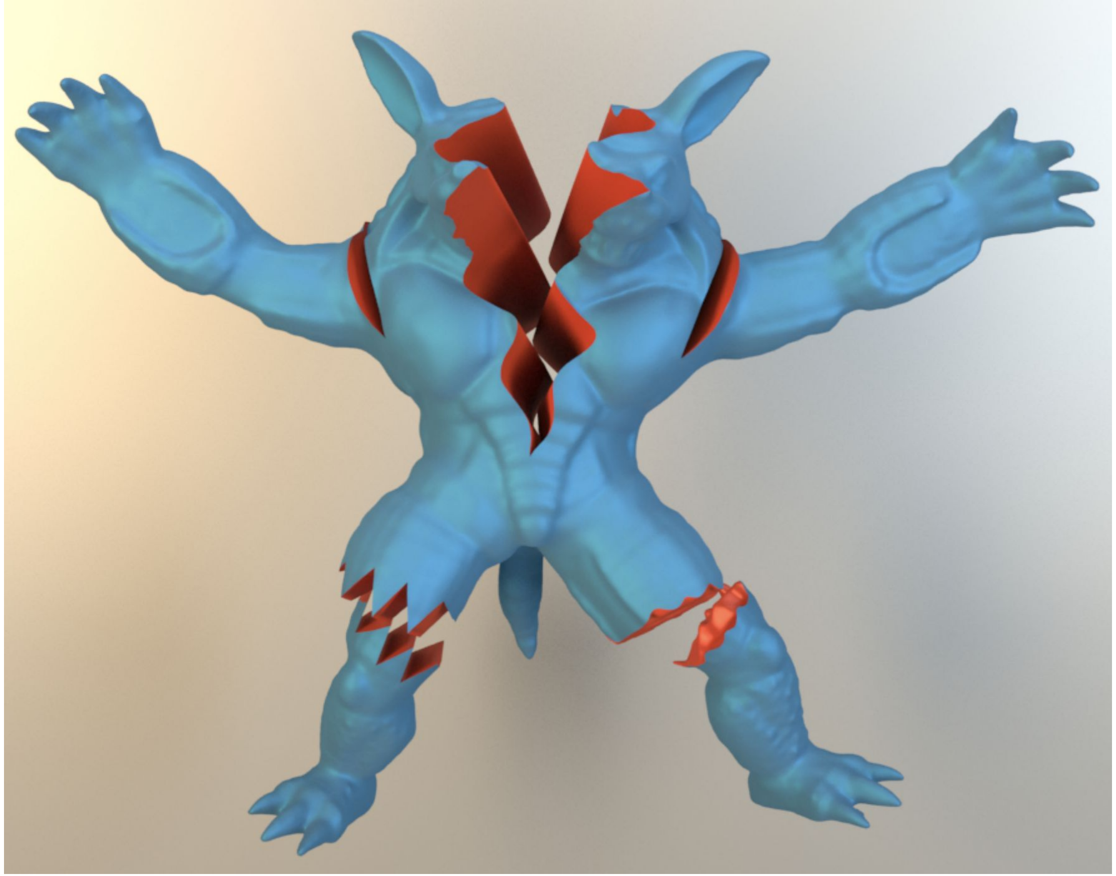


*(no remeshing)*



# Readings for Tuesday *(pick one)*

- "Robust eXtended Finite Elements for Complex Cutting of Deformables", Koschier, Bender, & Thuerey, SIGGRAPH 2017



- Lots of paper space dedicated to comparison with prior techniques
- Non-planar cuts
- Can it be used for hollow objects?
- Paper titles, acronyms, importance of good names

# Readings for Tuesday *(pick one)*

- "Multi-species simulation of porous sand and water mixtures", Pradhana, Gast, Klar, Fu, Teran, Jiang, and Museth, SIGGRAPH 2017.

- Lots of prior knowledge in multiple domains and familiarity with prior work expected to understand this paper & the contributions
- Intuitive to have 2 grids / 2 simulations, interact with each other
- Very complex problem, requires proper attention to subtle physical properties to mimic real-world behaviors
- Good writing style, a paragraph was included to describe each equation
- Choosing an appropriate mathematical model to reproduce a particular physical phenomena seems to be an art – not strictly science!

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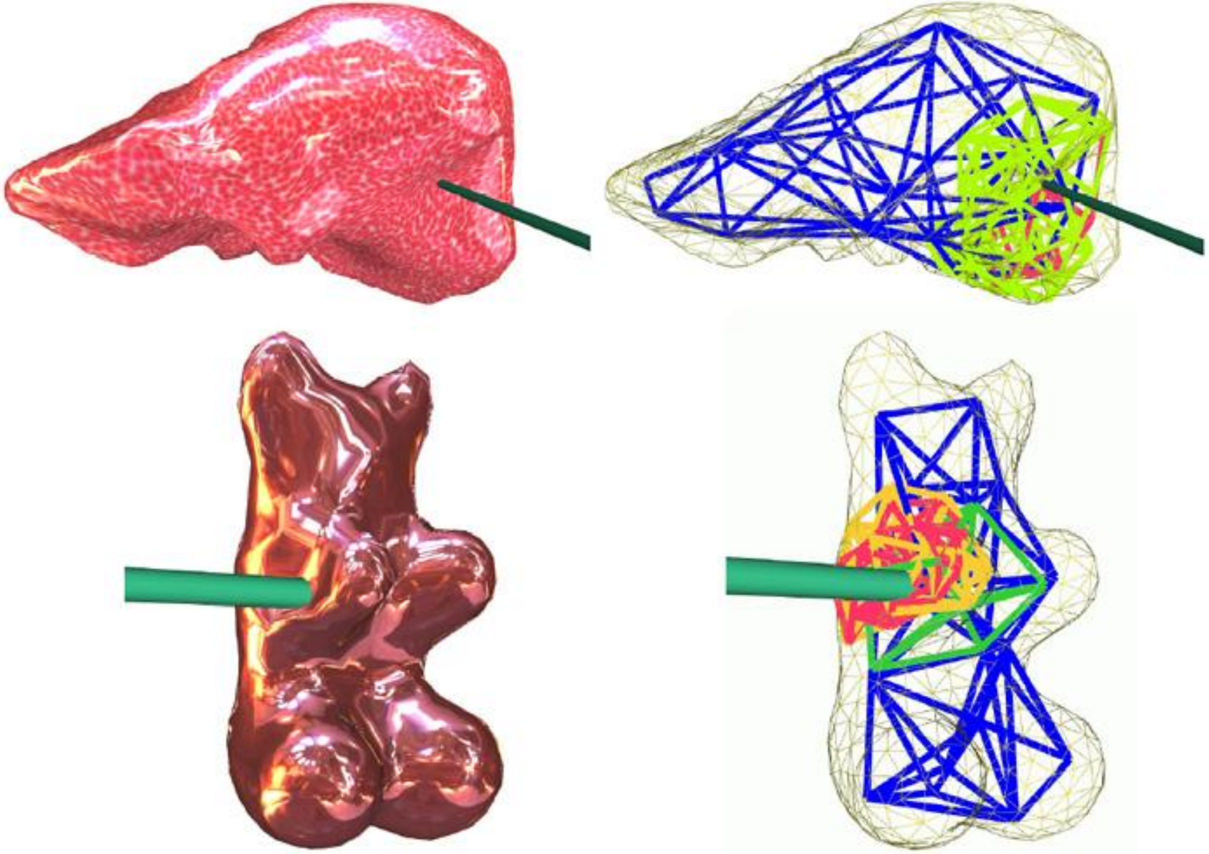
# “Dynamic Real-Time Deformations using Space & Time Adaptive Sampling” Debunne, Desbrun, Cani, & Barr, SIGGRAPH 2001

- Level of Detail
- Interactive shape deformation

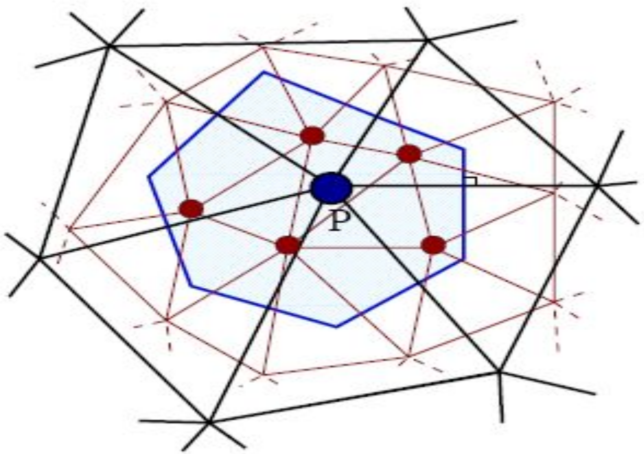


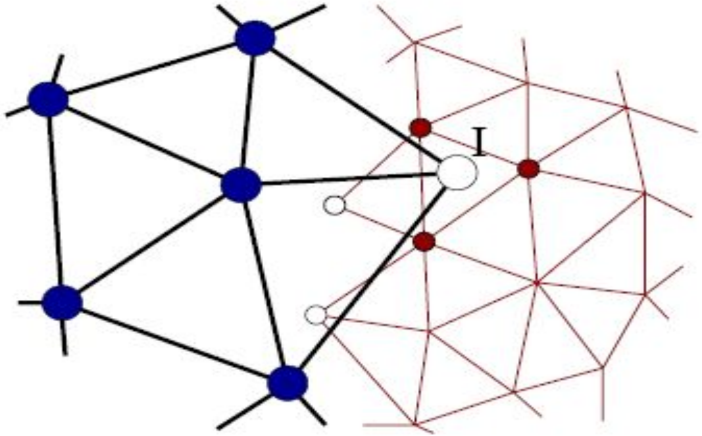
- Use high-resolution model only in areas of extreme deformation

# Multi-Resolution Deformation

*Debunne, Desbrun, Cani, & Barr, SIGGRAPH 2001*

- Use Voronoi diagrams to match parent & child vertices.
- Interpolate values for inactive interface vertices from active parent/child vertices

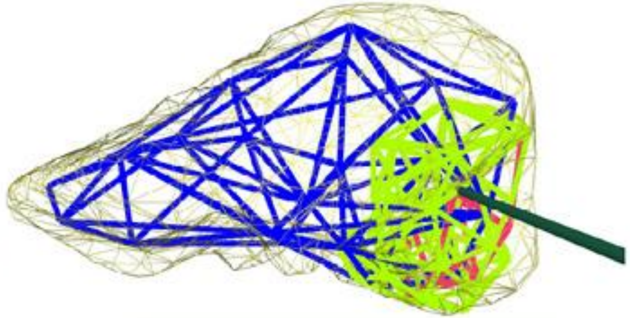


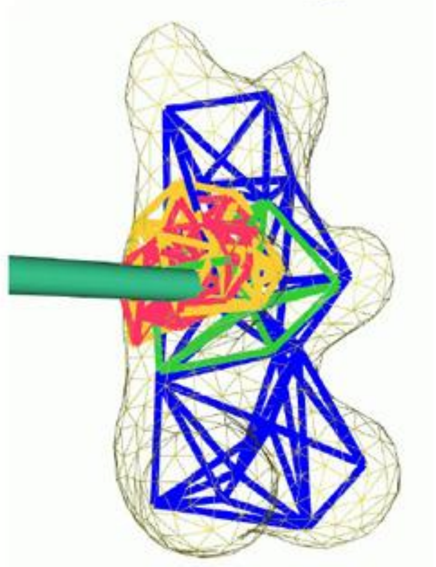

- *Need to avoid interference of vibrations between simulations at different resolutions*

# Pre-Computation & Simulation

- FEM matrix pre-computed
- Level of detail coupling pre-computed for rest topology



- *Limitation: Not appropriate for applications that need to change connectivity of elements E.g.:*
    - *Cloth that is cut or torn*
    - *Surgery simulation*



# Today

- Worksheet: Fluid Velocity & Incompressibility
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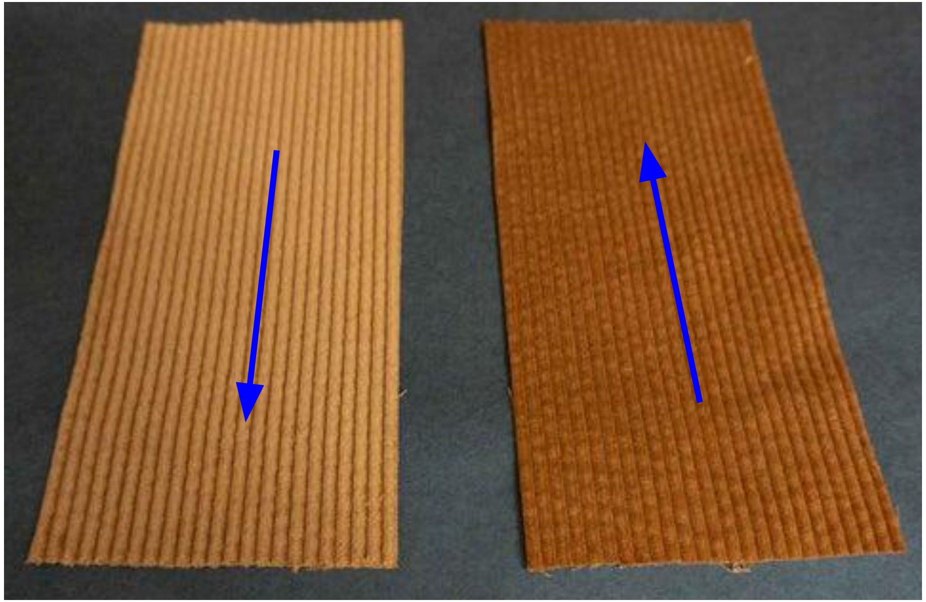
# Miscellaneous Definitions

- *Isotropic*: is a property which does not depend on the direction.

- *Anisotropic*: is a property which is directionally dependent.

*wood grain will impact strength & appearance*

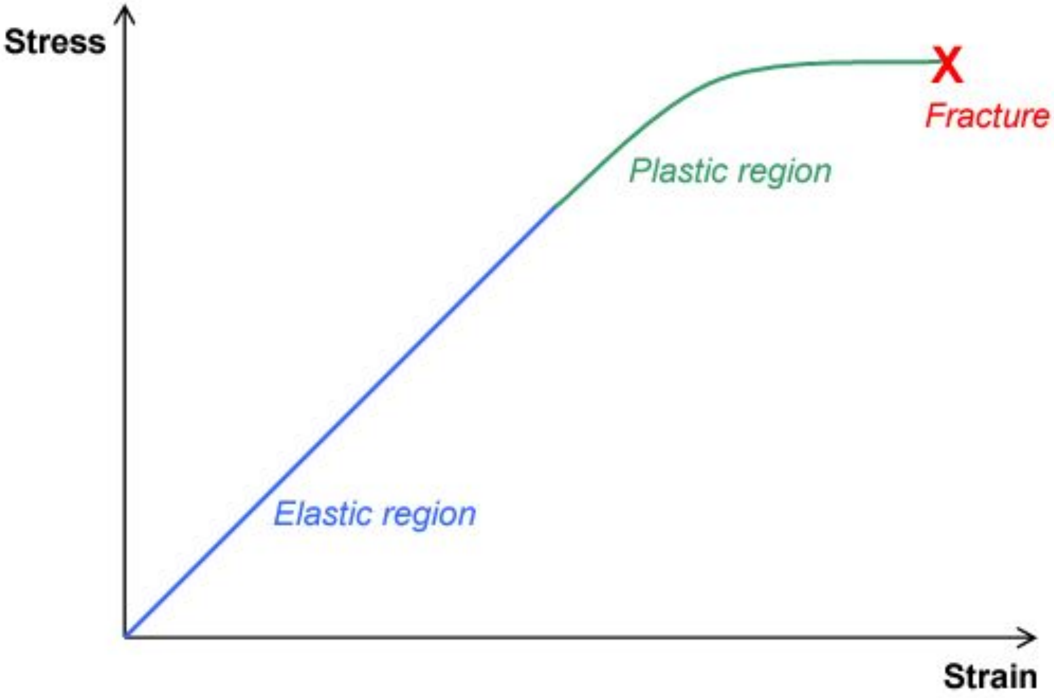


*Same corduroy fabric!*
*Just oriented with nap rotated 180°!*

*https://missmaudesewing.co.nz/blogs/miss-maude-musings-1/sewing-with-corduroy-fabrics*

# Miscellaneous Definitions

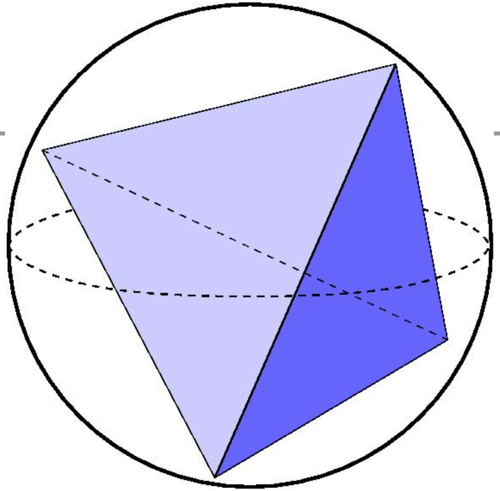
- *Elastic Deformation*: Once the forces are no longer applied, the object returns to its original shape.
- *Plastic Deformation*: An object in the plastic deformation range will first have undergone elastic deformation, which is reversible, so the object will return part way to its original shape.

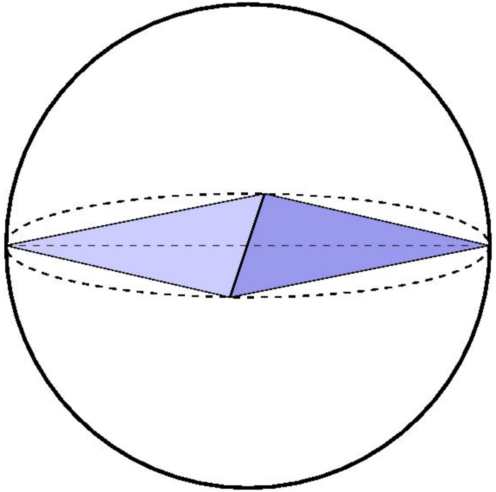


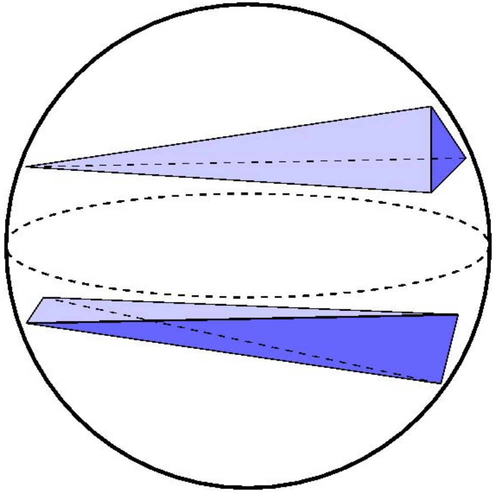

http://en.wikipedia.org/wiki/Image:Stress-strain1.png

# Miscellaneous Definitions

- *Degenerate/Ill-conditioned Element*:
  a.k.a. how “equilateral” are the elements?
  - Ratio of volume$^2$ to surface area$^3$
  - Smallest *solid* angle
  - Ratio of volume to volume of smallest circumscribed sphere

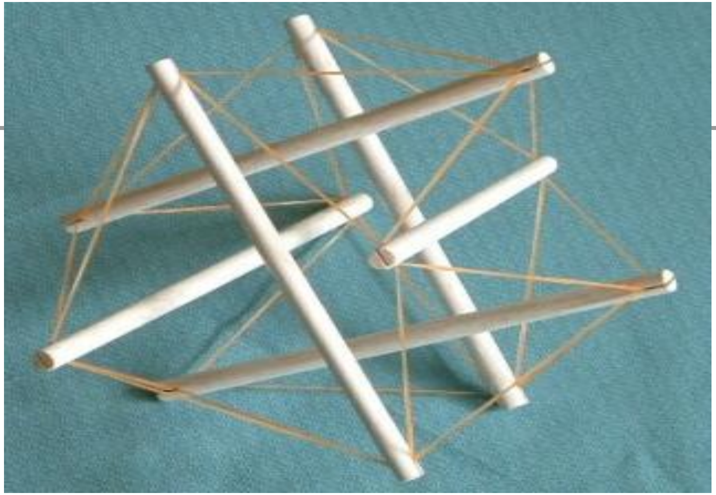






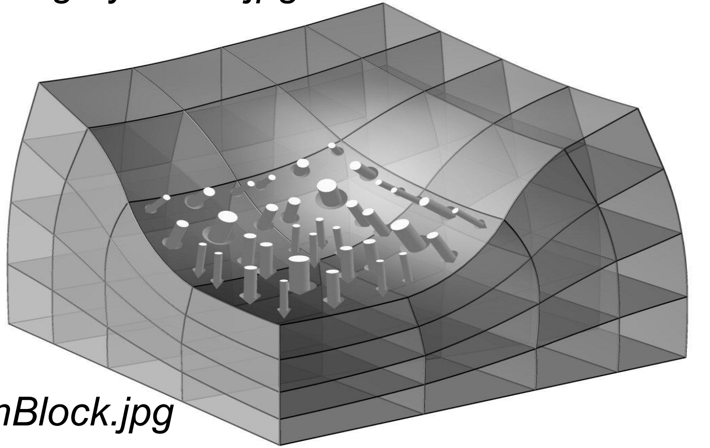
# Miscellaneous Definitions

- *Tension*: The direction of the force of tension is parallel to the string, away from the object exerting the stretching force.



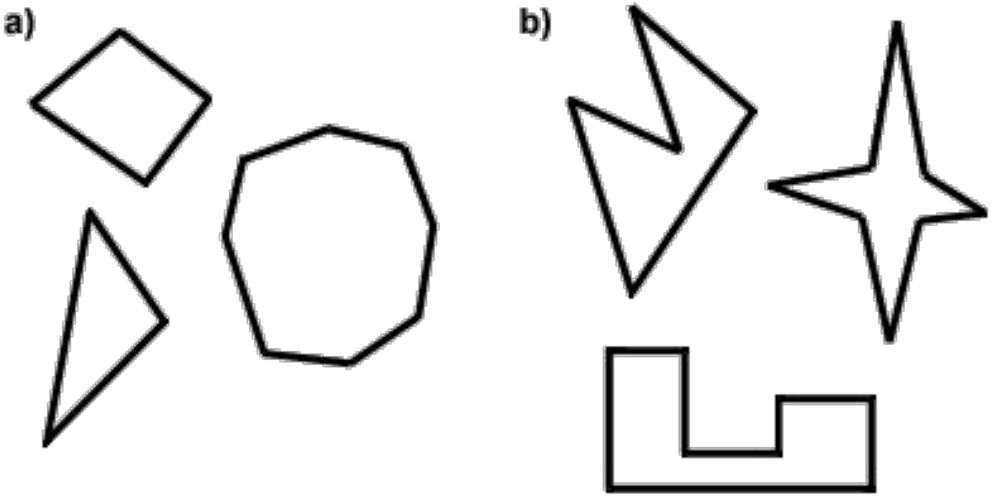
*http://fig.cox.miami.edu/~cmallery/255/255chem/tensegrity.sticks.jpg*

- *Compression*: resulting in reduction of volume

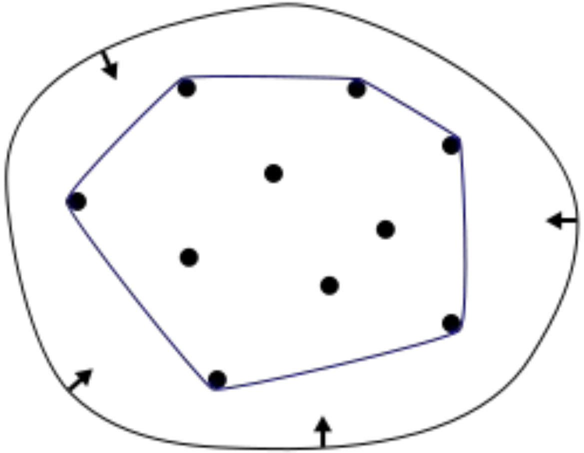


*http://www.aero.polimi.it/~merlini/SolidMechanics-FiniteElasticity/CompressionBlock.jpg*

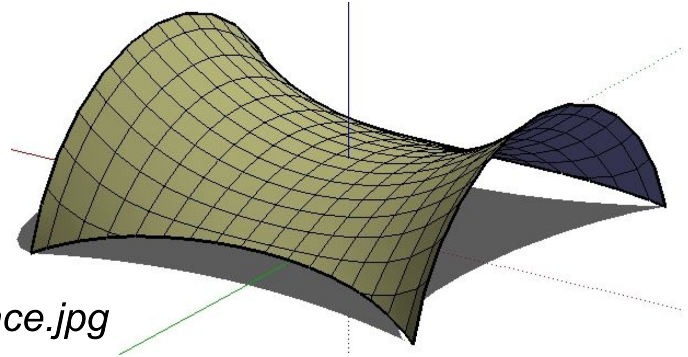
# Miscellaneous Definitions: Convex vs. Non-Convex




*http://img.sparknotes.com/figures/B/b333d91dc e2882b2db48b8ad670cd15a/convexconcave.gif*



*http://en.wikipedia.org/wiki/File:ConvexHull.svg*



*http://www.tensile-structures.de/Bilder/SaddleSurface.jpg*

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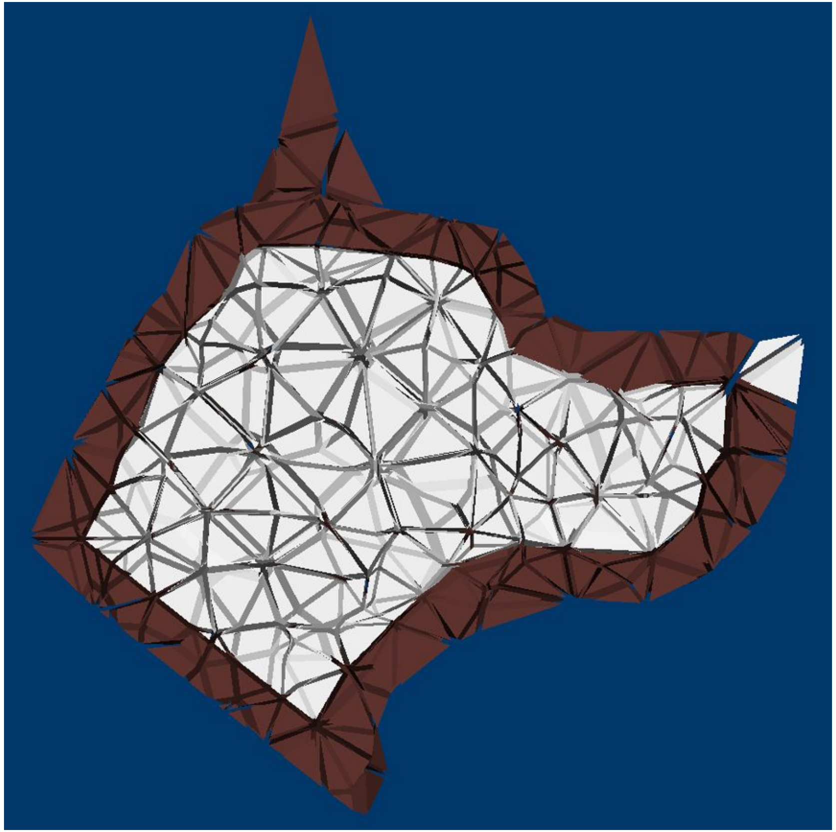
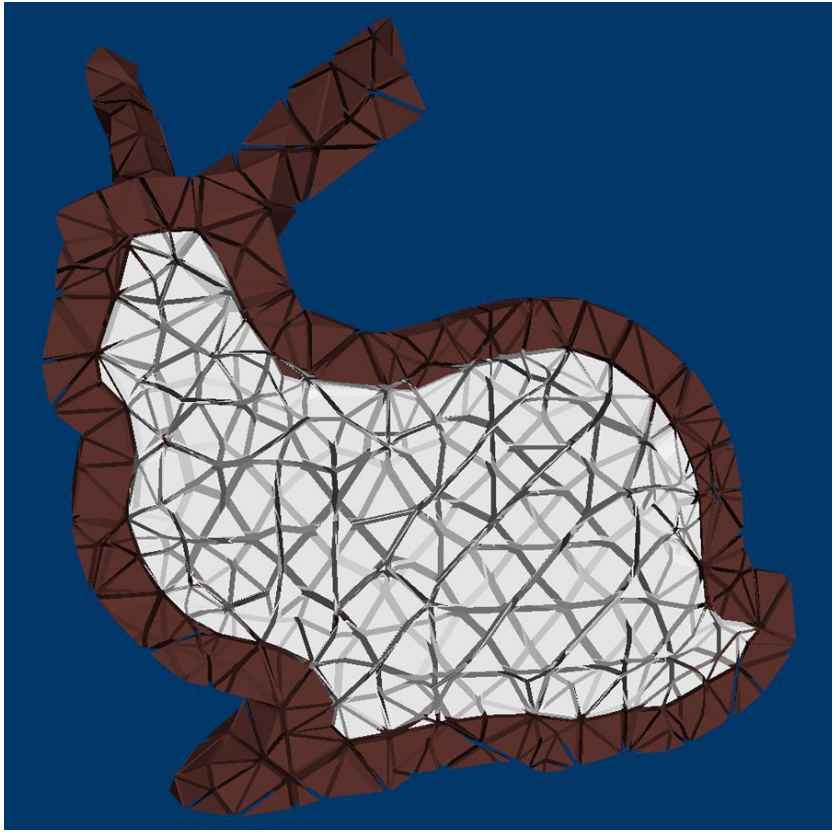
# Multiple Materials

Mueller, Dorsey, McMillan, Jagnow, & Cutler
*Stable Real-Time Deformations*
Symposium on Computer Animation 2002





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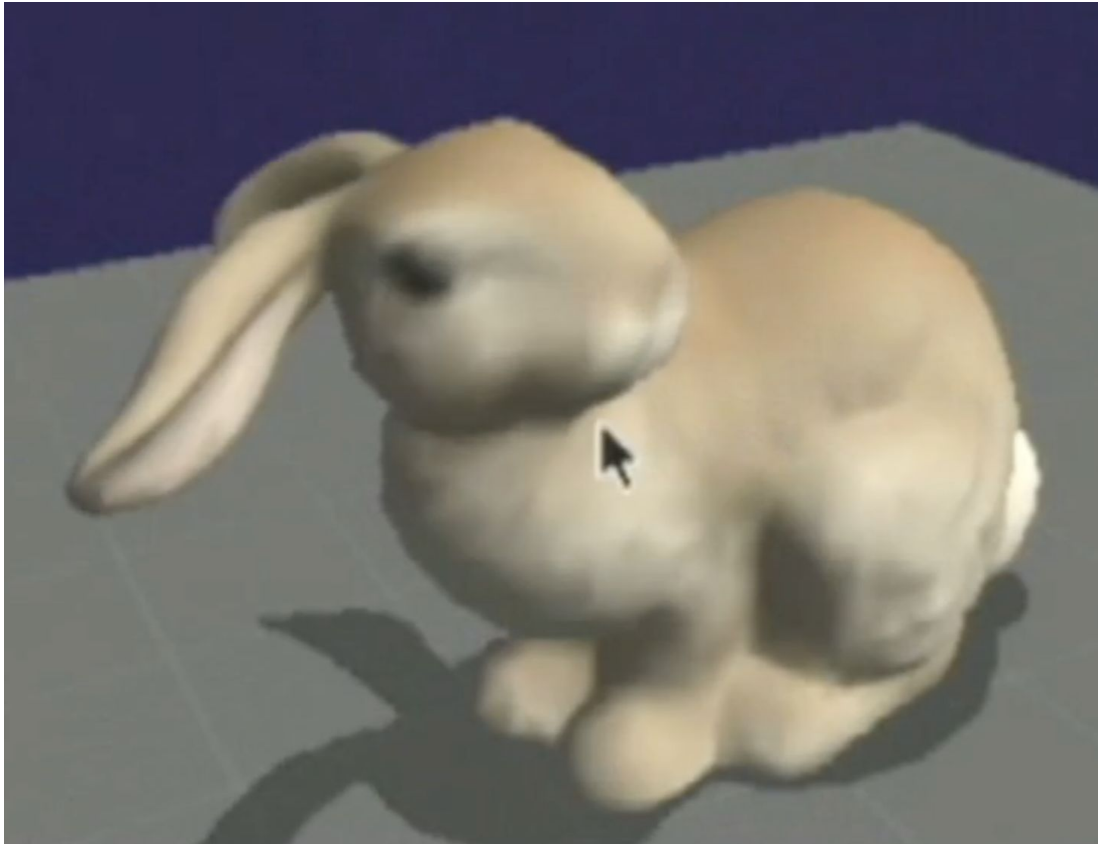


# Multiple Materials

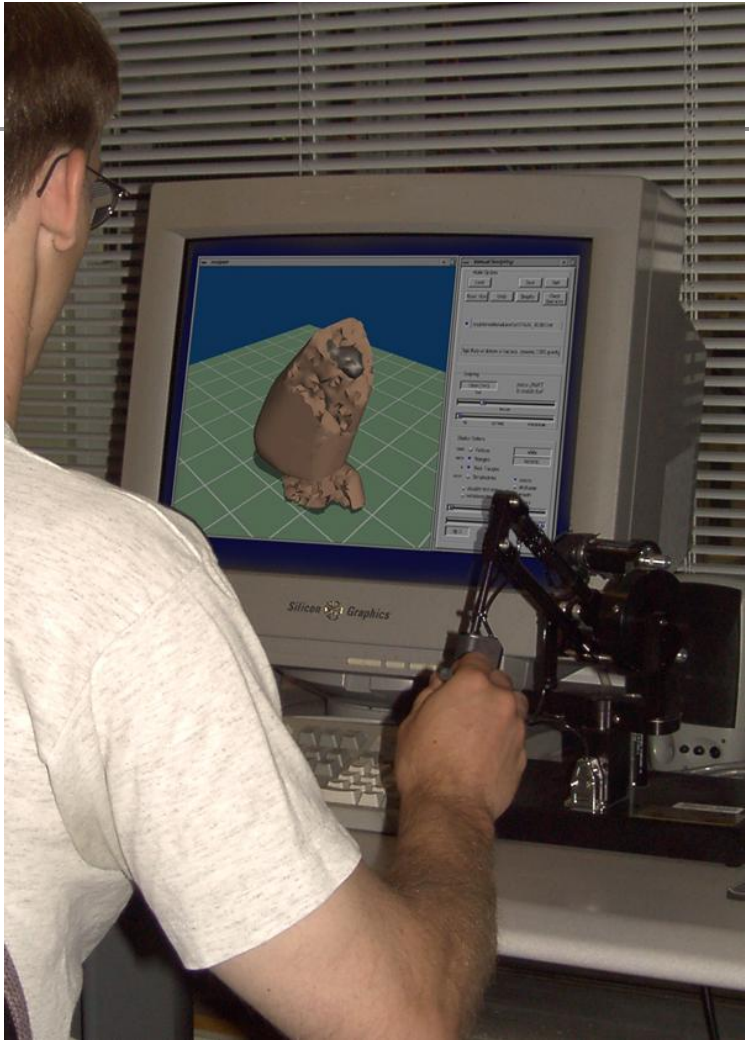
*Images from Cutler et al. 2002*

Image from Cutler et al. 2002

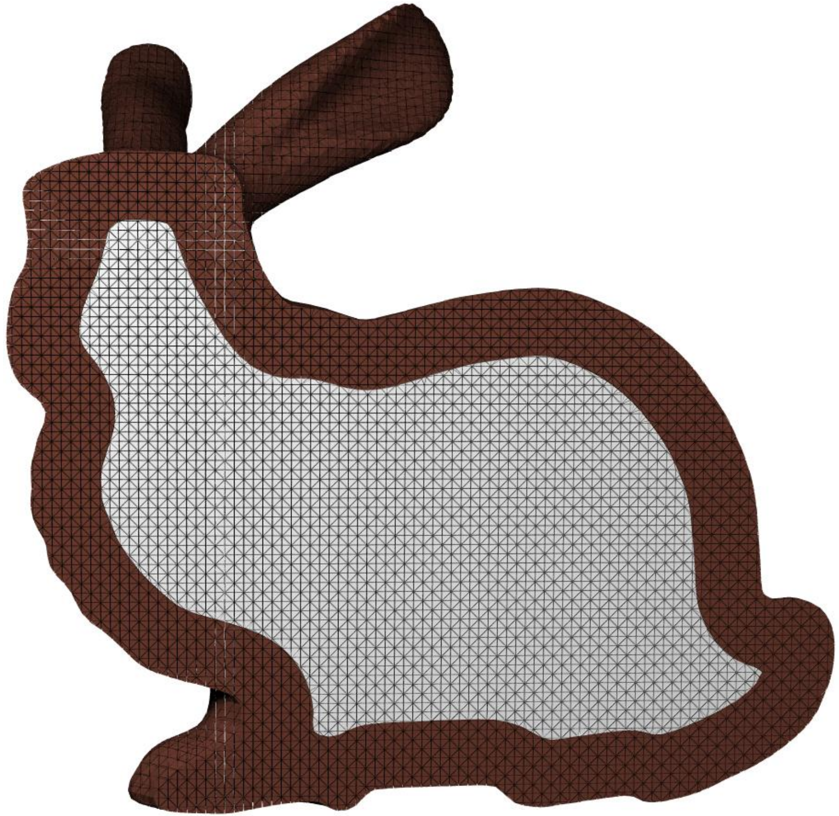
Image from Cutler et al. 2002

# Haptic Device

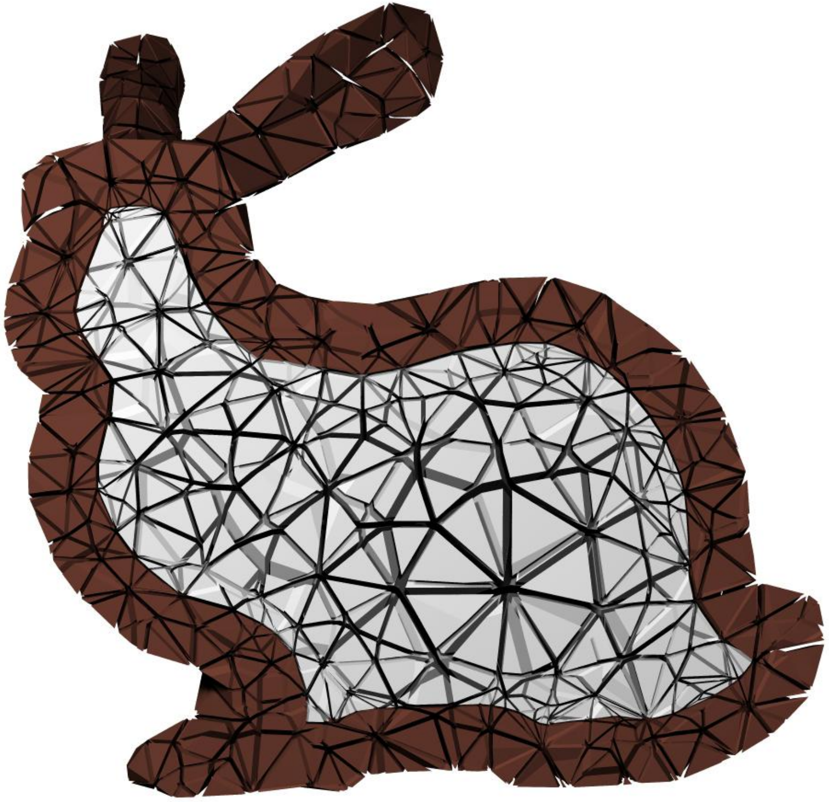
- "3D mouse" + force feedback
- 6 DOF (position & orientation)
- *requires 1000 Hz refresh*
  (*visual only requires ~30 Hz*)

# 3D Mesh Simplification



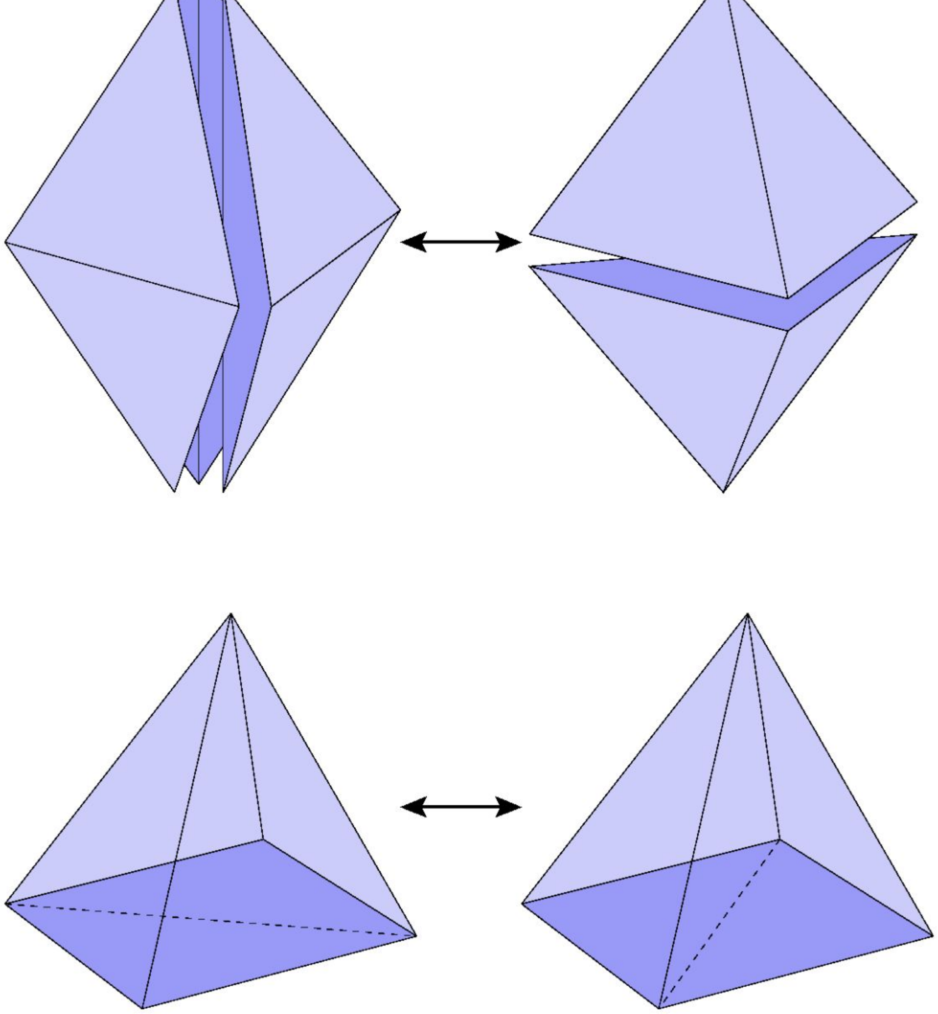
*1,050K tetras  (133K faces)*



*10K tetras  (3K faces)*

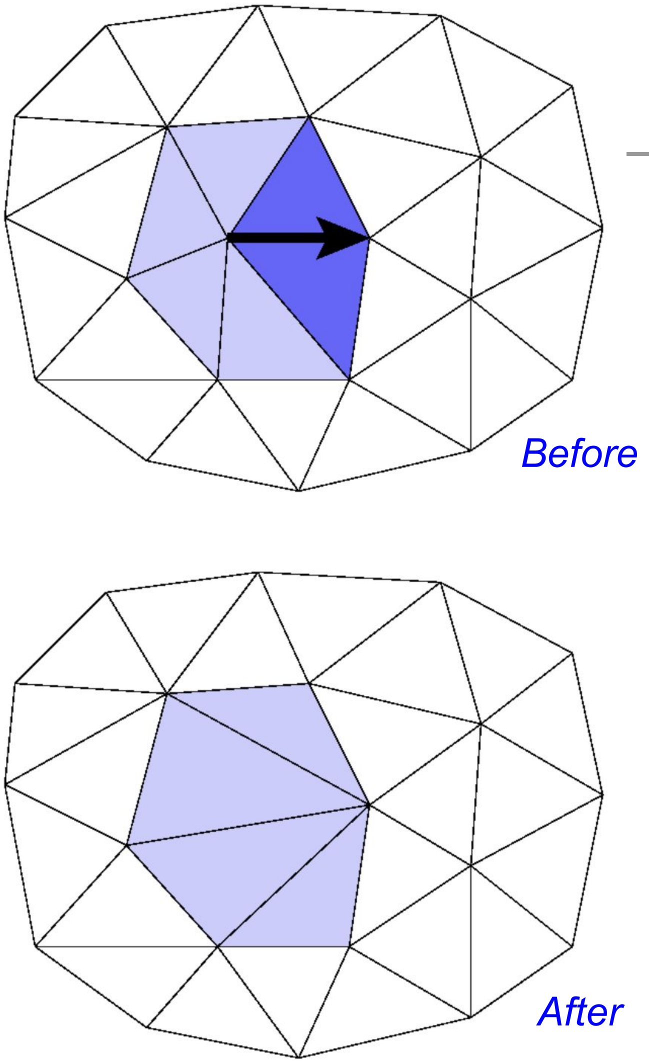
# 3D Mesh Operations

- Tetrahedral Swaps
  - Choose the configuration with the best local element shape
- Edge Collapse
- Vertex Smoothing
- Vertex Addition



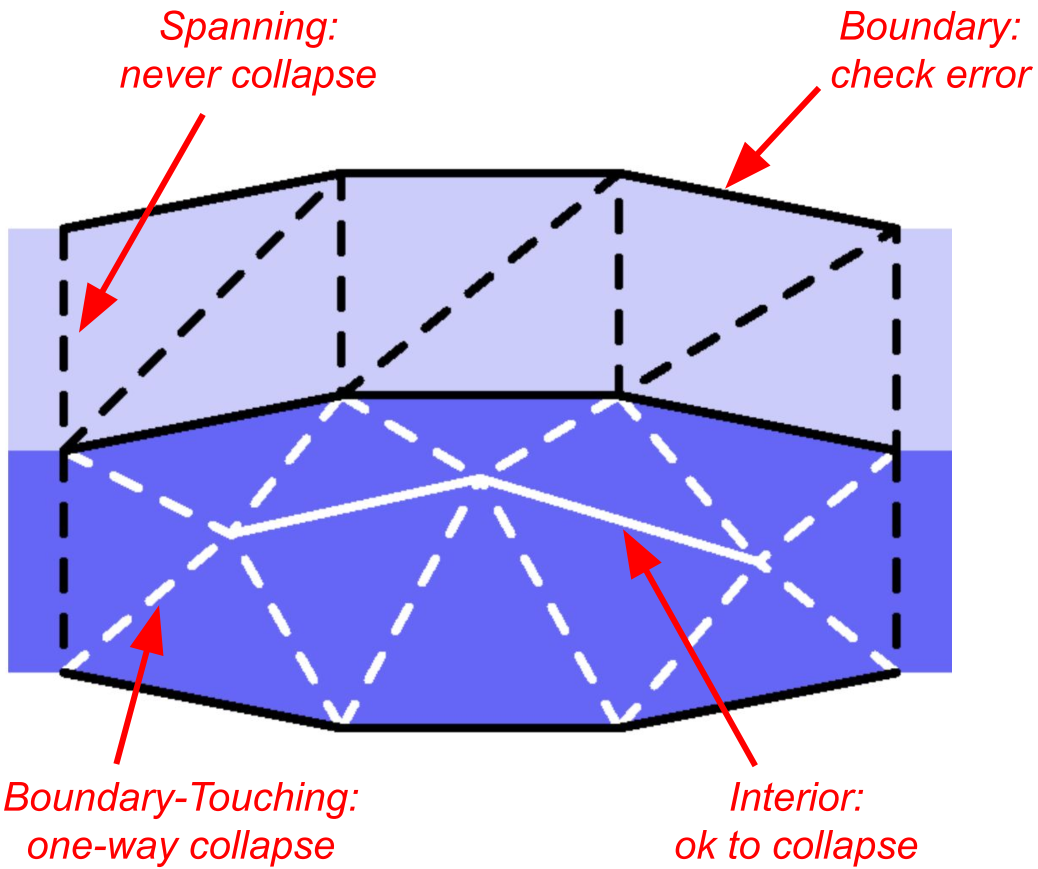
# 3D Mesh Operations

- Tetrahedral Swaps
- Edge Collapse
  - Delete a vertex & the elements around the edge
- Vertex Smoothing
- Vertex Addition



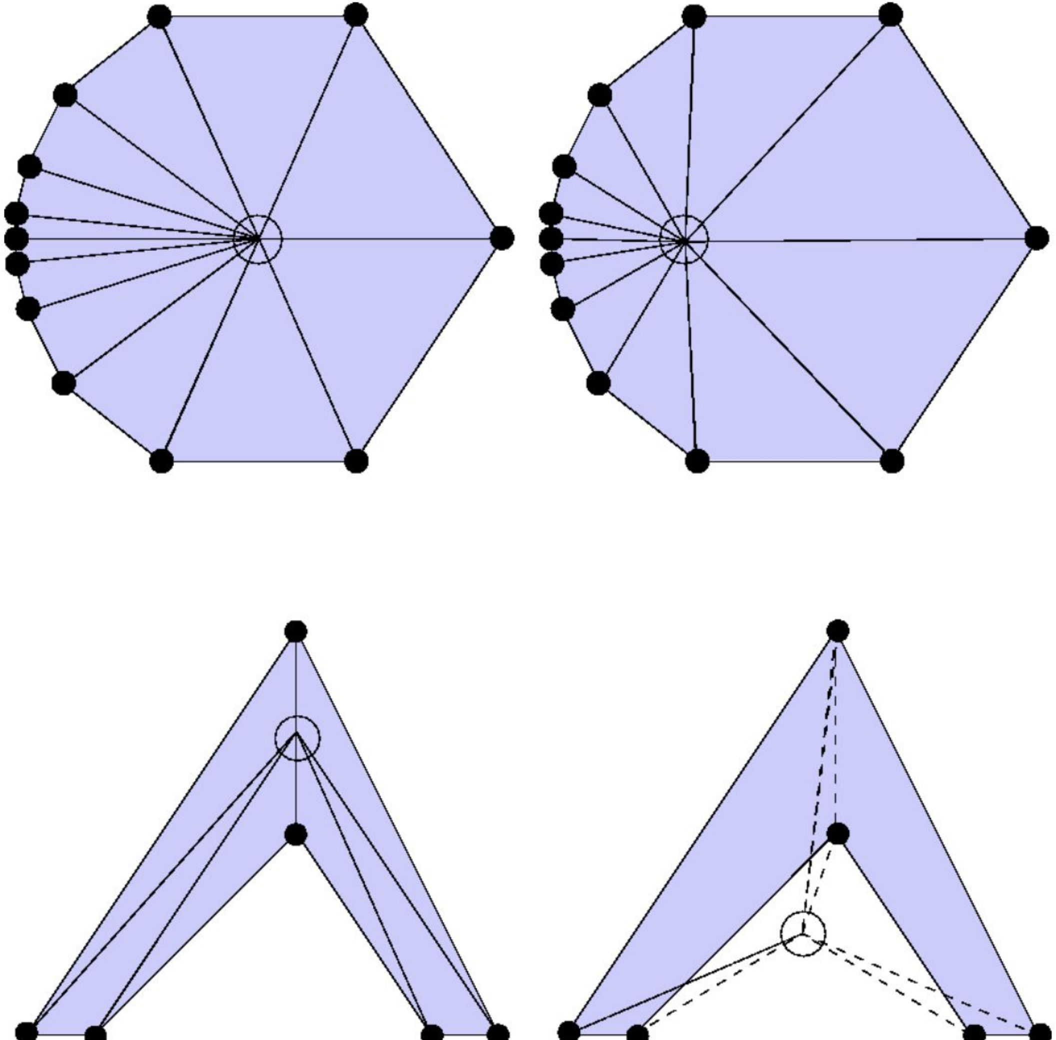
# Prioritizing Edge Collapses

- Preserve topology
  - *Thin layers should not pinch together*
- Collapse weight
  - *Edge length + boundary error*
- No negative volumes
- Local element quality does not significantly worsen



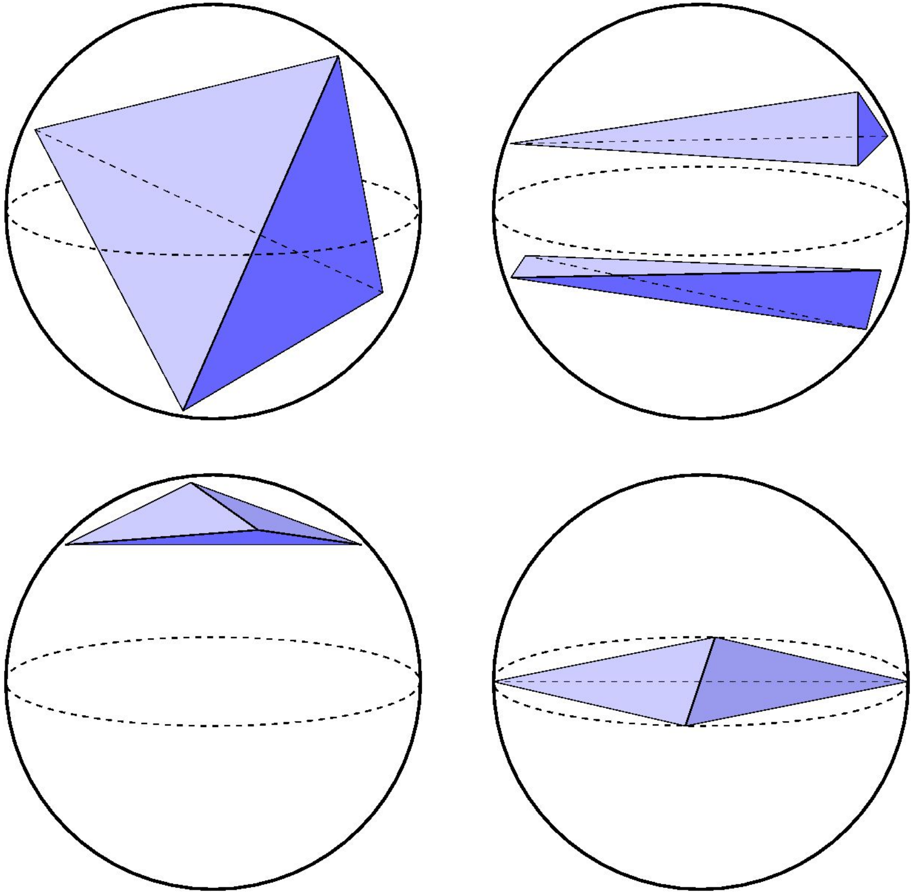
# 3D Mesh Operations

- Tetrahedral Swaps
- Edge Collapse
- Vertex Smoothing
    - Move a vertex to the centroid of its neighbors
    - Convex or concave, but avoid negative-volume elements
- Vertex Addition

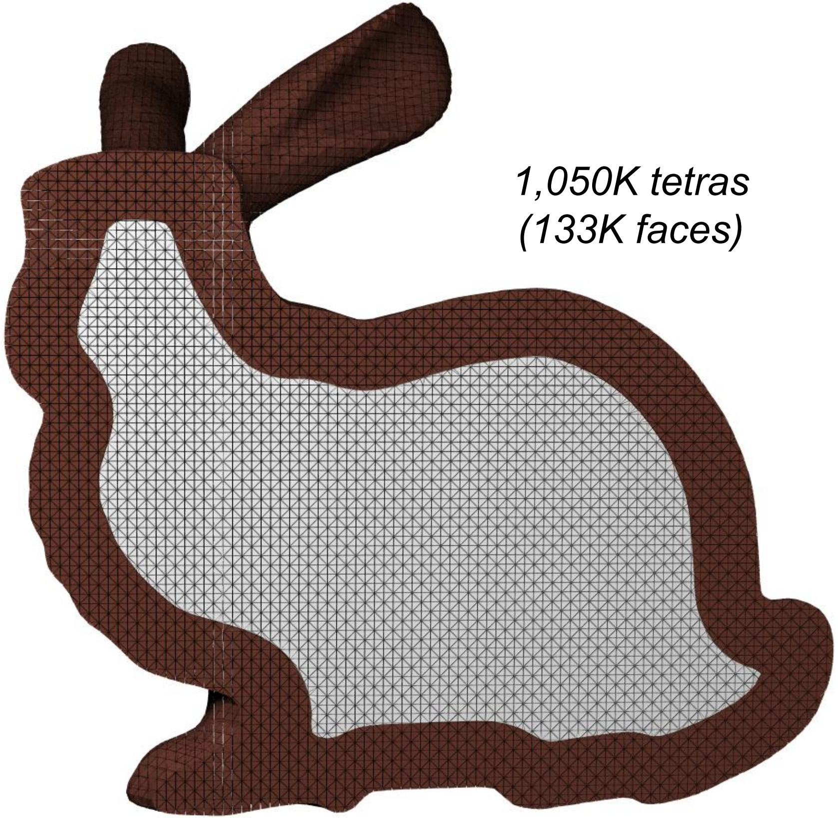


# 3D Mesh Operations

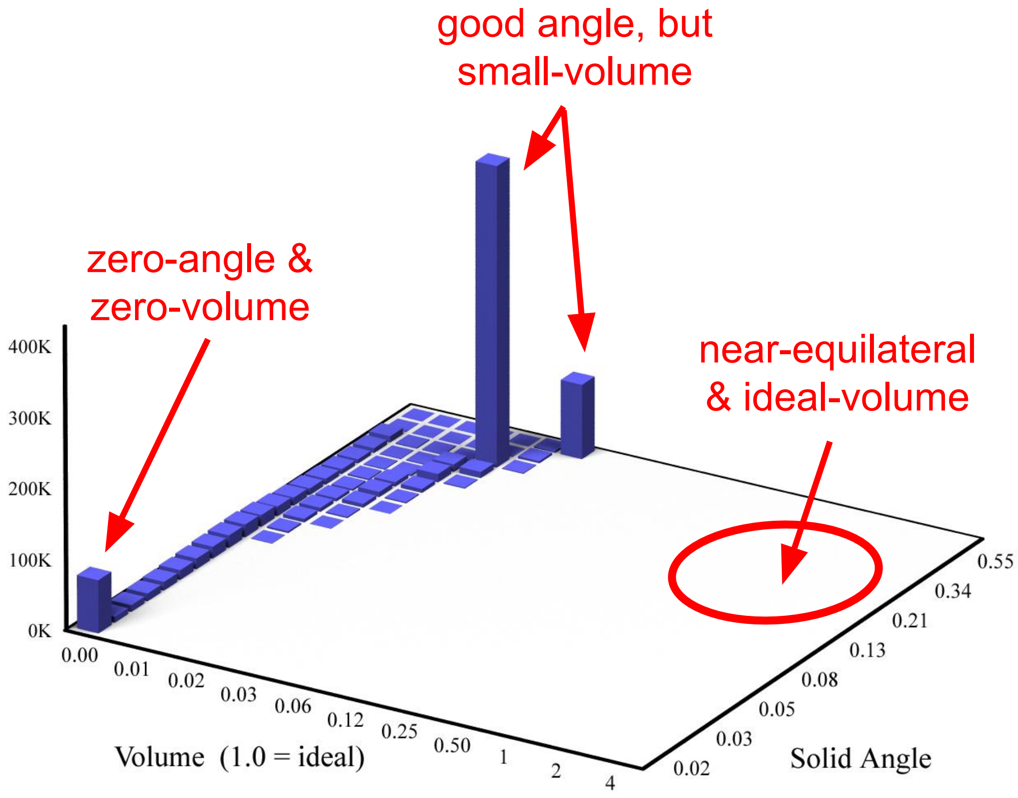
- Tetrahedral Swaps
- Edge Collapse
- Vertex Smoothing
- Vertex Addition
  - At the center of a tetra, face, or edge
  - Useful when mesh is simplified, but usually needs further element shape improvement



# Visualization of Tetrahedra Quality



*1,050K tetras*
*(133K faces)*



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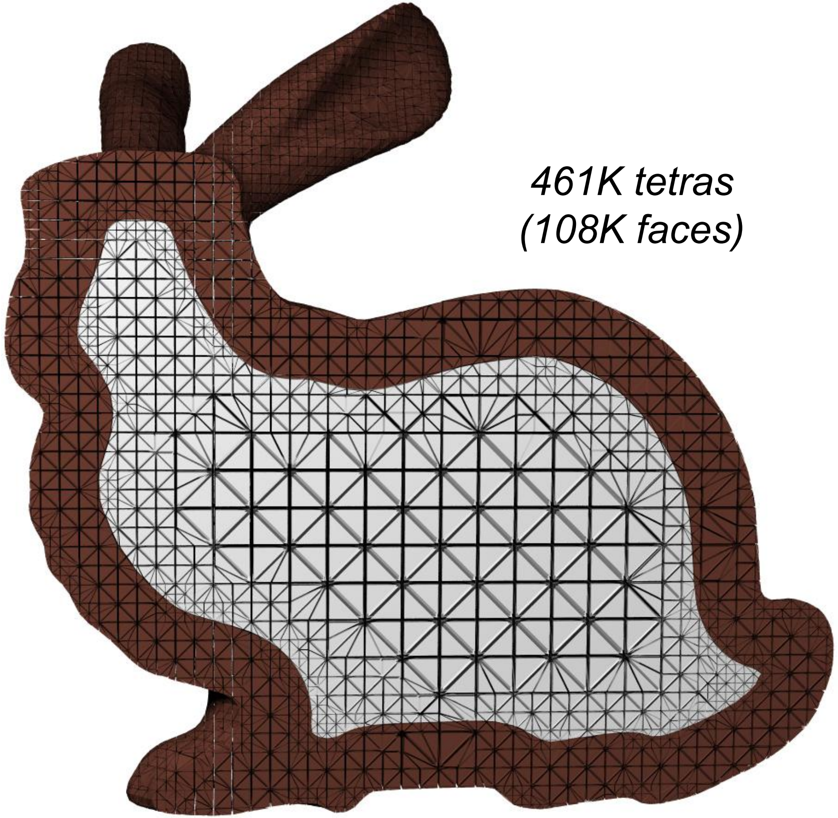
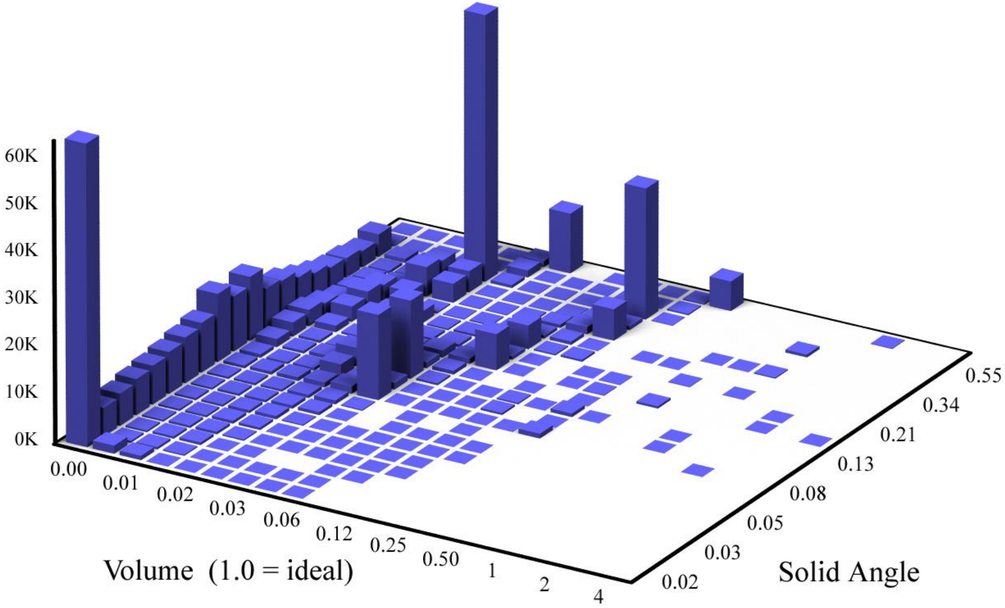
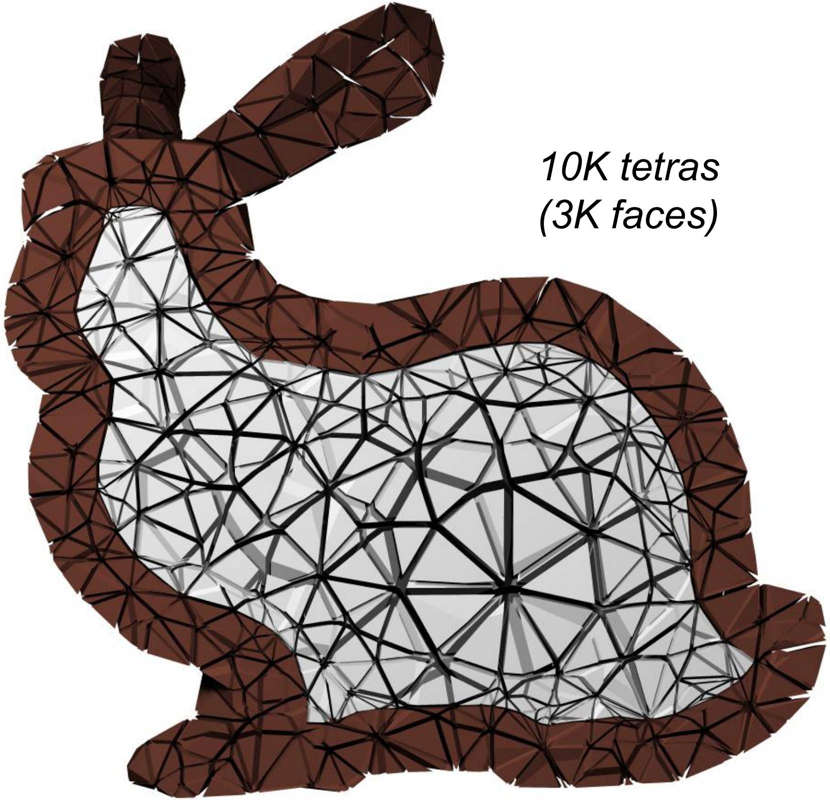


*461K tetras (108K faces)*

Octree or Adaptive Distance Field (ADF)

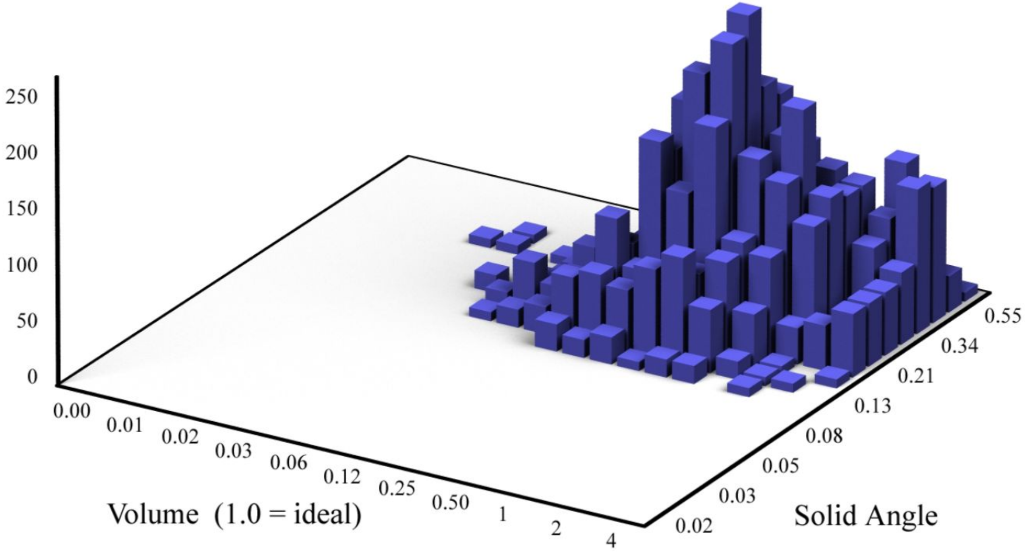


# Visualization of Tetrahedra Quality

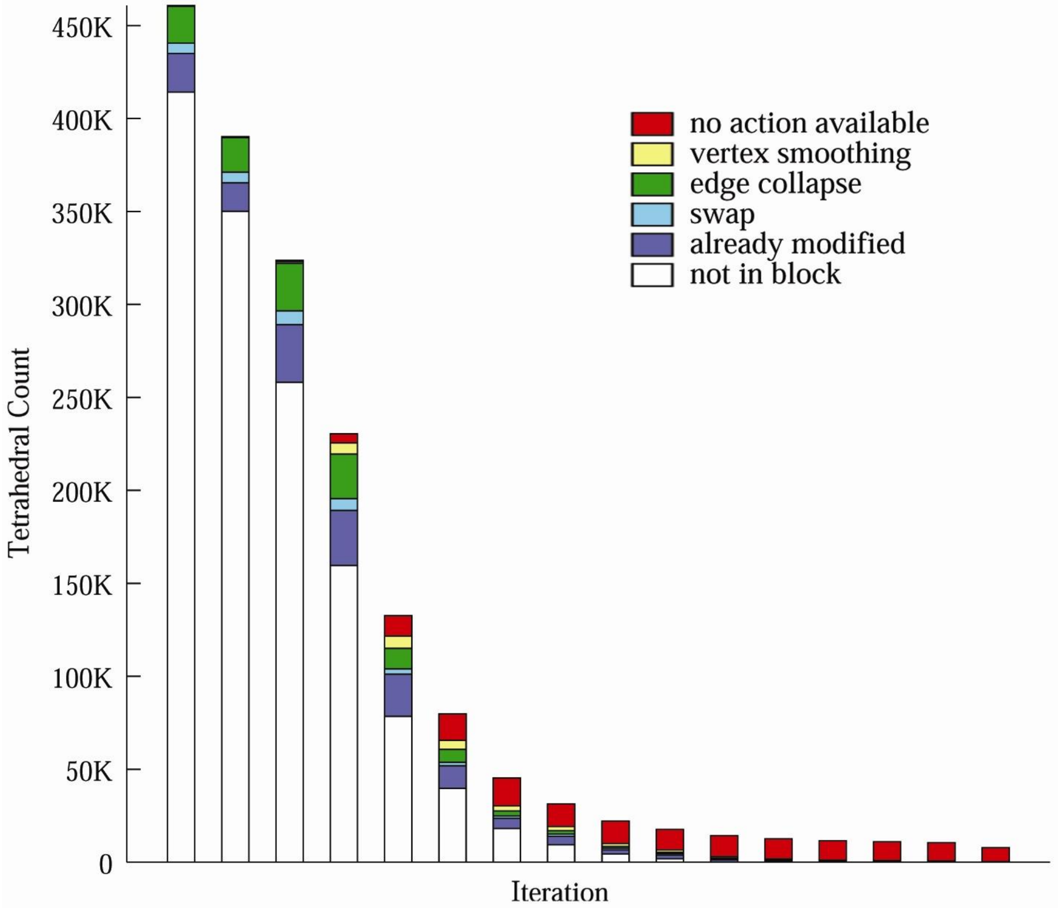


*10K tetras*
*(3K faces)*

*After Simplification*
*& Mesh Improvement*



# Visualization of Simplification Algorithm



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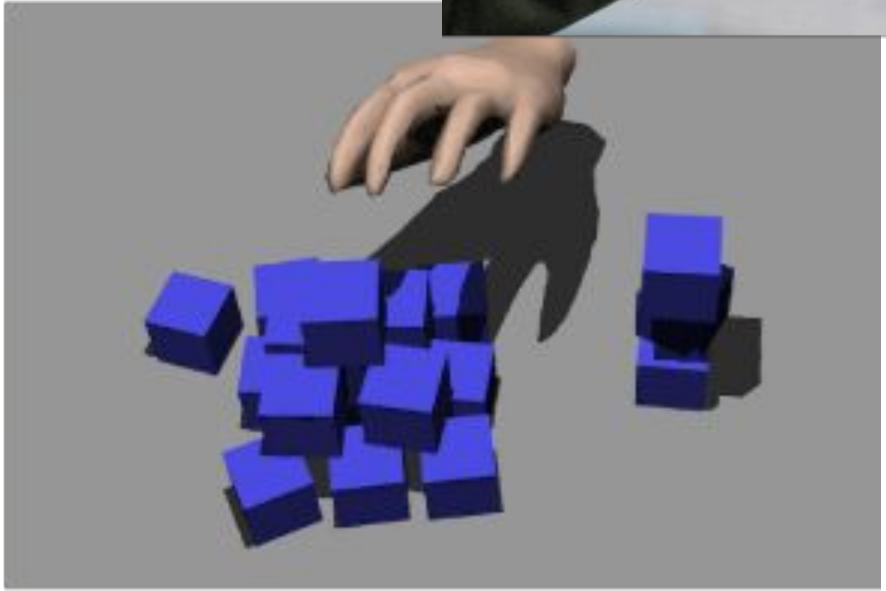
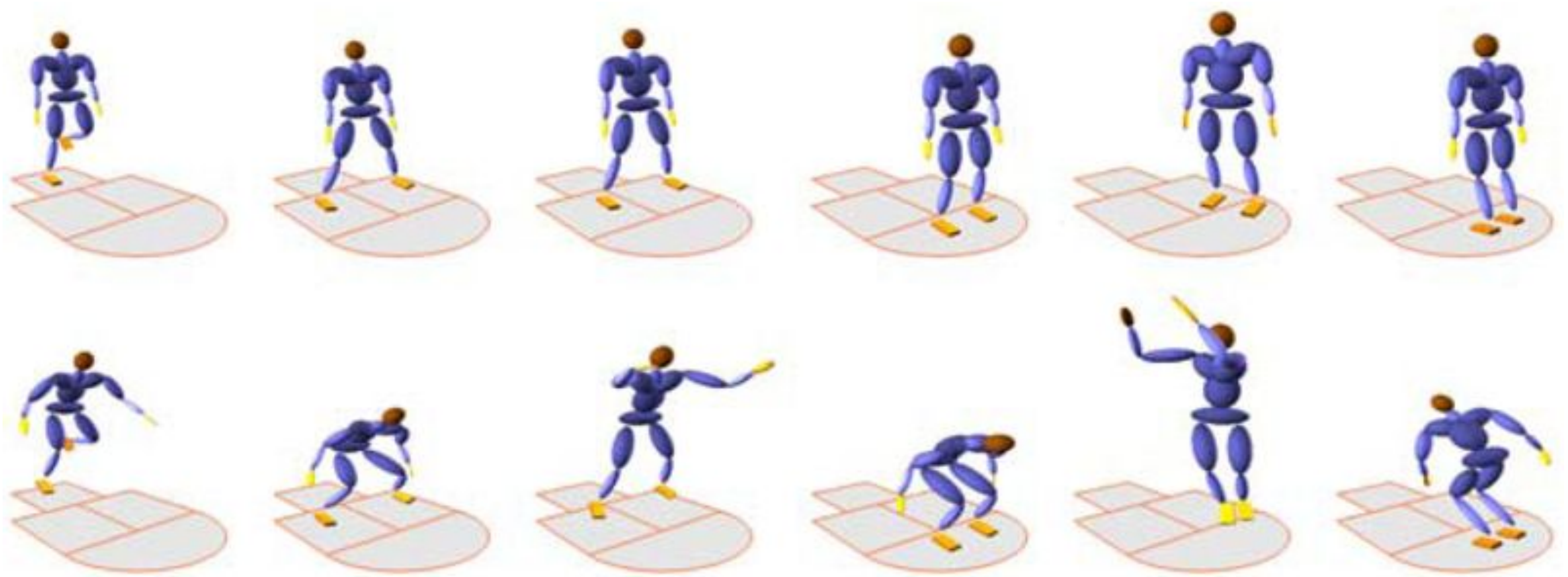
# Reading for Friday: *(pick one)*

- "Real-Time Hand-Tracking with a Color Glove"
  SIGGRAPH 2009, Wang & Popović



# Reading for Friday: *(pick one)*

- “Synthesis of Complex Dynamic Character Motion from Simple Animation”, Liu & Popović, 2002



# Reading for Friday: *(pick one)*

- "Artist-Directed Dynamics for 2D Animation", Bai, Kaufman, Liu, & Popović, SIGGRAPH 2016

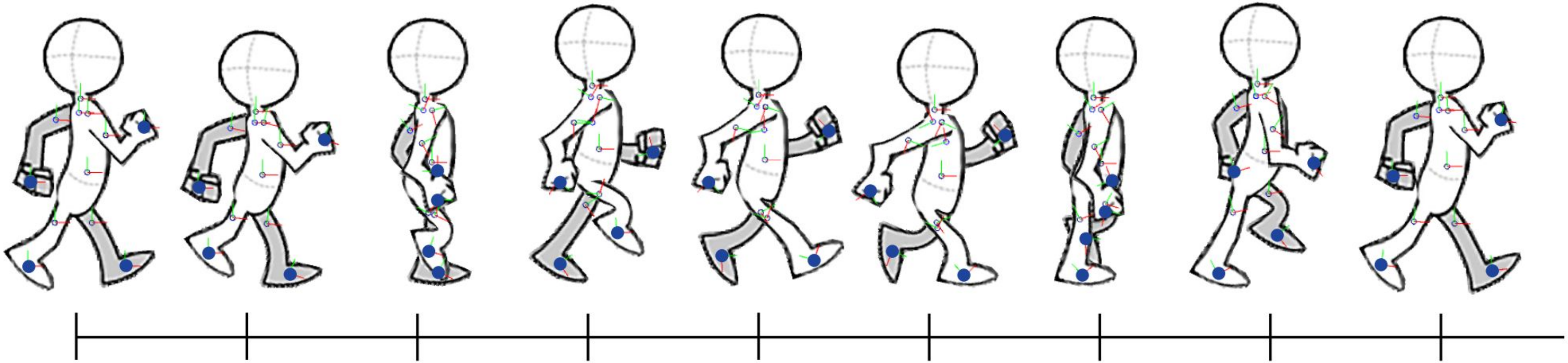


**Figure 6:** ***Keyframes used in the articulated character walk example.*** *The artist only specifies keyframes for a subset of handles (handles at hands and feet) which are shown as blue dots. Nine keyframes are used to create a walking cycle. Their timing is visualized by the black lines at the bottom. The artworks are adapted from Angryanimator.com (http://www.angryanimator.com/)*