CSCI 4530/6530 Advanced Computer Graphics

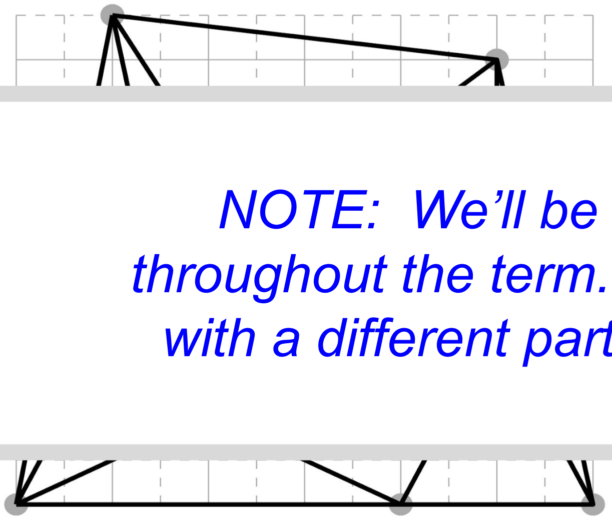
https://www.cs.rpi.edu/~cutler/classes/advancedgraphics/S25/

Lecture 4: Subdivision Surfaces

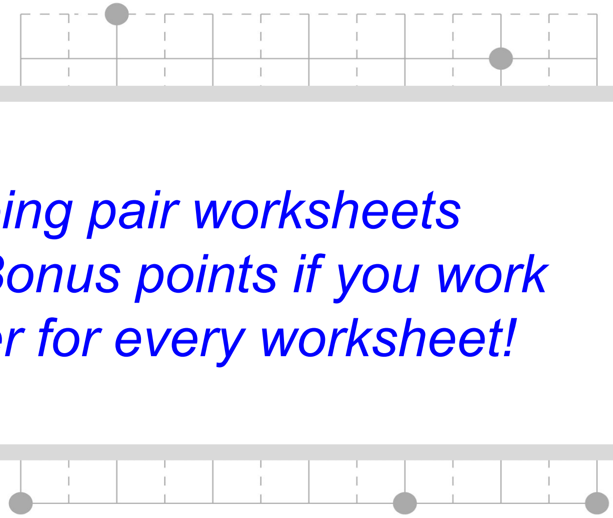
https://imgur.com/gallery/1jwOQms

Worksheet: Shortest Edge Collapse

- Perform a sequence of 3 edge collapses, one-at-a-time
- Always collapse the shortest edge that does not result in a zero area or “flipped”/upside-down triangle
- Replacement vertex should be at the midpoint of the edge



NOTE: We’ll be doing pair worksheets throughout the term. Bonus points if you work with a different partner for every worksheet!



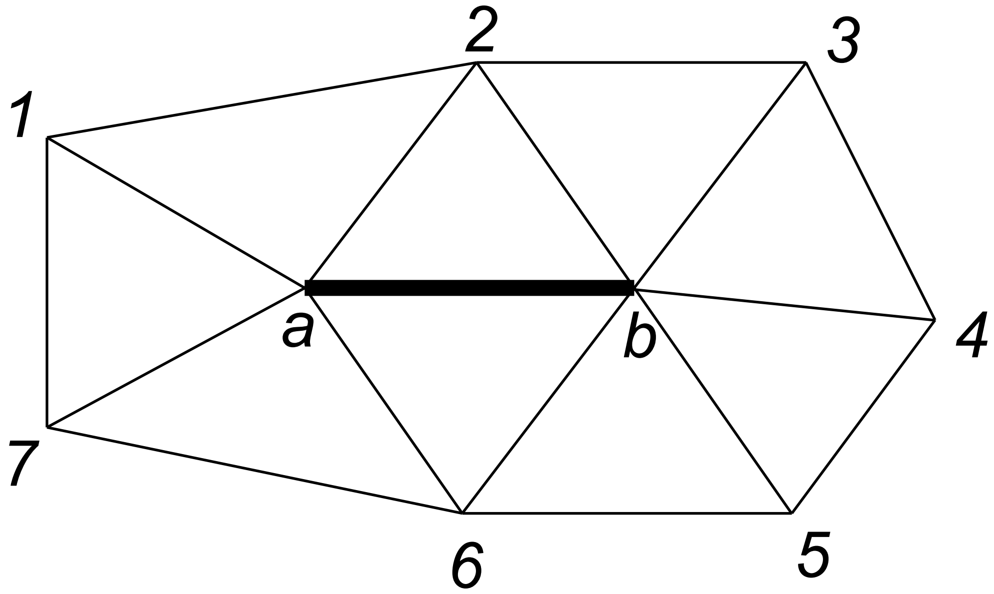
Geri's Game

Pixar Animation Studios, 1997

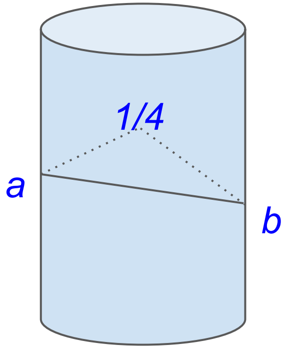
Questions on Homework 1?

HW1 Progress Post due Saturday @ midnight!

- What's an *illegal edge collapse*?
- To be legal, the ring of neighboring vertices *must be unique* (have no duplicates)!



What if vertex 1 is the same as vertex 4?



Notes about Homework Autograding

- Homework Autograding is run on a Linux desktop machine
- Automated:
  - Keyboard & mouse commands
  - Reasonable pauses (sleep)
  - Screenshots
- Will have longer wait times than other courses…
  - not parallelized (one student at a time)

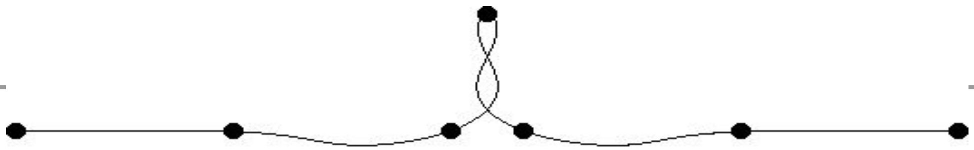
Don't panic if autograding takes a while or gets stuck.
Post on the forum if you experience problems.

Misc. Announcements

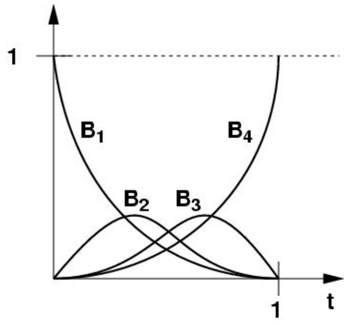
- Homework 1 Progress Post
  - Deadline extended to Saturday 1/18 @ midnight (no late days)
  - Post 1 image + short written description of progress
- Homework 1 Deadline
  - Thursday 1/18 @ midnight (max 2 late days allowed)

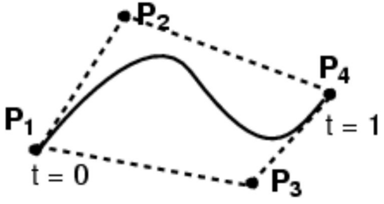
- Barb’s Office Hours - Mondays 1-3pm, Lally 302
- Nyx’s Office Hours - Wednesdays noon-2pm, AE 118
- Mingyi’s Office Hours - Thursdays 6-8pm, CII 3112
- *Also immediately after lecture on Tuesdays & Fridays*

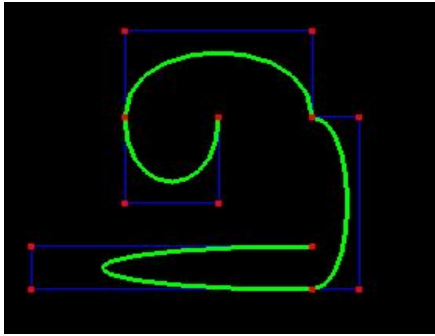
Last Time?

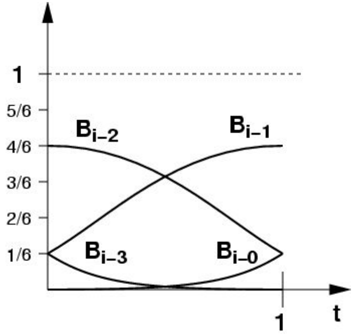


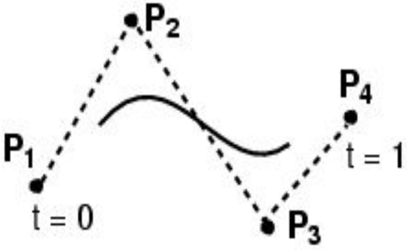
- Curves & Surfaces
- Continuity Definitions
  - C^0, G^1, C^1, … C^∞
- Interpolation vs. Approximation Splines
- Cubic Bézier & BSpline

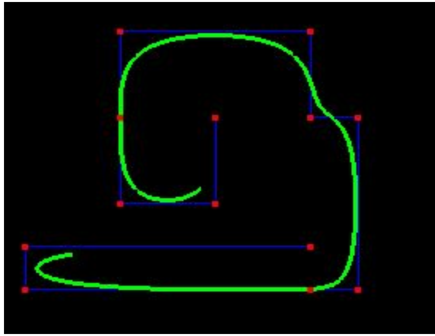


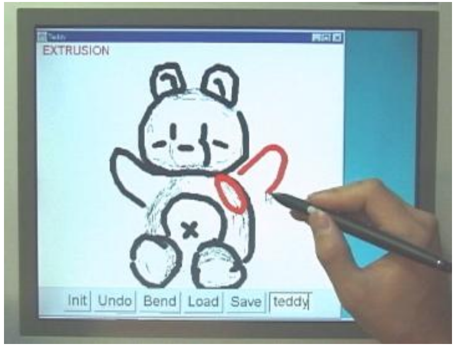












Today

- Worksheet: Shortest Edge Collapse
- From Last Time: Bézier Splines → Bézier Surfaces
- Papers for Today
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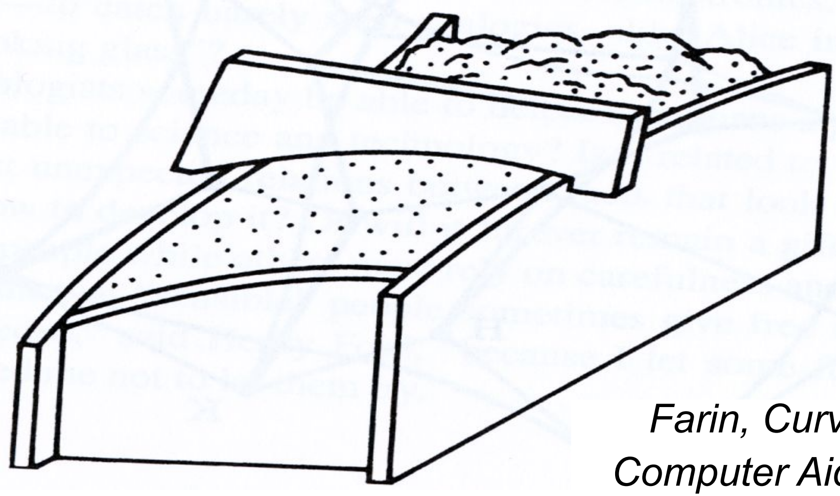
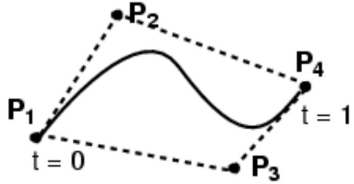
Spline Surface via Tensor Product

- Of two vectors:

$$\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \otimes \begin{bmatrix} b_1 & b_2 & b_3 & b_4 \end{bmatrix} = \begin{bmatrix} a_1b_1 & a_2b_1 & a_3b_1 \\ a_1b_2 & a_2b_2 & a_3b_2 \\ a_1b_3 & a_2b_3 & a_3b_3 \\ a_1b_4 & a_2b_4 & a_3b_4 \end{bmatrix}$$

- Similarly, we can define a surface as the tensor product of two curves....

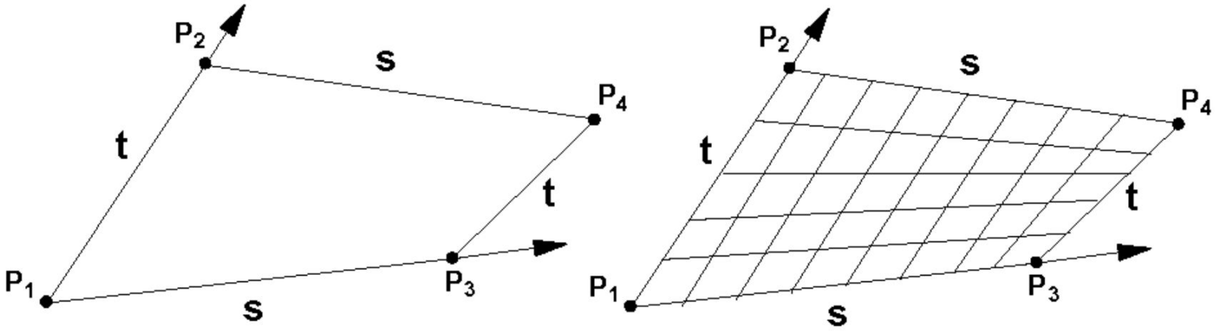




Farin, Curves and Surfaces for Computer Aided Geometric Design

Bilinear Patch

- 1D Linear Interpolation: $\mathbf{L}\,(P_1, P_2, \alpha) = (1 - \alpha)\,P_1 + \alpha\,P_2$
- 2D Bilinear Interpolation: $\mathbf{Q}\,(s,t) = \mathbf{L}\,(\,\mathbf{L}\,(P_1, P_2, t),\ \mathbf{L}(P_3, P_4, t), s)$



- Bilinear Interpolation creates non-planar quadrilaterals (if P_1, P_2, P_3, P_4 are not co-planar)
- But will this help us model smooth surfaces?
- Do we have control of the derivative at the edges?

Ruled Surfaces in Art & Architecture

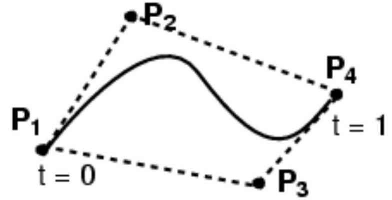
http://www.bergenwood.no/wp-content/media/images/frozenmusic.jpg

Chiras Iulia
Astri Isabella
Matiss Shteinerts

http://www.lonelyplanetimages.com/images/399954

Antoni Gaudi
Children's School
Barcelona

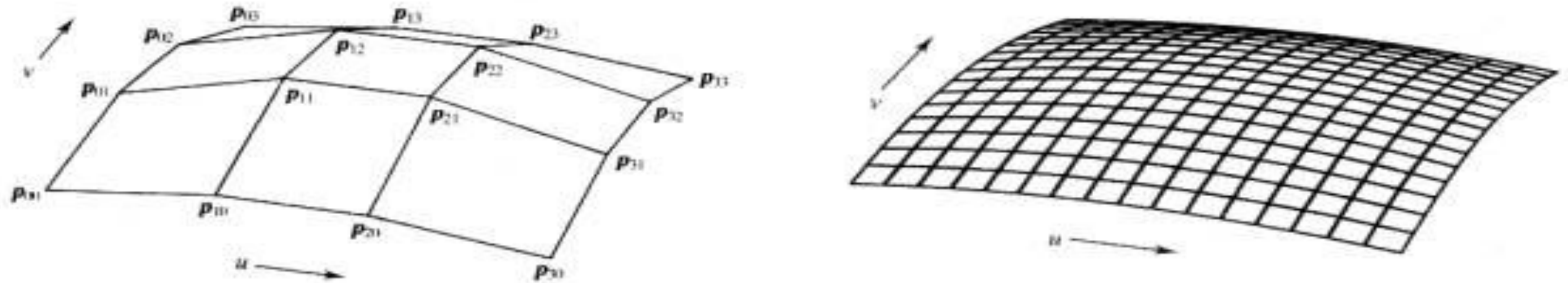
Bicubic Bézier Patch



Bézier Curve: **CB** $(P_1, P_2, P_3, P_4, \alpha)$

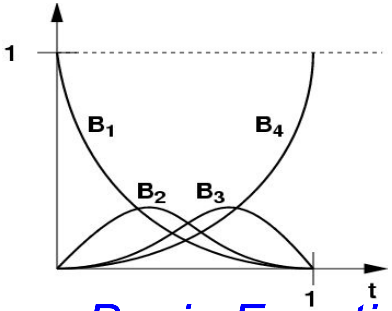
Bézier Surface:

$$\boldsymbol{Q}(s,t) = \boldsymbol{CB}\big(\boldsymbol{CB}(P1, P2, P3, P4, t), \; \boldsymbol{CB}(P1, P2, P3, P4, t), \; \boldsymbol{CB}(P1, P2, P3, P4, t), \; \boldsymbol{CB}(P1, P2, P3, P4, t), s \big)$$

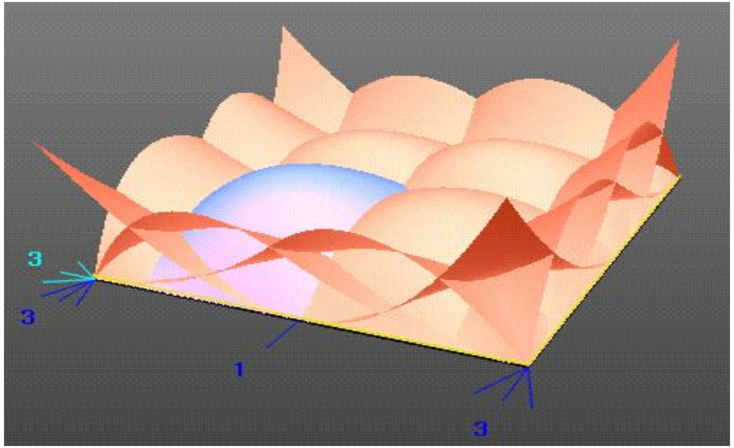


(a) (b)

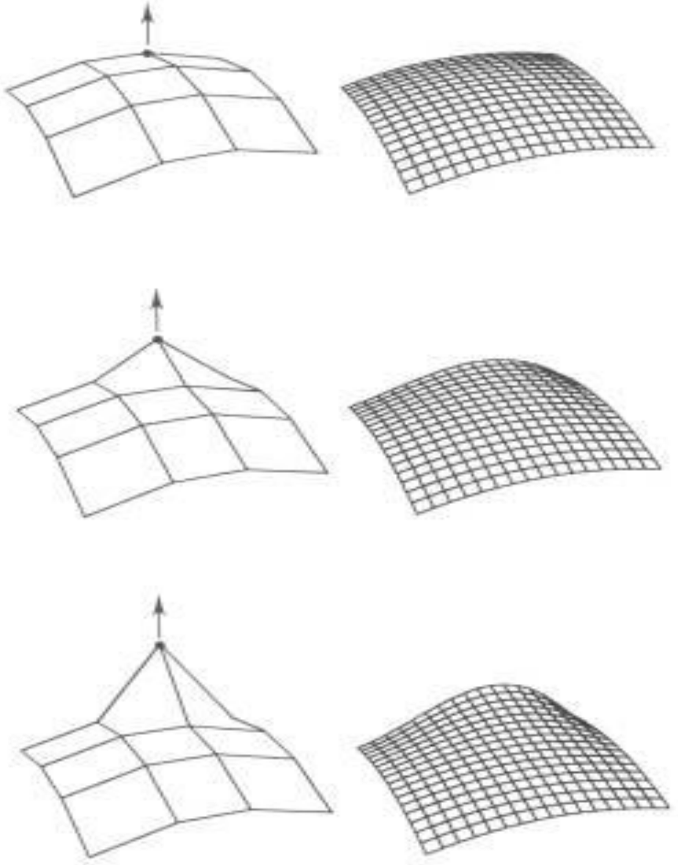
Editing Bicubic Bézier Patches



Curve Basis Functions

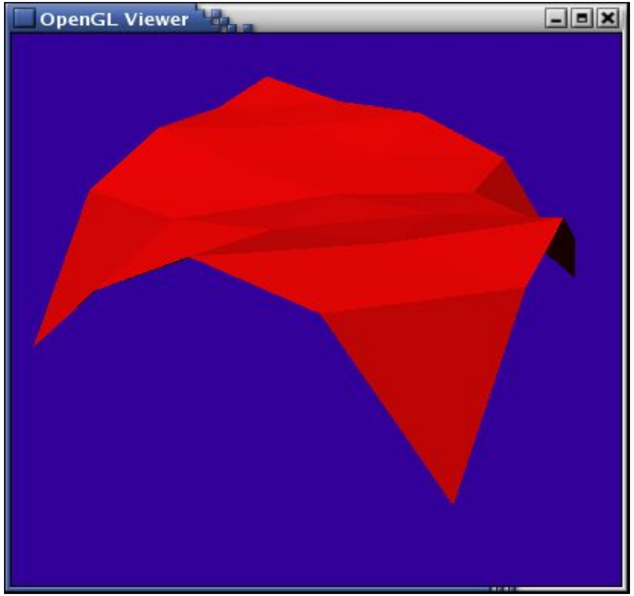


Surface Basis Functions

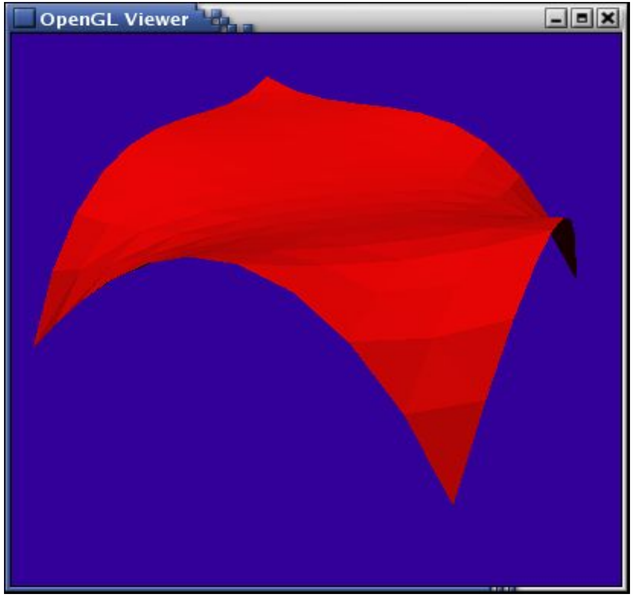


Bicubic Bézier Patch Tessellation

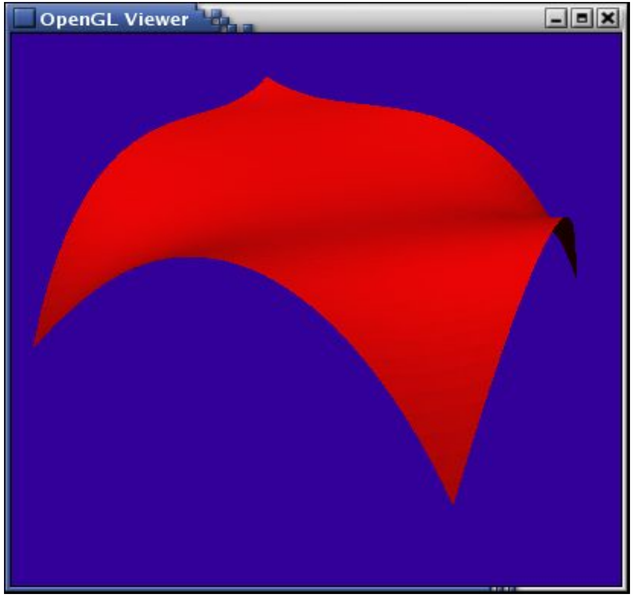
- Given 16 control points and a tessellation resolution, we can create a triangle mesh



resolution:
5x5 vertices



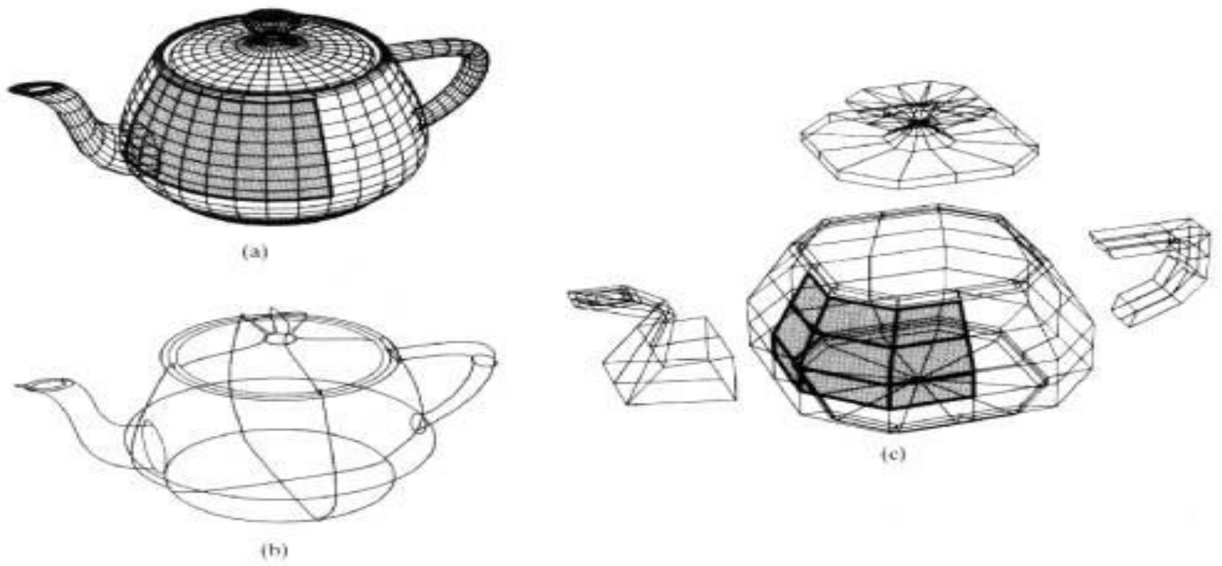
resolution:
11x11 vertices



resolution:
41x41 vertices

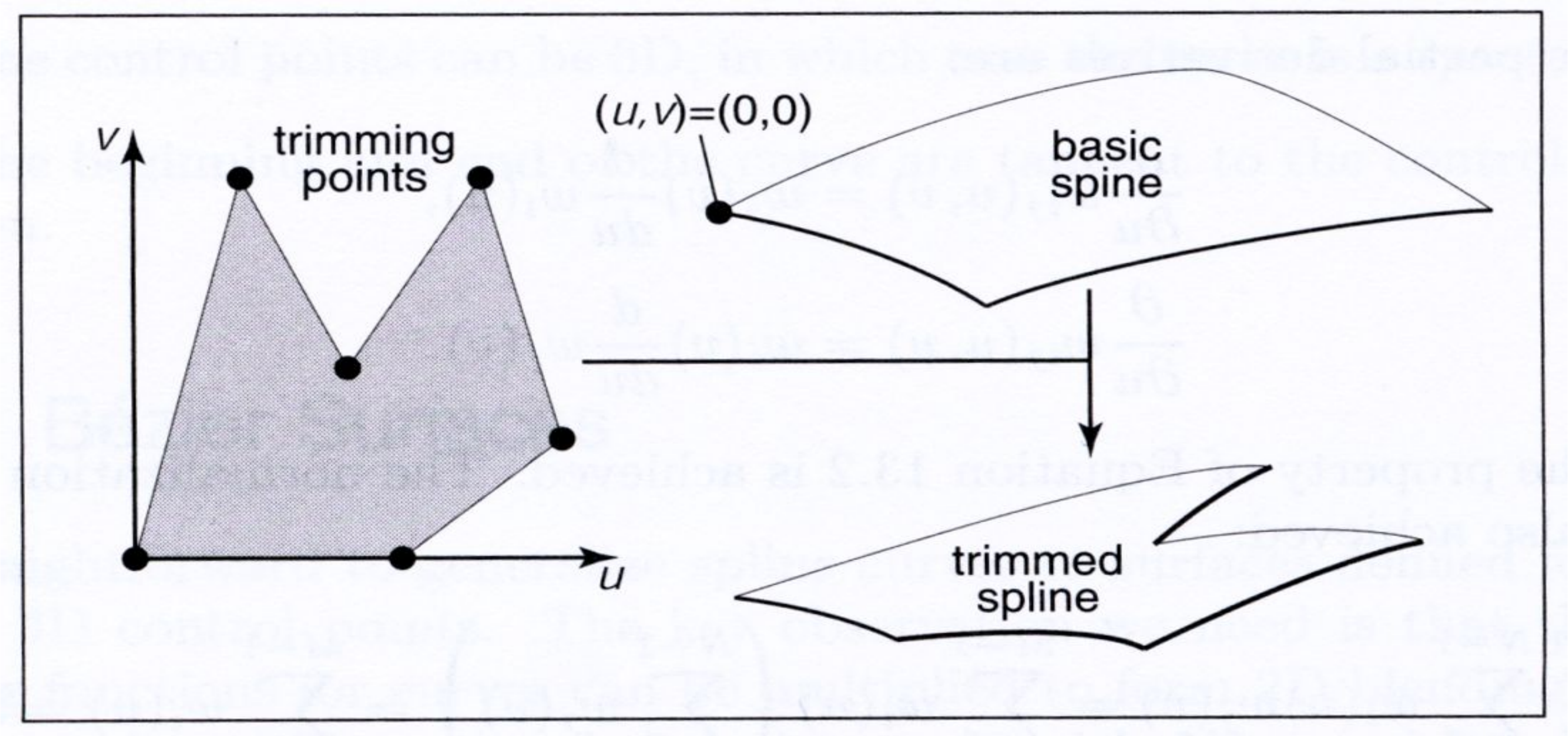
Modeling with Bicubic Bézier Patches

- Original Teapot specified with Bézier Patches:



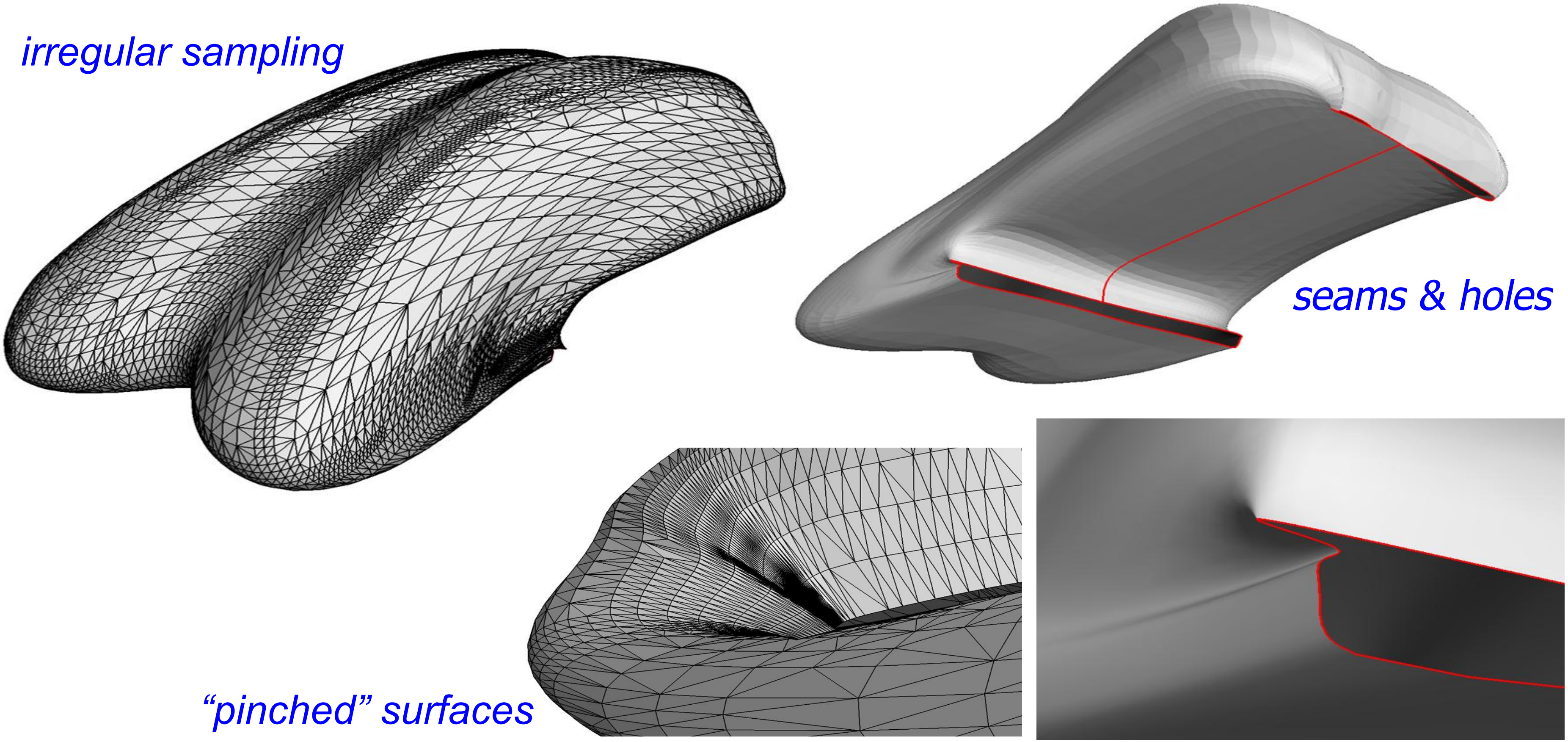
- But it’s not "watertight": it has intersecting surfaces at spout & handle, no bottom, a hole at the spout tip, a gap between lid & base

Trimming Curves for Patches



Shirley, Fundamentals of Computer Graphics

Spline-Based Modeling Headaches



Questions?

- Bézier Patches?

  or

- Triangle Mesh?

Henrik Wann Jensen

Today

- Worksheet: Shortest Edge Collapse
- From Last Time: Bézier Splines → Bézier Surfaces
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Reading for Today

- DeRose, Kass, & Truong, "Subdivision Surfaces in Character Animation", SIGGRAPH 1998

Quad Meshes
more common
in artistic practice
(e.g. Pixar's Geri's Game)

Figure 5: Geri's hand as a piecewise smooth Catmull-Clark surface. Infinitely sharp creases are used between the skin and the finger nails.

Subdivision Surfaces in Character Animation

- Catmull Clark Subdivision Rules
- Semi-sharp vs.
  Infinitely-sharp creases
- Mass-Spring Cloth *(next week)*
- Hierarchical Mesh
  for Collision
- Texturing Subdivision Surfaces

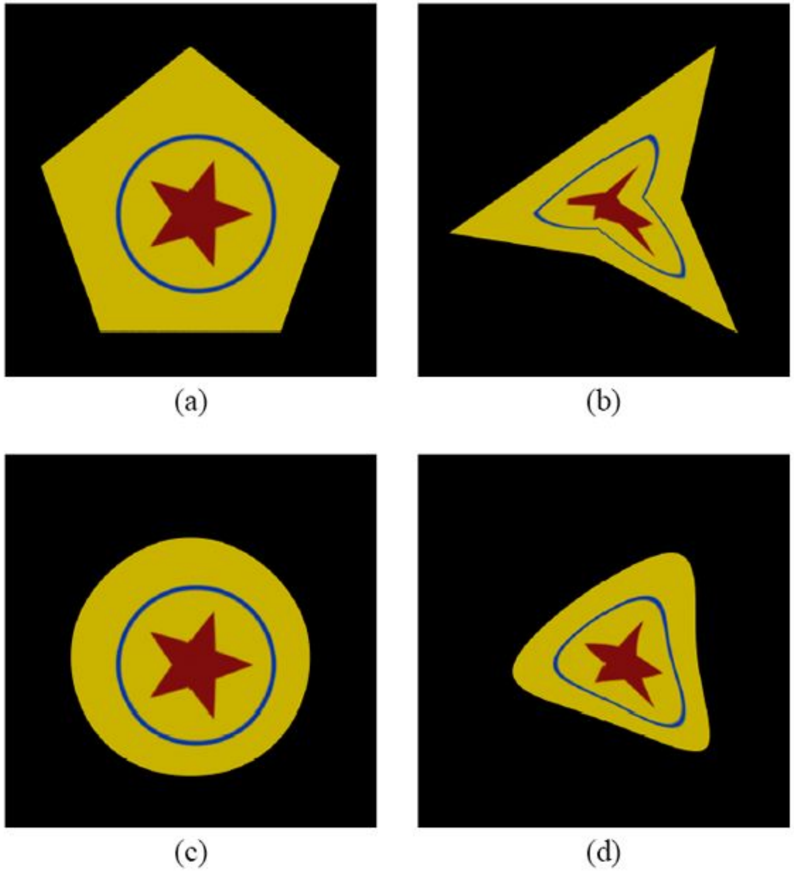
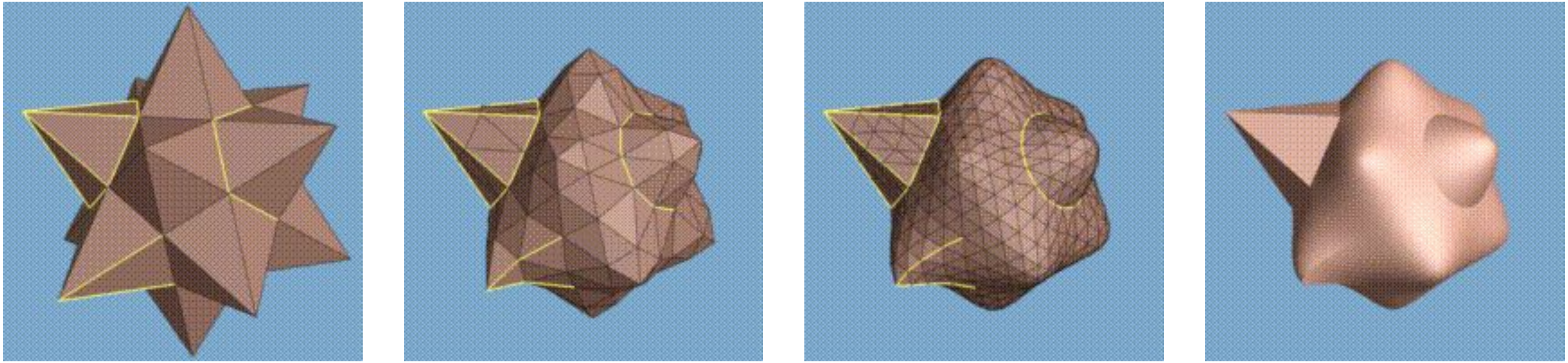


Figure 11: (a) A texture mapped regular pentagon comprised of 5 triangles; (b) the pentagonal model with its vertices moved; (c) A subdivision surface whose control mesh is the same 5 triangles in (a), and where boundary edges are marked as creases; (d) the subdivision surface with its vertices positioned as in (b).

Reading for Today

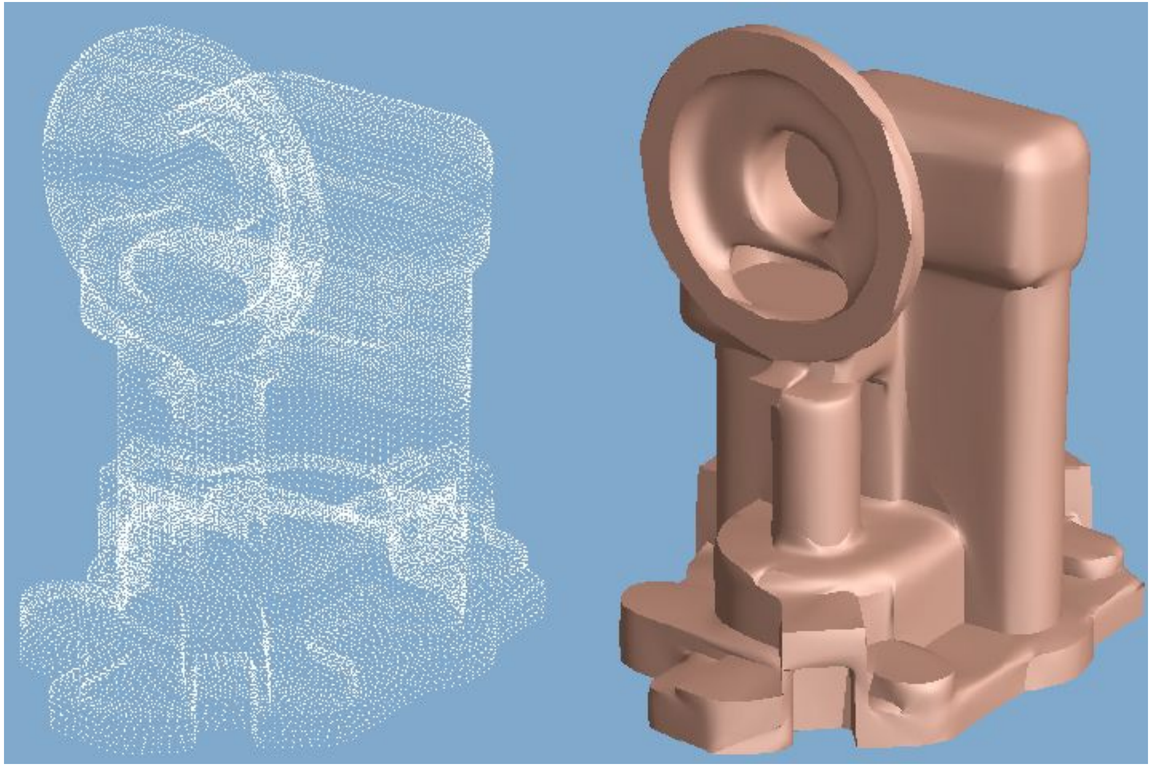
- Hoppe et al.,
  "Piecewise Smooth
  Surface Reconstruction"
  SIGGRAPH 1994

Triangle meshes
directly applies
to HW1!



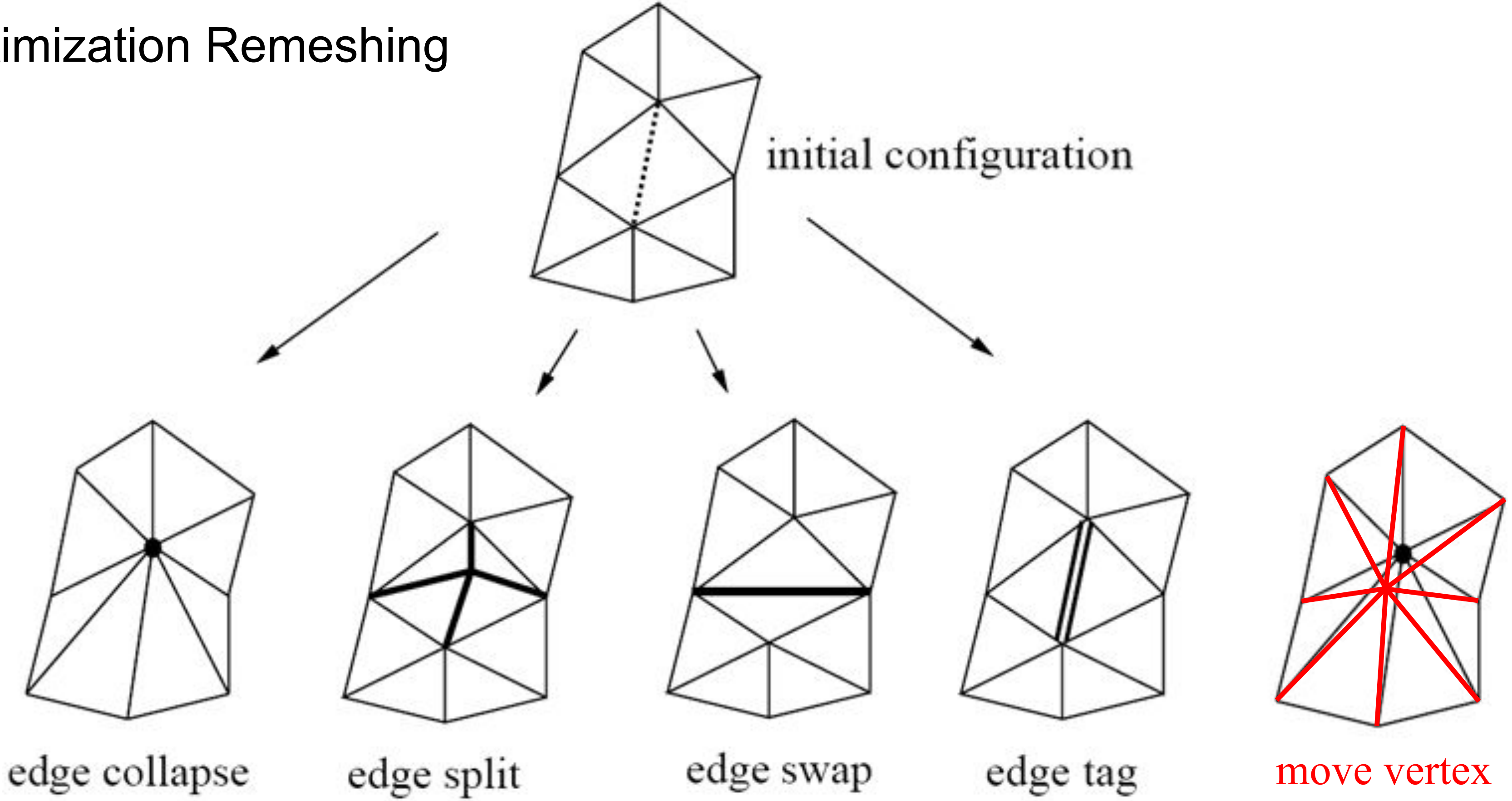
Piecewise Smooth Surface Reconstruction

- From input: scanned mesh points
  - Estimate topological type (genus)
  - Mesh optimization (a.k.a. simplification)
  - Smooth surface optimization



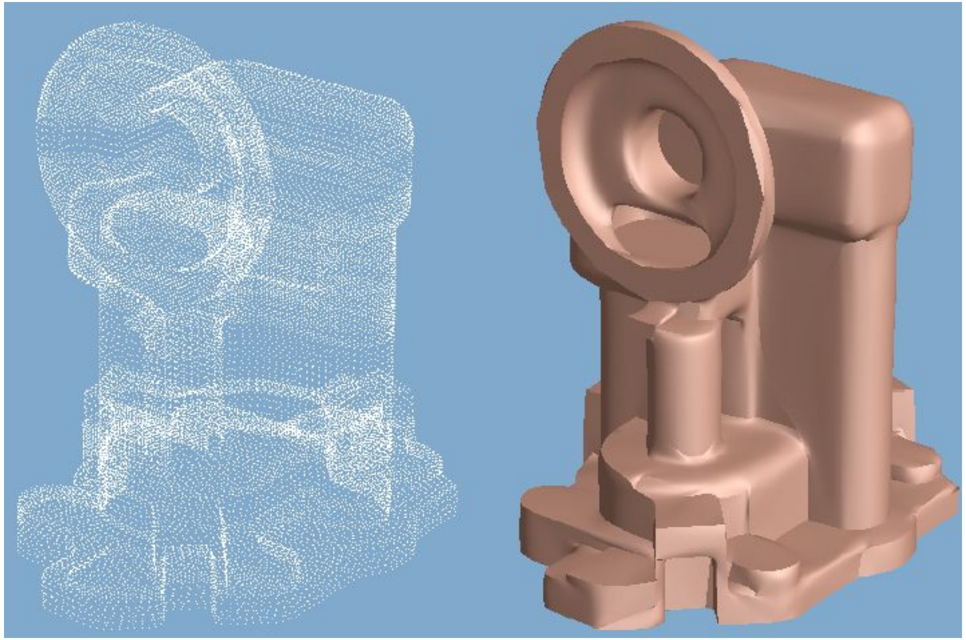
Piecewise Smooth Surface Reconstruction

- Optimization Remeshing



Piecewise Smooth Surface Reconstruction

- Crease subdivision masks *decouple* behavior of surface on either side of crease
- Crease rules cannot model a cone
- Optimization can be done locally
  - subdivision control points have only local influence
- Results
  - Noise?
  - Applicability?
  - Limitations?
  - Running Time



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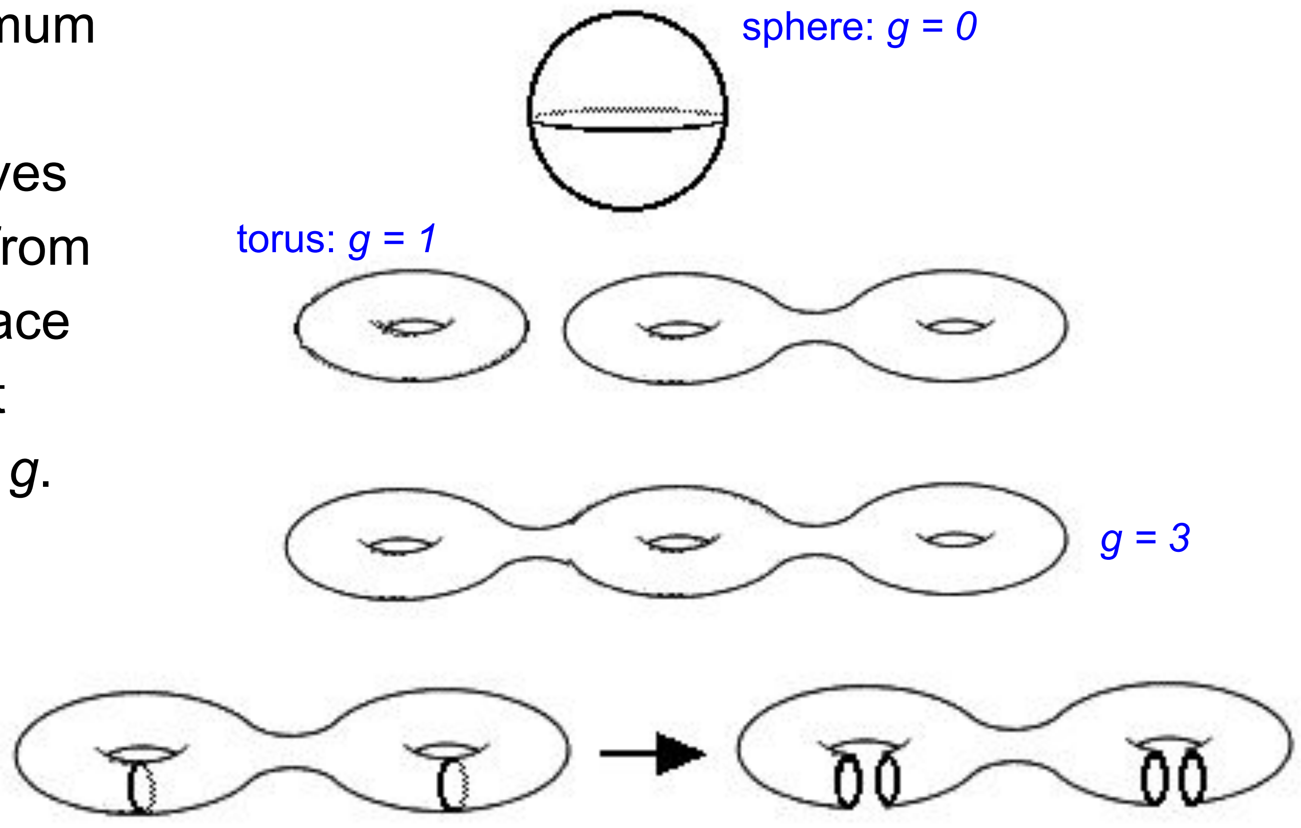
Misc. Mesh/Surface Vocabulary

- *Genus:* The maximum number of disjoint simple closed curves which can be cut from an orientable surface of genus *g* without disconnecting it is *g*.



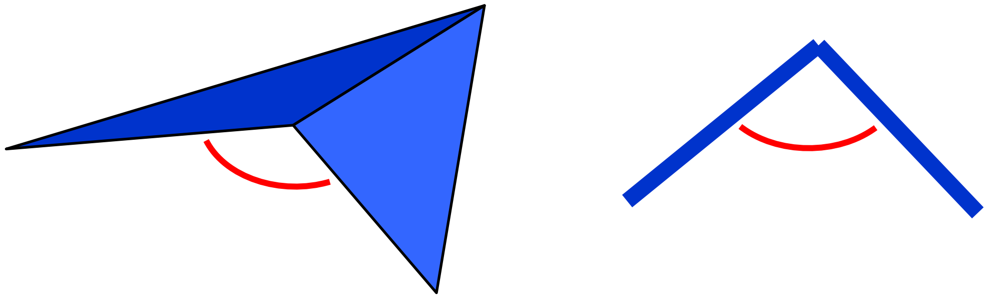
Misc. Mesh/Surface Vocabulary

- *Homeomorphic/Topological equivalence*:
  a continuous stretching and bending of the object into a new shape

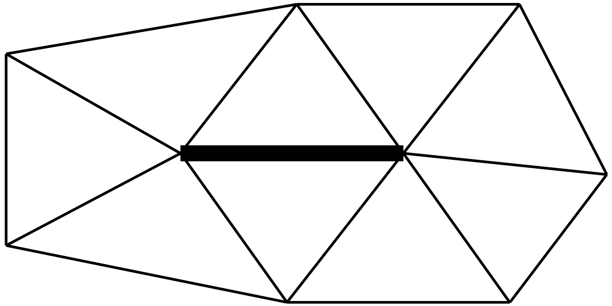
http://en.wikipedia.org/wiki/Image:Mug_and_Torus_morph.gif

Misc. Mesh/Surface Vocabulary

- *Dihedral Angle*:
  - *"looking down the edge" between two faces…*
  - the angle between the planes of two triangular faces

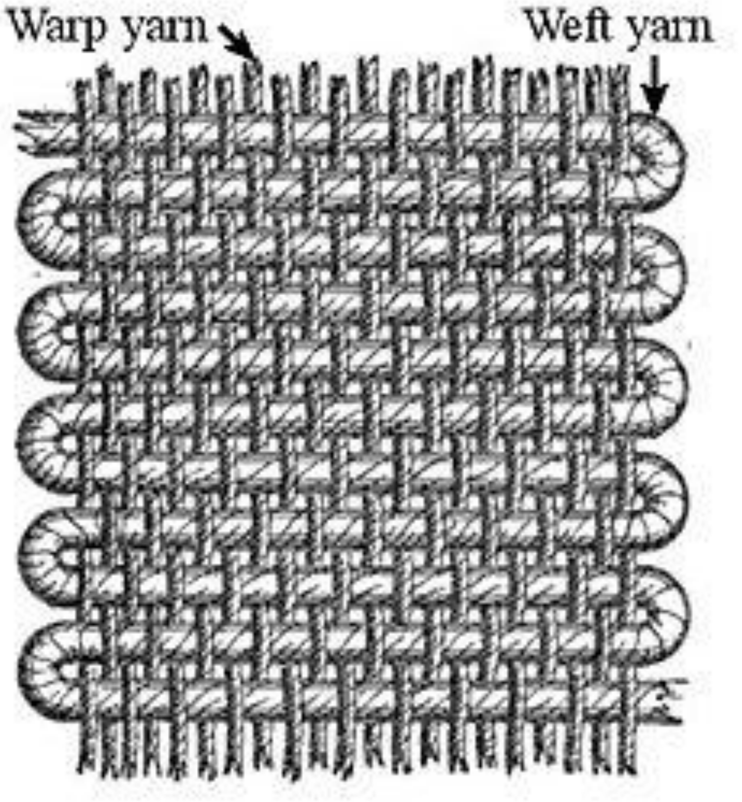


- *Valence (a.k.a. degree)*:
  the number of edges incident to the vertex



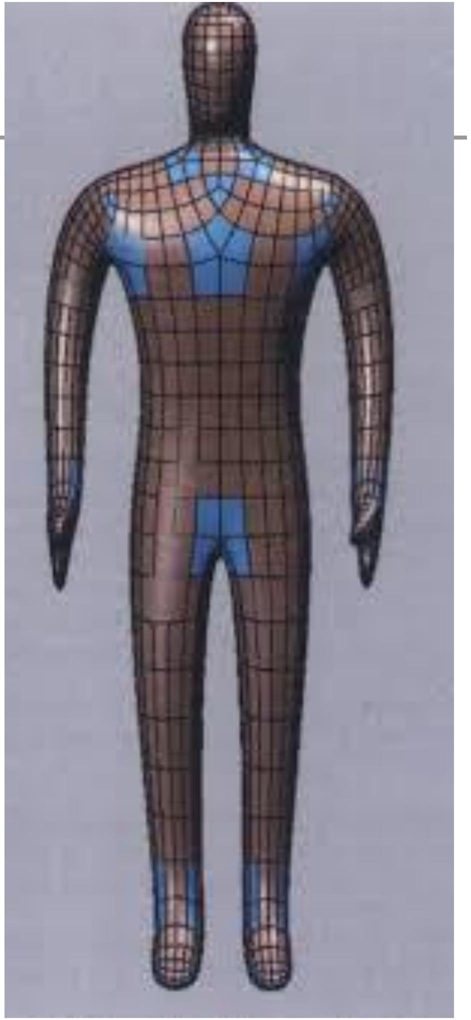
Misc. Mesh/Surface Vocabulary

- *Warp & weft:*
  Yarns used in weaving.
  Because the weft does not have to be stretched in the way that the warp is, it can generally be less strong.

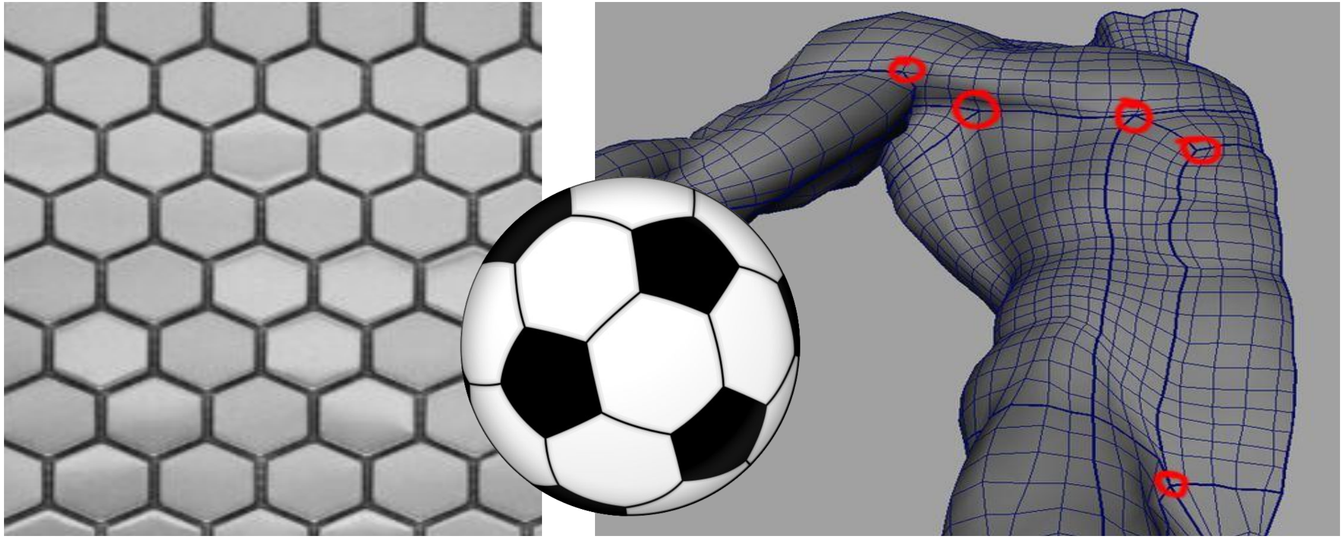


http://en.wikipedia.org/wiki/Weft

Misc. Mesh/Surface Vocabulary



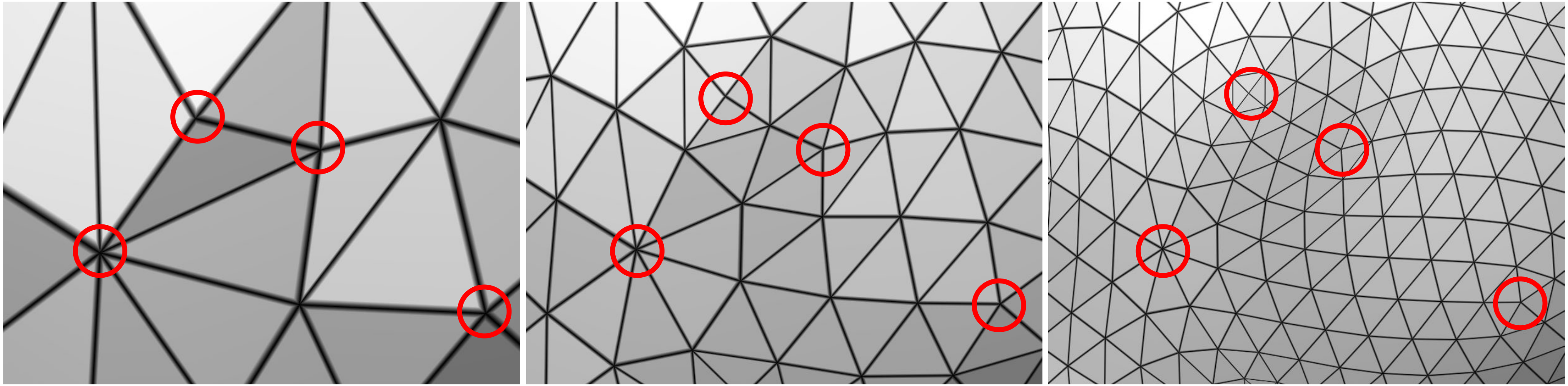
- Extraordinary Vertex
  - Quad mesh: vertices w/ valence ≠ 4
  - Hex mesh: vertices w/ valence ≠ 3
  - Tri mesh: vertices w/ valence ≠ 6



Misc. Mesh/Surface Vocabulary

- Extraordinary Vertex
  - Quad mesh: vertices w/ valence ≠ 4
  - Hex mesh: vertices w/ valence ≠ 3
  - Tri mesh: vertices w/ valence ≠ 6

How does valence change with each subdivision iteration?



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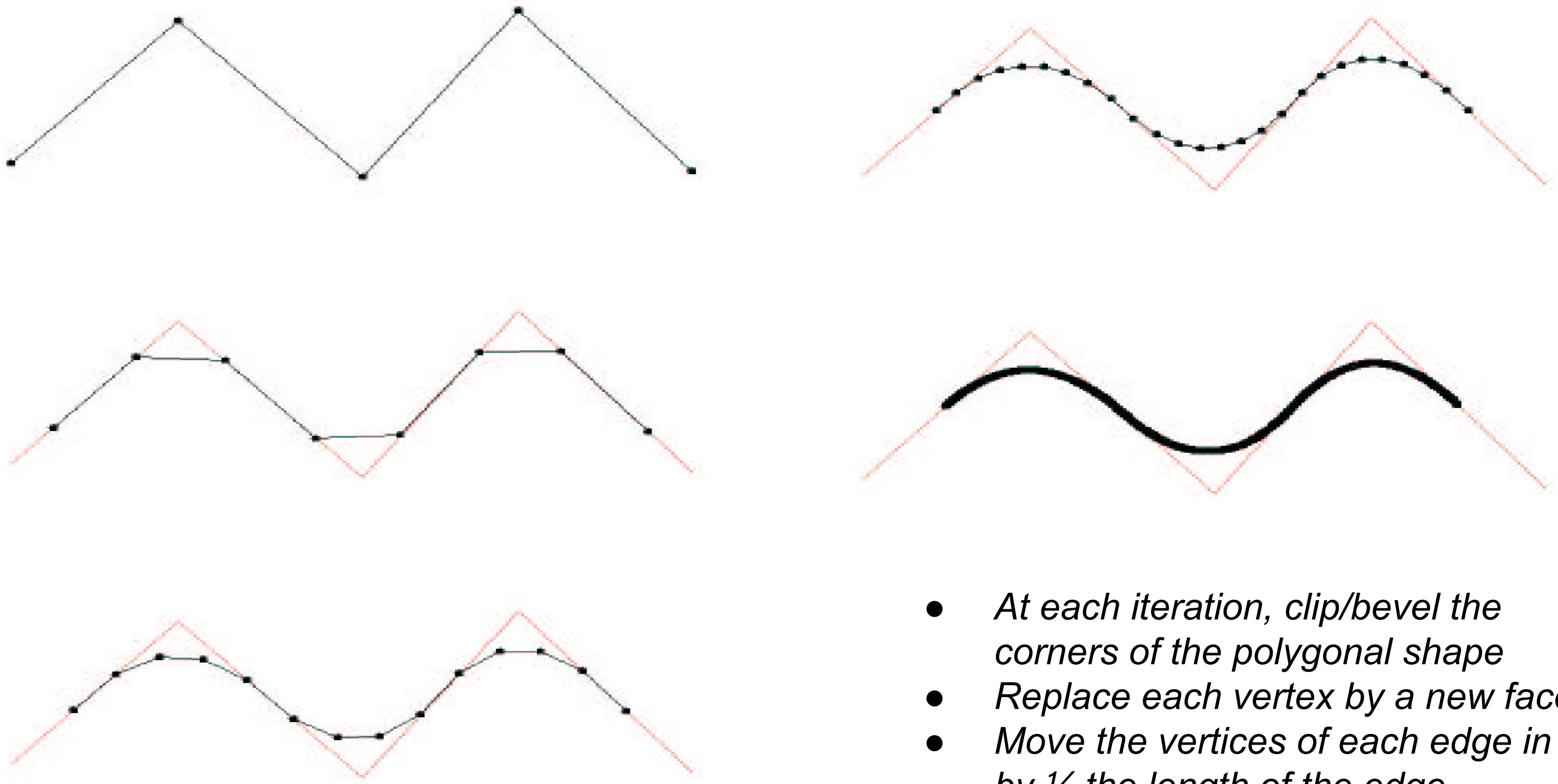
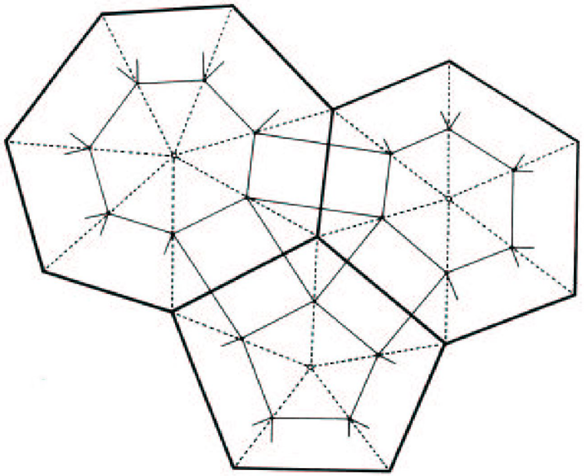
Chaikin's Algorithm

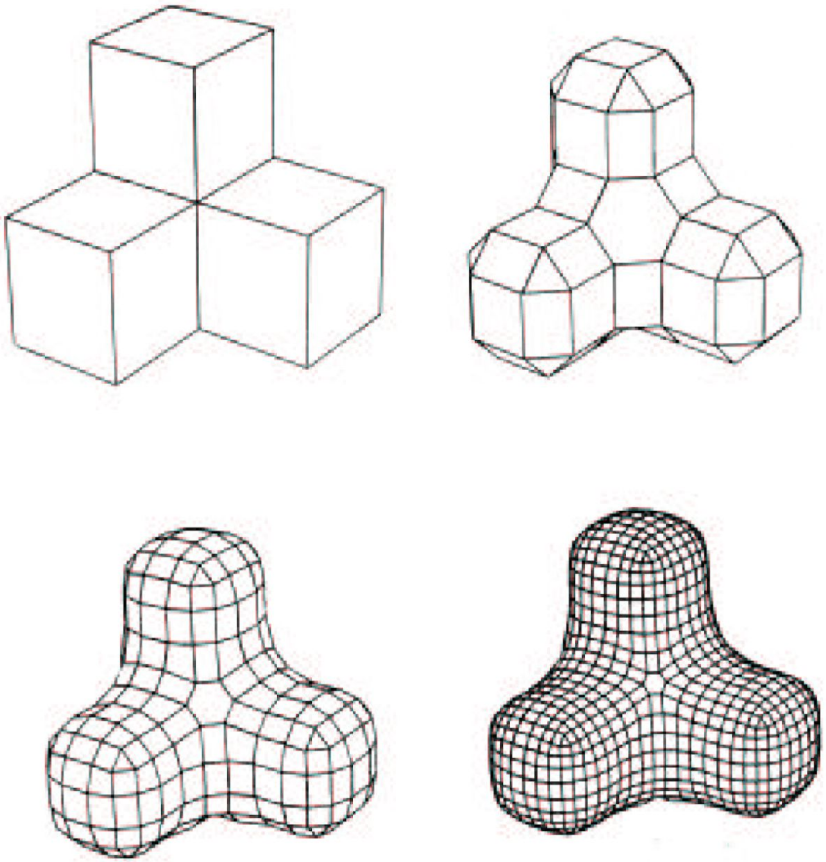


- *At each iteration, clip/bevel the corners of the polygonal shape*
- *Replace each vertex by a new face*
- *Move the vertices of each edge in by ¼ the length of the edge*

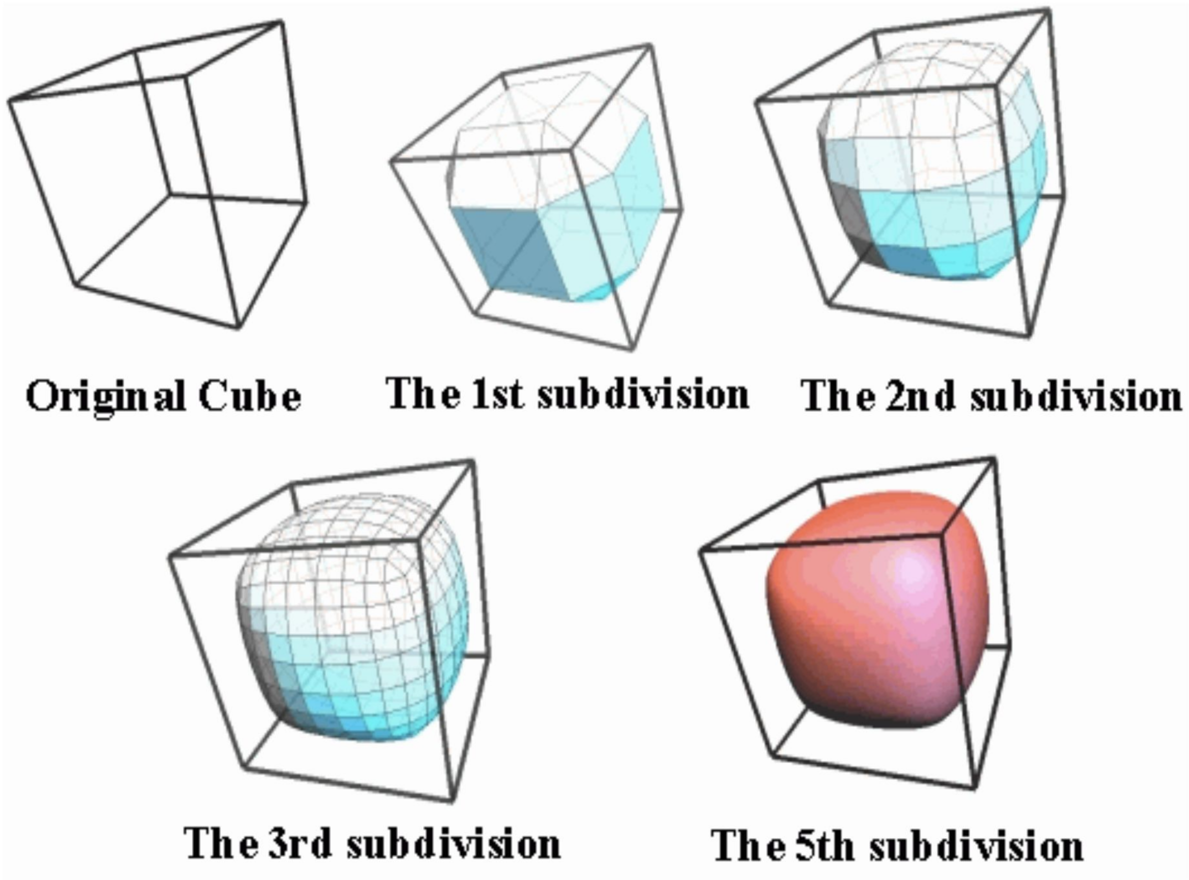
Doo-Sabin Subdivision

- *Shrink each original face:*
  move every vertex halfway to face center
- *Add a new face at each vertex:*
  edge count = original vertex valence
- *Add a quad at each original edge*



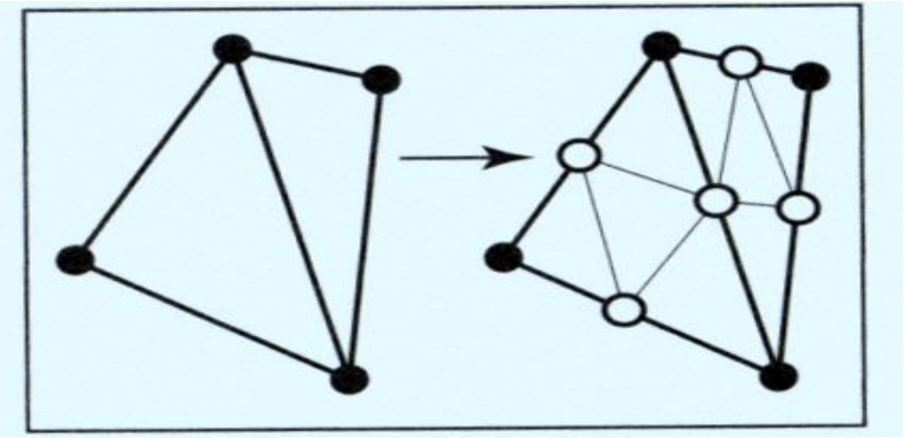


Doo-Sabin Subdivision

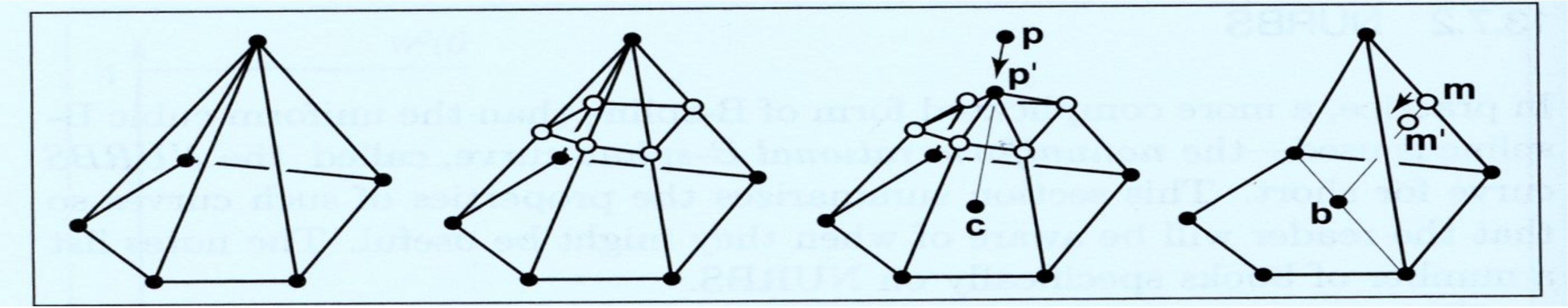


Does a cube turn into a sphere with Doo-Sabin subdivision?

Loop Subdivision



- Add a vertex in the middle of each edge
- Replace each triangle with 4 triangles
- Adjust the positions of both old & new vertices…

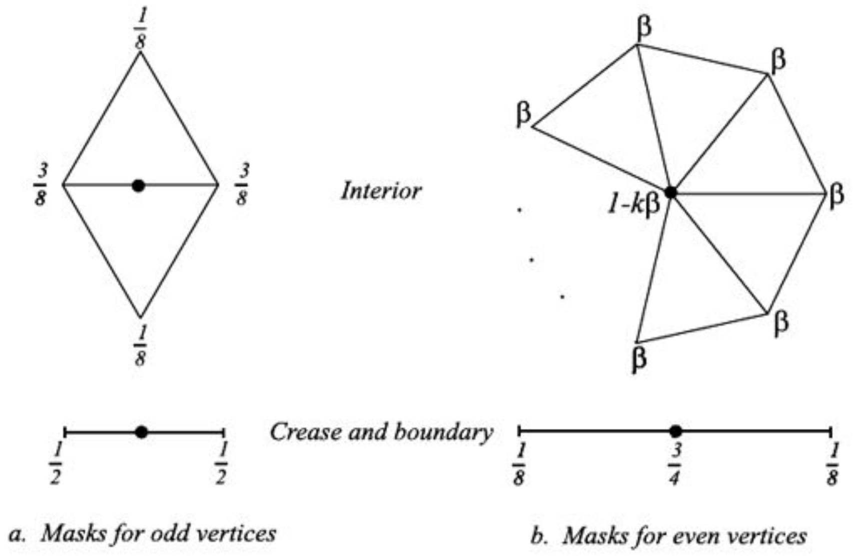


Shirley, Fundamentals of Computer Graphics

Loop Subdivision

Subdivision Rules. The masks for the Loop scheme are shown in Figure 4.3. For boundaries and edges tagged as *crease* edges, special rules are used. These rules produce a cubic spline curve along the boundary/crease. The curve only depends on control points on the boundary/crease.

(new vertices)

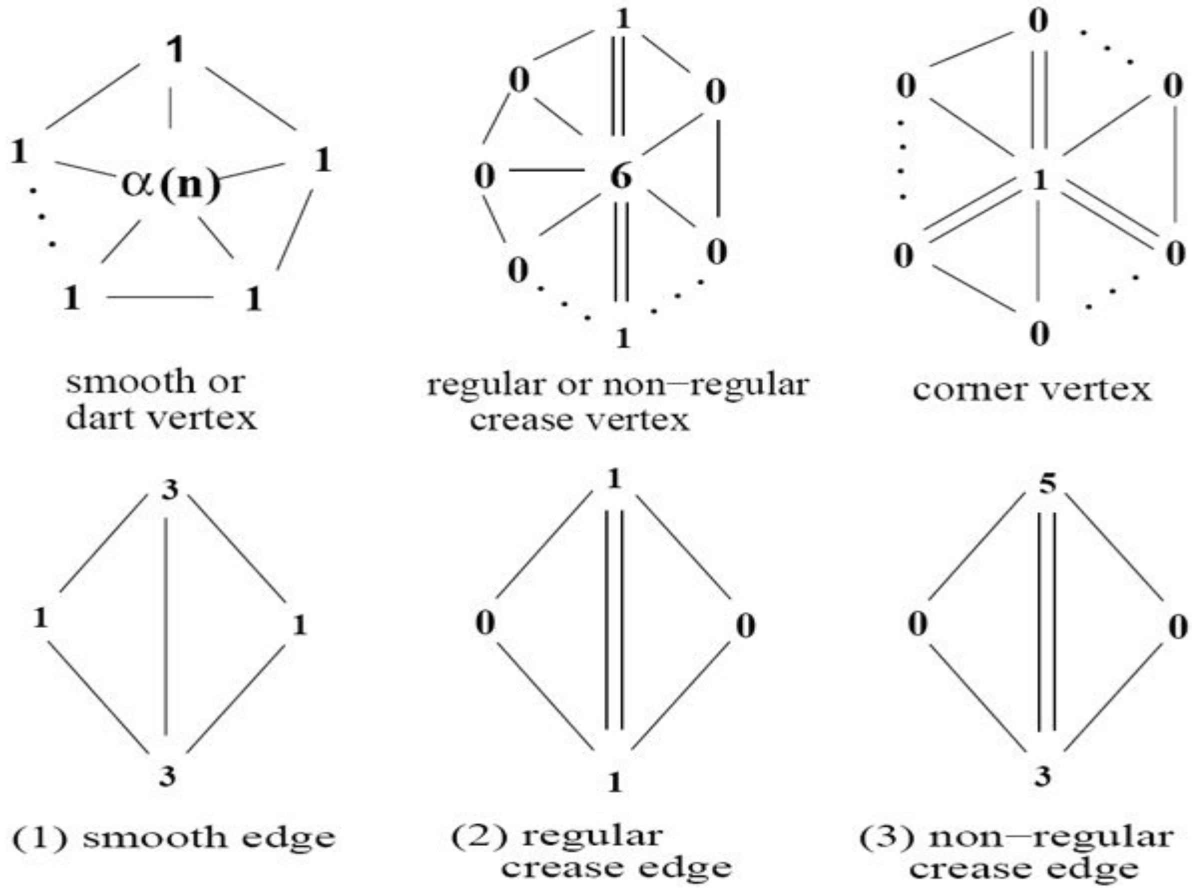


(old/existing vertices)

Figure 4.3: *Loop subdivision: in the picture above,* β *can be chosen to be either* $\frac{1}{n}(5/8-(\frac{3}{8}+\frac{1}{4}\cos\frac{2\pi}{n})^2)$ *(original choice of Loop [16]), or, for* $n>3$, $\beta=\frac{3}{8n}$ *as proposed by Warren [33]. For* $n=3$, $\beta=3/16$ *can be used.*

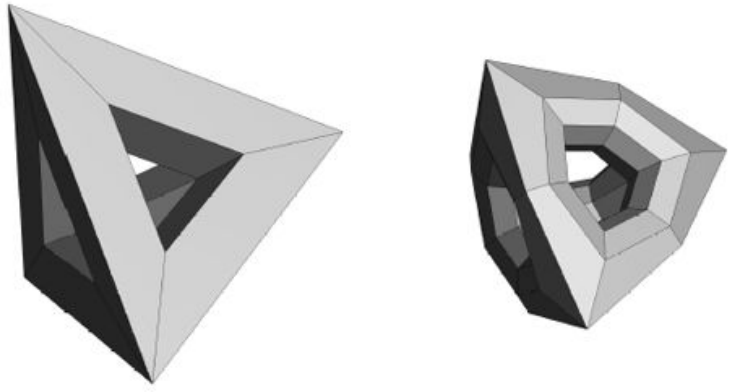
Adding Creases to Loop Subdivision

- Vertex & edge masks
- Limit masks
  - Position
  - Tangent



Hoppe et al.,
“Piecewise Smooth Surface Reconstruction”
SIGGRAPH 1994

Catmull Clark Subdivision



(a) (b)

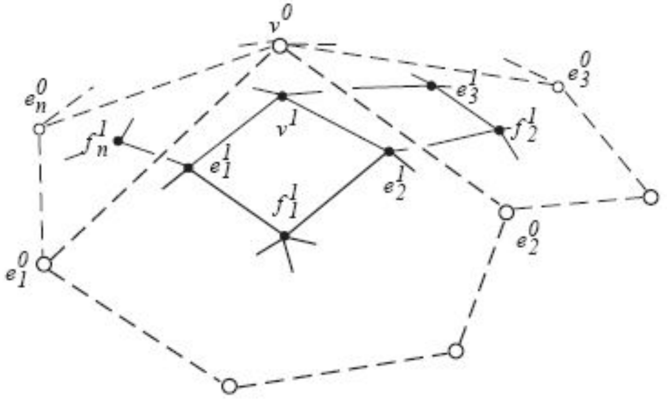
"Subdivision Surfaces in Character Animation", DeRose, Kass & Truong, SIGGRAPH 1998

$$e_j^{i+1} = \frac{v^i + e_j^i + f_{j-1}^{i+1} + f_j^{i+1}}{4}, \tag{1}$$

where subscripts are taken modulo the valence of the central vertex v^0. (The valence of a vertex is the number of edges incident to it.) Finally, a vertex point v^i is computed as

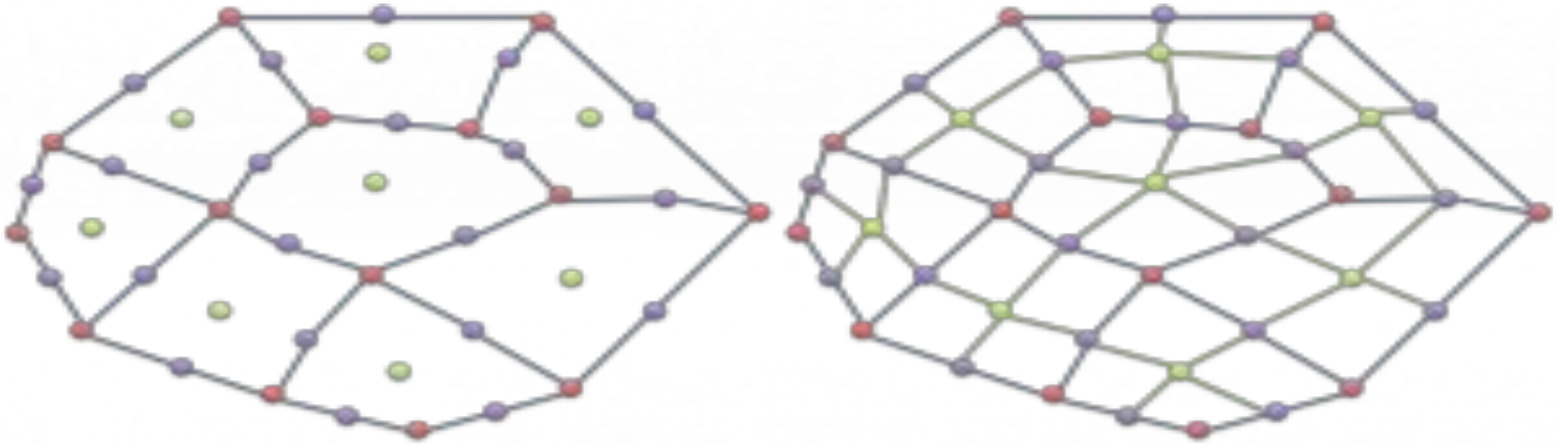
$$v^{i+1} = \frac{n-2}{n} v^i + \frac{1}{n^2} \sum_j e_j^i + \frac{1}{n^2} \sum_j f_j^{i+1} \tag{2}$$

Vertices of valence 4 are called ordinary; others are called extraordinary.



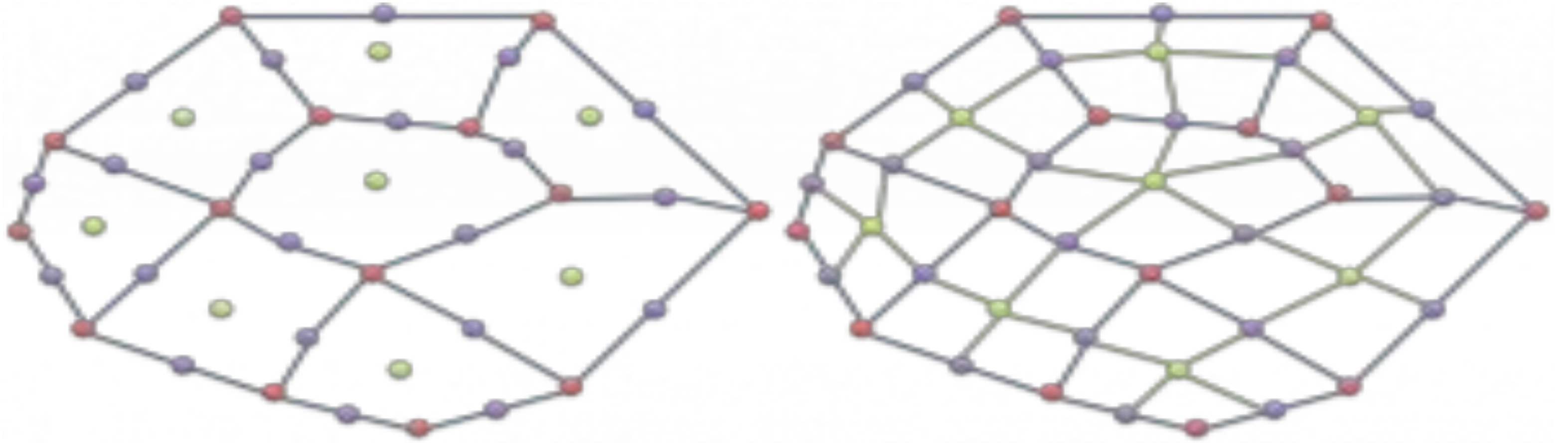
Catmull Clark Subdivision

- Add a vertex in the middle of each original edge
- Add a vertex in the middle of each original face
- Connect each new edge vertex to each new face vertex
- *NOTE: The mesh contains only quads after 1 iteration.*

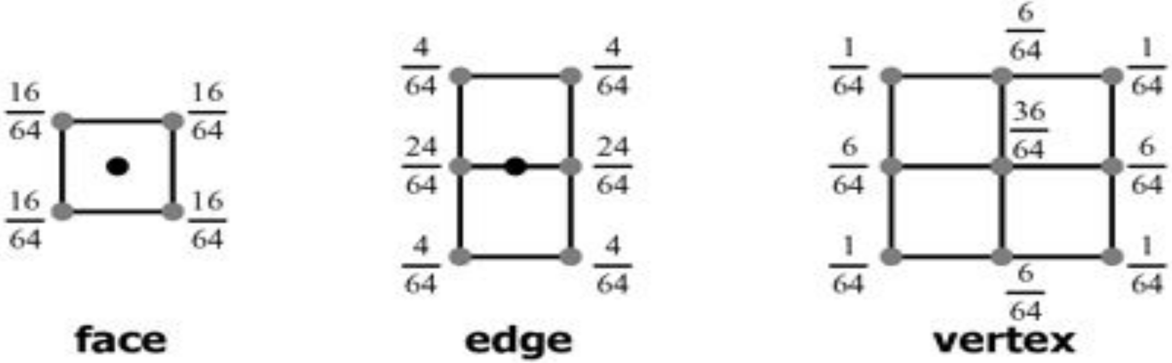


https://team.inria.fr/virtualplants/teaching/informatique-graphique-2016/tp4-instructions/

Catmull Clark Subdivision

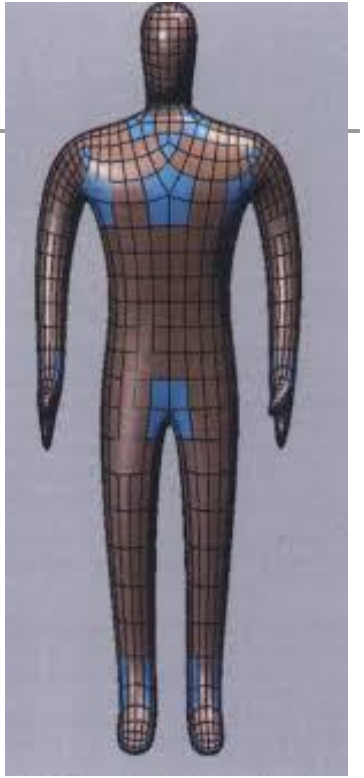


Adjust/average the position of every vertex (old & new) using these masks:

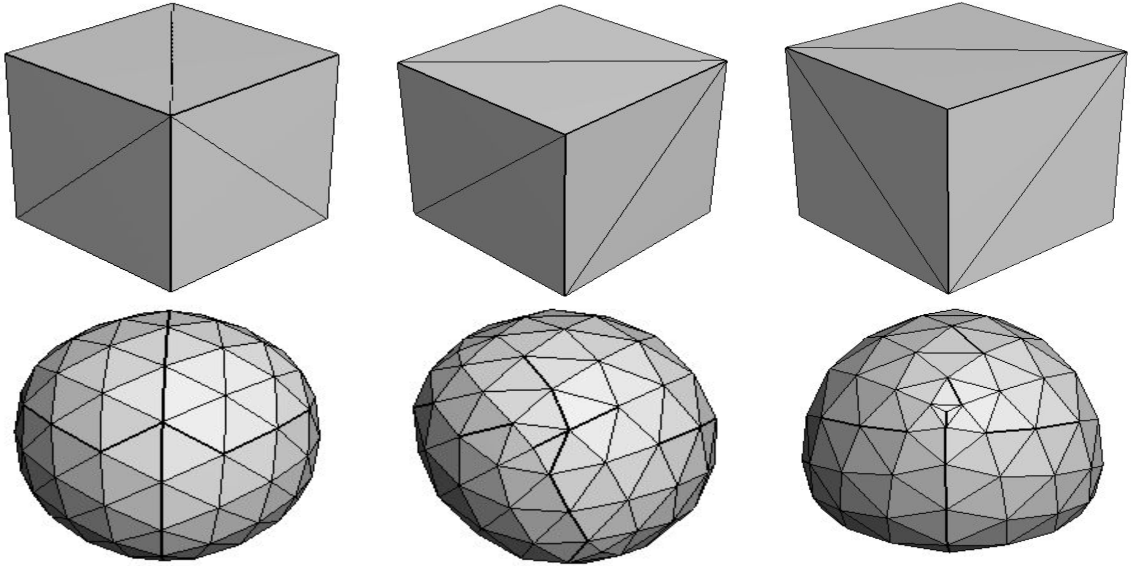


http://www.cl.cam.ac.uk/teaching/2005/AdvGraph/exercise2.html

Catmull Clark is preferred by Artists



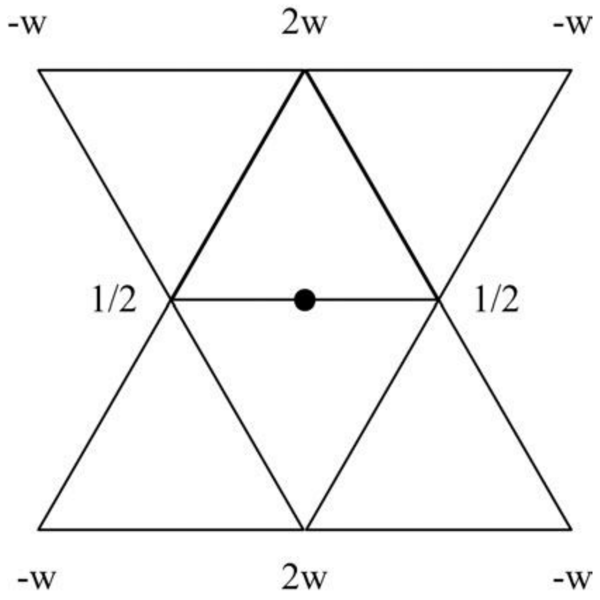
- Catmull-Clark is based on quadrilaterals
  - Like NURBS, specifically cubic BSplines
  - Implicit adjacency in subdivided microgeometry
  - *Quads are better than triangles for symmetric objects*

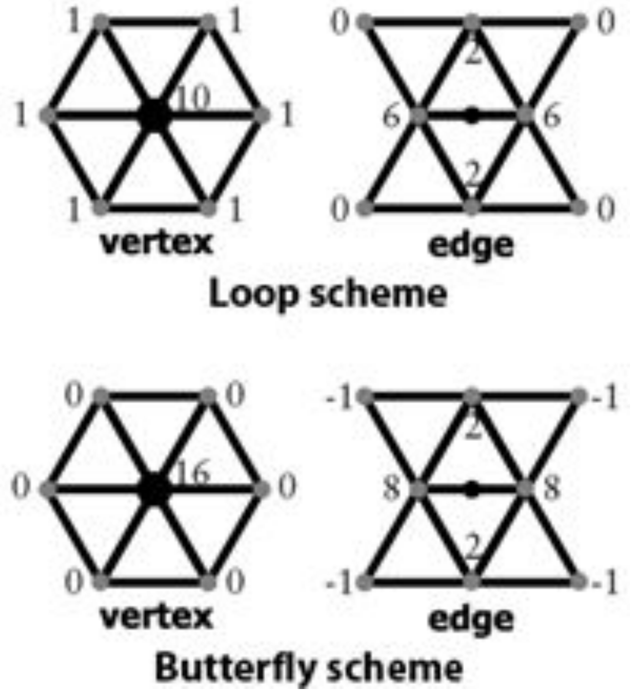


Does a cube turn into a sphere with Loop subdivision? What about with Catmull-Clark subdivision?

Butterfly Subdivision

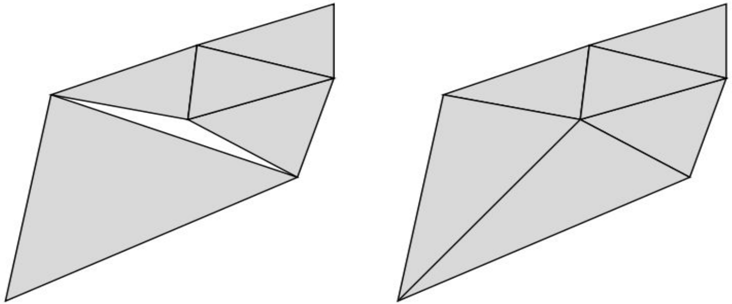
- Triangle-based subdivision
- Alternate scheme to Loop



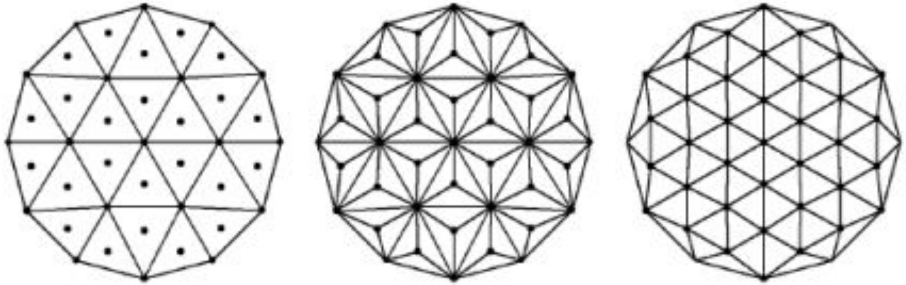


http://www.cl.cam.ac.uk/teaching/2005/AdvGraph/exercise2.html

√3 Subdivision Kobbelt, SIGGRAPH 2000



Adaptive Subdivision (Loop): Need to close gaps between different levels of refinement

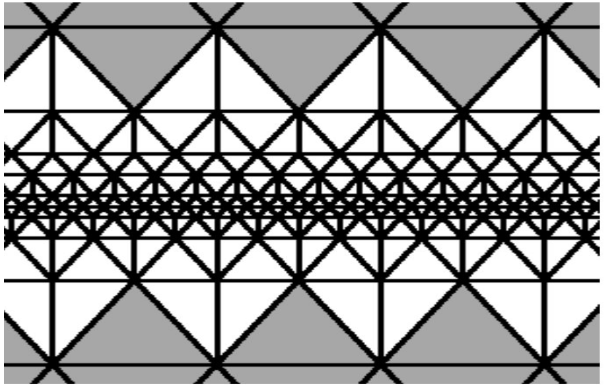


the split operation places a midvertex at the centre of each triangle

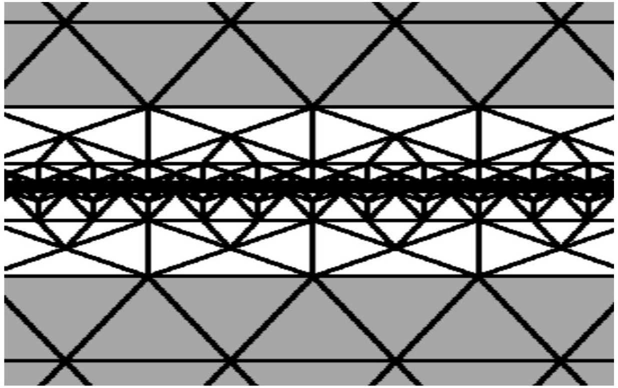
joining the midvertex to the vertices of the triangle realises the 1-to-3 split

after smoothing each old vertex, edges are flipped to connect pairs of midvertices

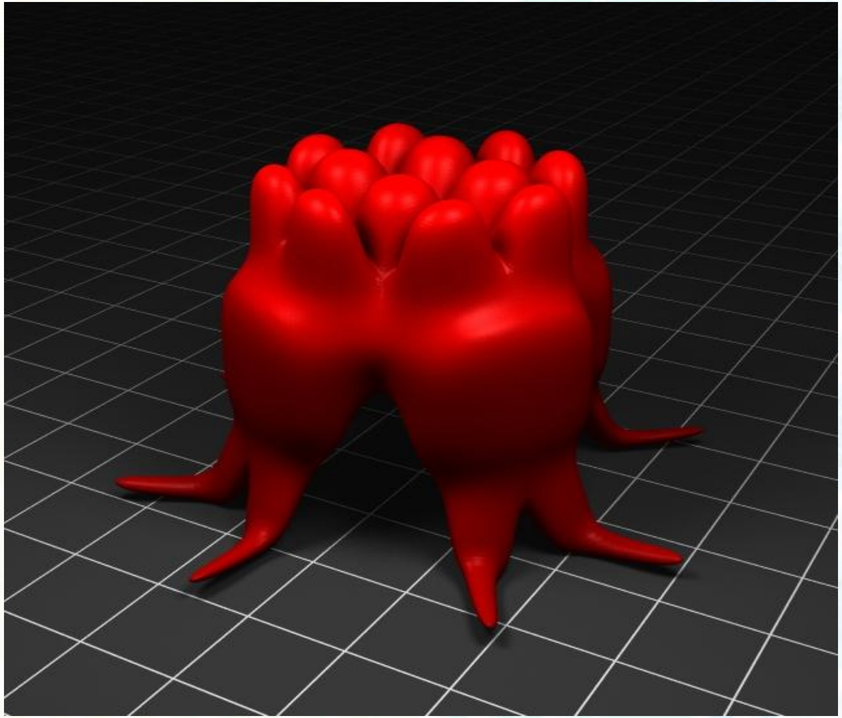
Loop: less localized refinement

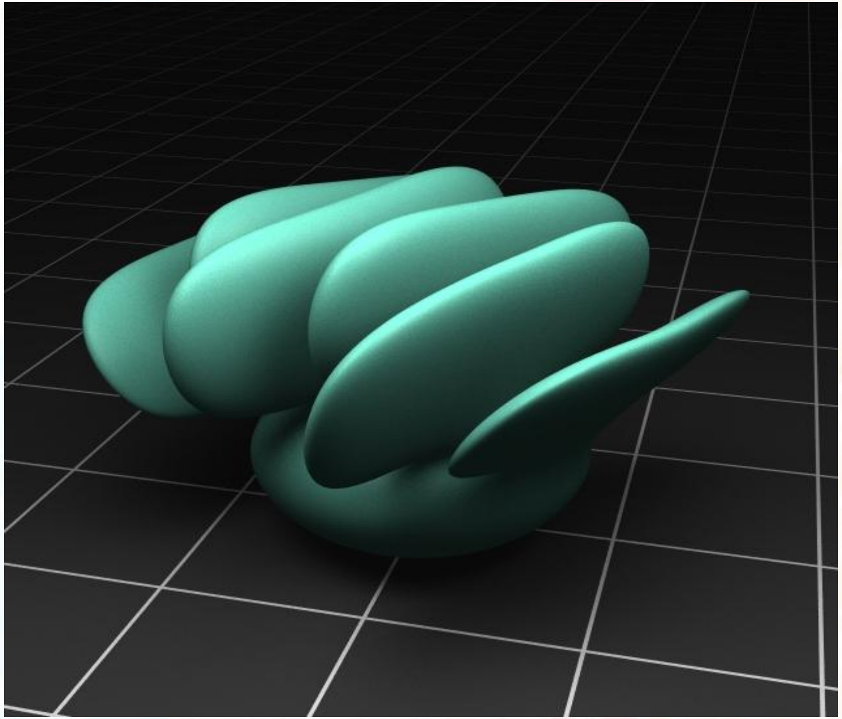


√3: more localized refinement



Questions?





Justin Legakis

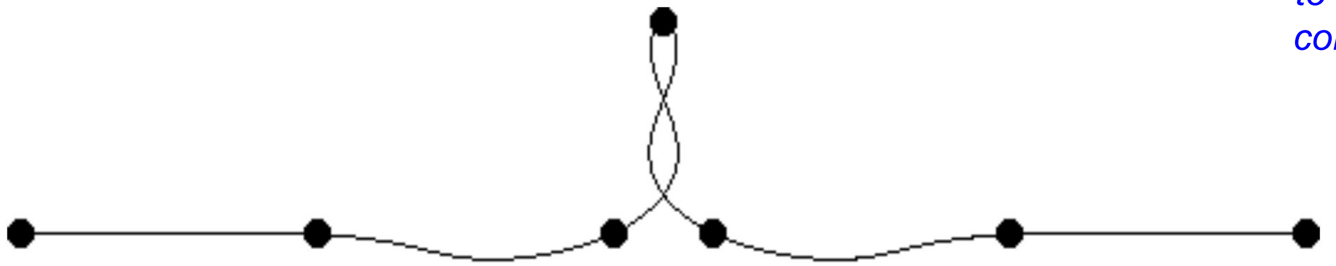
Today

- Worksheet: Shortest Edge Collapse
- From Last Time: Bézier Splines → Bézier Surfaces
- Papers for Today
  - “Subdivision Surfaces in Character Animation”
  - “Piecewise Smooth Surface Reconstruction”
- Misc. Mesh/Surface Vocabulary
- Subdivision Surface “Zoo”
- Interpolating Subdivision
- Papers for Next Time

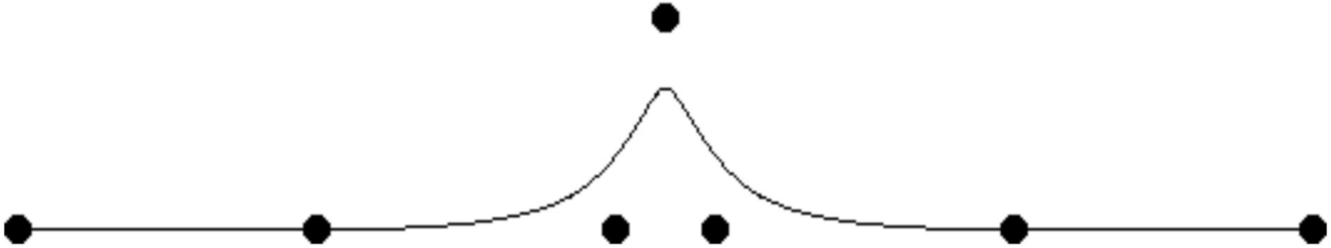
Interpolation vs. Approximation Curves

- Interpolation Curve:
  over constrained → lots of (undesirable?) oscillations

Artists would like to have this direct control of surface!

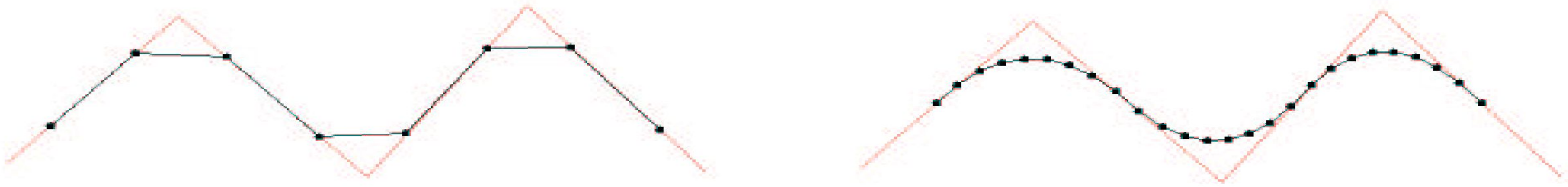


- Approximation Curve – more reasonable?

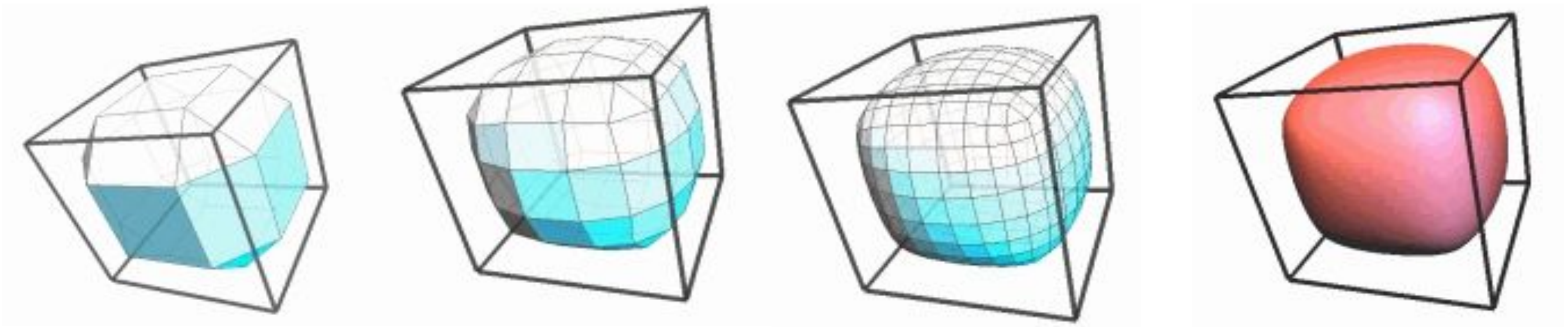


Interpolating Subdivision

- Chaikin:



- Doo-Sabin:



Surface will pass through the centroids of each edge/face

Interpolating Subdivision

- *Interpolation* vs. *Approximation* of control points
- Handle arbitrary topological type
- Reduce the "extraneous bumps & wiggles"

"Efficient, fair interpolation using Catmull-Clark surfaces", Halstead, Kass & DeRose, SIGGRAPH 1993

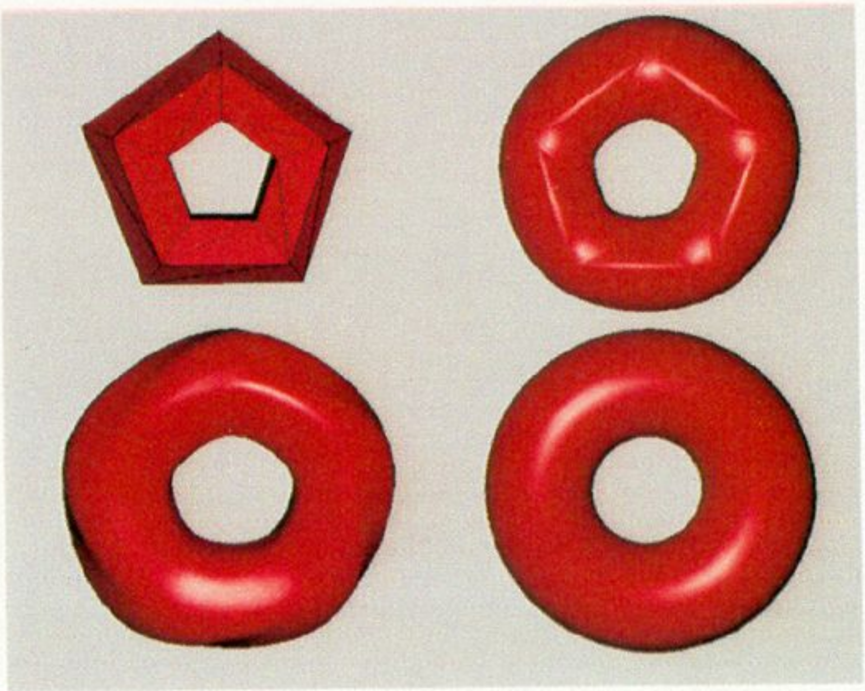
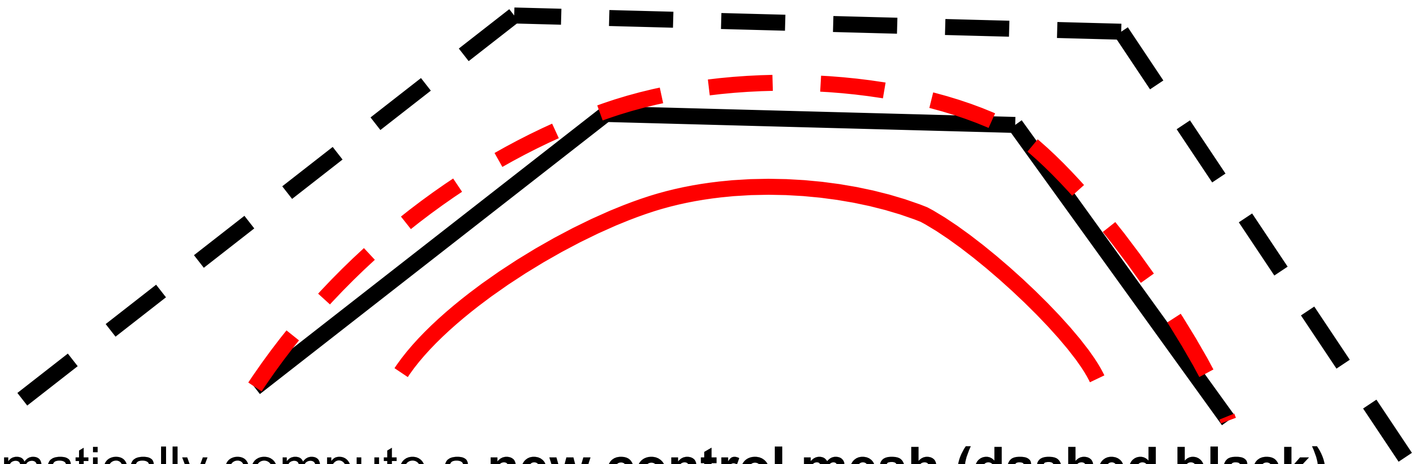


Figure 4: Interpolating a coarsely polygonized torus. Upper left: original mesh. Upper right: Shirman-Séquin interpolation[14]. Lower left: Interpolating Catmull-Clark surface. Lower right: Faired interpolating Catmull-Clark surface.

Interpolation of Catmull-Clark Surfaces

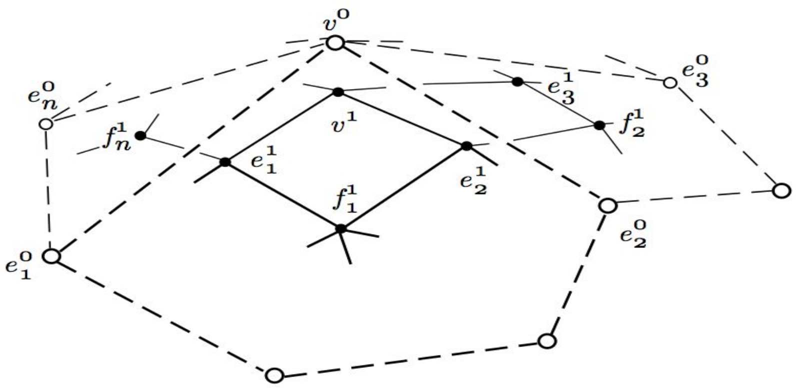
- Artist draws an **input control mesh (solid black)**
  - Catmull Clark subdivision of the input will be **non-interpolating (solid red)**



- Idea: Automatically compute a **new control mesh (dashed black)** such that Catmull Clark of the new mesh ***will interpolate (dashed red)*** the vertices of the original mesh!

Vertex Position in Limit

- V_n stores the center vertex & surrounding as a big column vector



$$V_n^{i+1} = \mathbf{S}_n V_n^i$$

- When n = 4:
  (n = valence)

$$\mathbf{S}_4 = \frac{1}{16} * \begin{pmatrix} 9 & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 6 & 6 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 6 & 1 & 6 & 1 & 0 & 1 & 1 & 0 & 0 \\ 6 & 0 & 1 & 6 & 1 & 0 & 1 & 1 & 0 \\ 6 & 1 & 0 & 1 & 6 & 0 & 0 & 1 & 1 \\ 4 & 4 & 4 & 0 & 0 & 4 & 0 & 0 & 0 \\ 4 & 0 & 4 & 4 & 0 & 0 & 4 & 0 & 0 \\ 4 & 0 & 0 & 4 & 4 & 0 & 0 & 4 & 0 \\ 4 & 4 & 0 & 0 & 4 & 0 & 0 & 0 & 4 \end{pmatrix}$$

$$V_n^{\infty} := \lim_{i \to \infty} \mathbf{S}_n^i V_n^1$$

Solve for New Positions

- Goal: Find the control mesh vertex positions, *x* (a column vector of 3D points), such that the position of the vertices in the limit match the input vertices, *b* (also a column vector of points)
- Use Least Squares to solve

$$\mathbf{A}x = b$$

  where A is a square matrix with the interpolation rules and connectivity of the mesh

- *See paper for extension to match limit normals*

Fairing

- Fairing: an additional part or structure added to an aircraft, tractor-trailer, etc. to smooth the outline and thus reduce drag
- Subdivide initial resolution twice so that all constrained vertex positions are independent

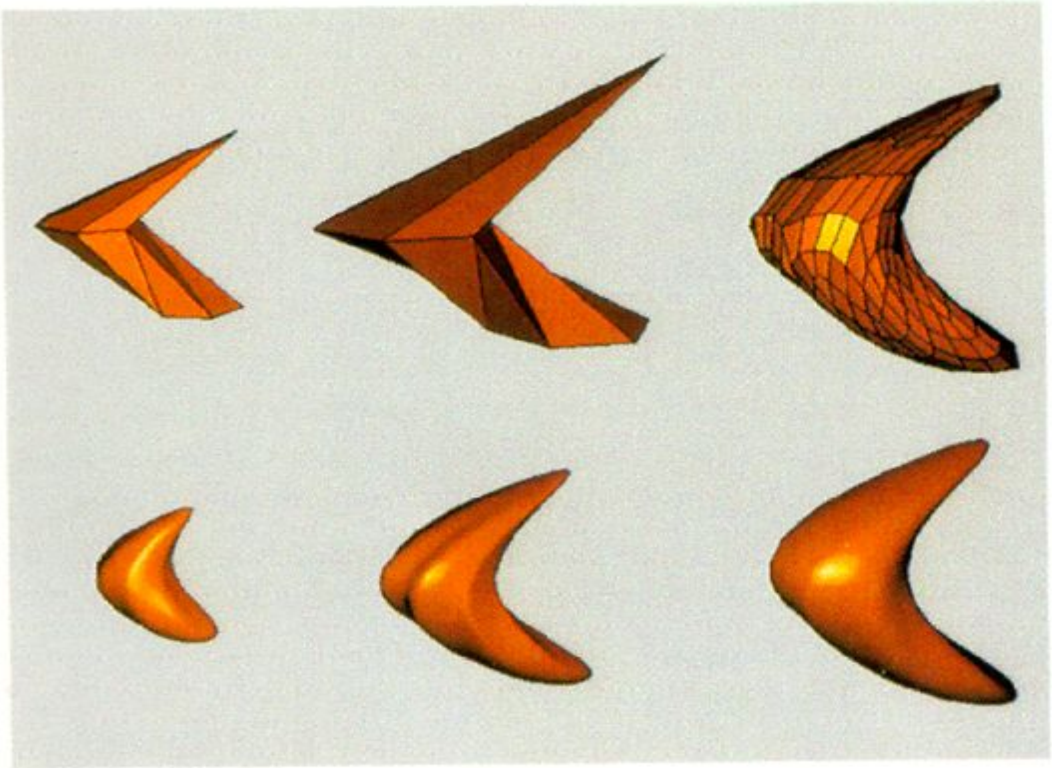


Figure 5: Top row: Original mesh, Interpolating mesh, Faired interpolating mesh. Bottom row: Corresponding Catmull-Clark surfaces. Interpolation introduces wiggles which are removed by fairing.

Questions?

Today

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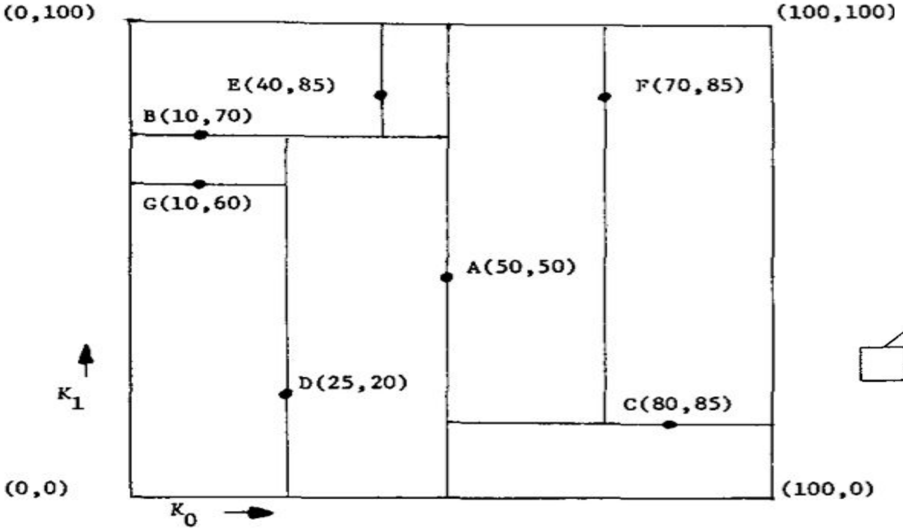
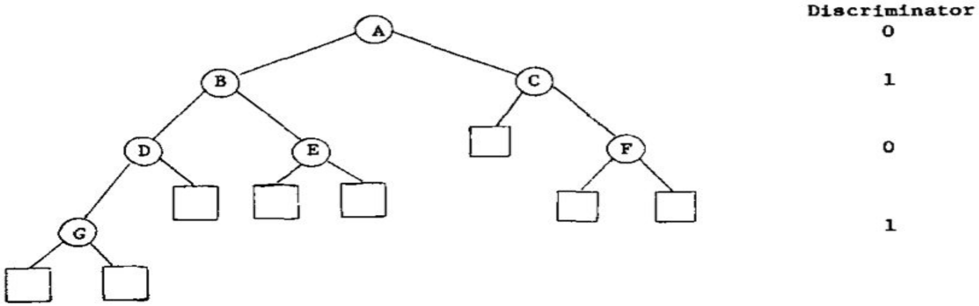
- Interpolating Subdivision
- Papers for Next Time

Reading for Next Time: *(pick one)*

"Multidimensional Binary Search Trees Used for Associative Searching", Bentley, Communications of the ACM, 1975

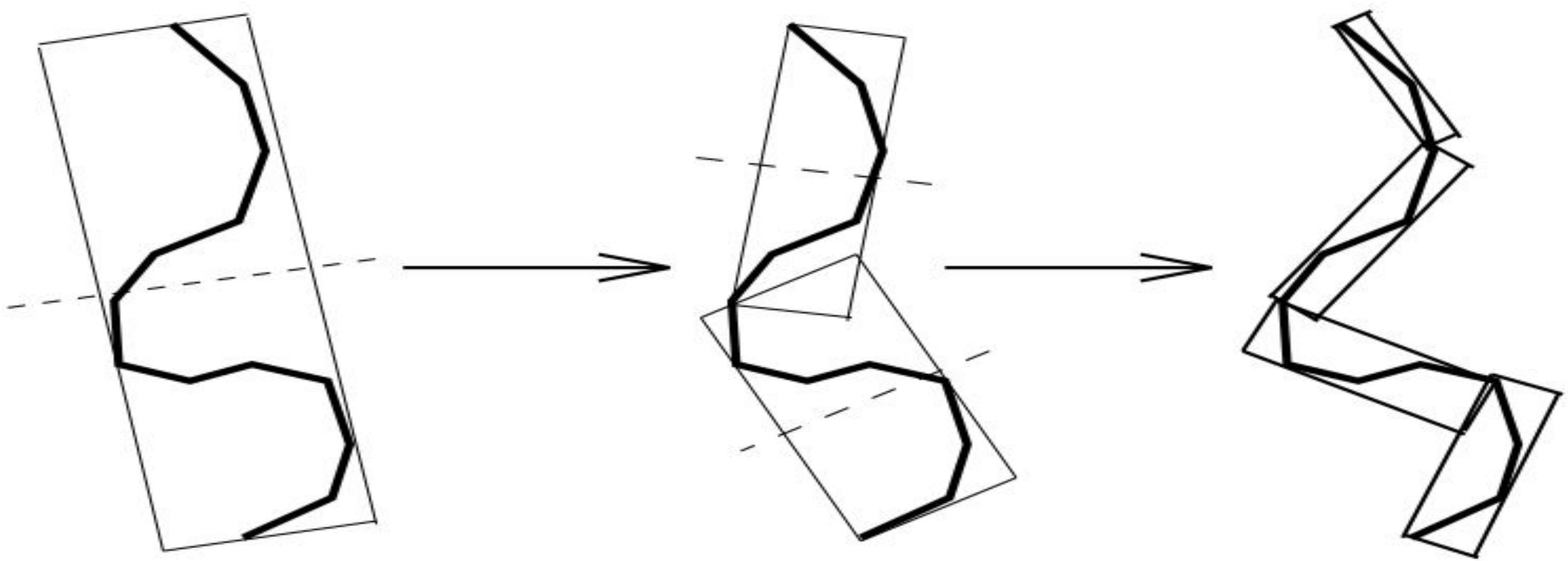




Need 2 volunteers to be "Discussants"
(read one or more of the 3 papers)

Reading for Next Time: *(pick one)*

"OBB-Tree: A Hierarchical Structure for Rapid Interference Detection", Gottschalk, Lin, & Manocha, SIGGRAPH 1996.



Reading for Next Time: *(pick one)*

"Visibility Preprocessing For Interactive Walkthroughs", Teller & Sequin, SIGGRAPH 1991.