

NOTE: Solutions for this assignment will be posted on the web page on Saturday October 16 so that you can see the solutions before the midterm on Tuesday October 19. No late assignments will be accepted once the solutions have been posted. Late solutions can be slipped under my office door, but keep in mind that you will have difficulty doing so once the Amos Eaton building is locked Friday evening.

1. (10 points) Using the predicates $Likes(x, y)$ (i.e. x likes y) and $Hates(x, y)$ (i.e. x hates y), translate the following English sentences into first order logic.
 - (a) Alice likes everyone that likes Bob.
 - (b) Bob likes everyone that Alice likes.
 - (c) There is someone who likes everyone that Alice hates.
 - (d) No one likes anyone that Alice hates.
 - (e) Not everyone hates the people that like Alice.

For example "Everyone who likes Bob likes Alice" becomes $\forall x Likes(x, Bob) \rightarrow Likes(x, Alice)$.

2. (8 points) Transform the following sentences from conjunctive normal form to implicative normal form:
 - (a) $C(x) \vee D(x)$
 - (b) $\neg A(x) \vee B(x) \vee C(x)$
 - (c) $\neg A(x) \vee B(x) \vee \neg C(x)$
 - (d) $\neg A(x) \vee \neg B(x) \vee \neg C(x)$

For example $\neg A(x) \vee B(x)$ becomes $A(x) \rightarrow B(x)$.

3. (12 points) Transform the following sentences into conjunctive normal form (use Universal elimination, Existential elimination, and And-elimination as necessary):
 - (a) $\forall x A(x) \rightarrow \forall y B(x, y) \rightarrow \neg C(y)$
 - (b) $\neg \forall x A(x) \rightarrow \neg B(x)$
 - (c) $\neg \exists x A(x) \wedge B(x) \wedge \neg C(x)$
 - (d) $\forall x (\exists y A(x, y) \wedge \neg B(y)) \rightarrow D(x)$

For example $\forall x A(x) \wedge B(x) \rightarrow C(x) \vee D(x)$ becomes $\neg A(x) \vee \neg B(x) \vee C(x) \vee D(x)$.

4. (30 points) For this question, you will use resolution to solve a problem based on the following facts :

Vegetarians don't eat meat (and someone who doesn't eat meat is a vegetarian). Bob eats anything that Alice eats. Bob is a vegetarian.

 - (a) Translate the above prose into first order logic statements. Use the predicates $Veg(x)$ (x is a vegetarian) and $Eats(x, y)$ (x eats y) and the constants Bob, Alice, and Meat.
 - (b) Transform your sentences from part (a) into conjunctive normal form (not implicative normal form!)
 - (c) Using the (generalized) resolution inference rule, show that Alice is a vegetarian. Give the details each time you use this inference rule (i.e. which sentences are you using? what is the substitution that applies and to which two terms?)
 - (d) Why can't this problem be solved using the generalized modus ponens inference rule?

5. (30 points) For this question, you will apply the forward chaining procedure with (generalized) modus ponens to the following information:

A single male under the age of 25 who drives a sportscar pays high premiums for auto insurance. Bob, who is under 25, likes to drive fast. Anyone who likes to drive fast and is rich drives a sportscar. Bob has a rich aunt, and anyone who has a rich aunt is rich.

- (a) Translate the above prose into first order logic using the predicates:

Single(x)	x is single
Under25(x)	x is under 25 years of age
Sportscar(x)	x drives a sportscar
Male(x)	x is male
Aunt(x, y)	x is the aunt of y
Rich(x)	x is rich
Fast(x)	x likes to drive fast
HighIns(x)	x pays high auto insurance premiums

- (b) Transform your sentences from part (a) into Horn sentences by applying either (1) Existential elimination and And-elimination or (2) Universal elimination.
- (c) Show how forward chaining works on this problem:
- Indicate the non-atomic sentence that are initially in the knowledge base.
 - Add each atomic sentence; whenever the (generalized) modus ponens rule is used, indicate which sentences are involved and the applicable substitution. Show what new sentences are reached and the forward chaining that occurs on those sentences (if any).