CSCI-1200 Data Structures — Fall 2024
Lecture 22 – Priority Queues

Review from Lecture 20 & 21

- STL's `for_each`
- Function Objects, a.k.a. *Functors*
- STL's `unordered_set` (and `unordered_map`)
- Hash functions as functors/function objects (or non-type template parameters, or function pointers)
- Collision resolution: separate chaining vs. open addressing

Today's Lecture

- STL Queue and STL Stack
- What's a Priority Queue?
- Definition of a Binary Heap
- A Priority Queue as a Heap
- Implementing Pop (Delete Min) and Push (Insert)
- A Heap as a Vector
- Building a Heap (all at once)
- Complexity Analysis
- Heap Sort

22.1 Additional STL Container Classes: Stacks and Queues

- We've studied STL vectors, lists, (BST/ordered) maps & sets, and unordered/hash maps & sets. These data structures provide a wide range of flexibility in terms of operations. One way to obtain computational efficiency is to consider a simplified set of operations or functionality.
- For example, with a hash table we give up the notion of a sorted table and gain in find, insert, & erase efficiency.
- 2 additional examples are:
  - **Stacks** allow access, insertion and deletion from only one end called the *top*
    - There is no access to values in the middle of a stack.
    - Stacks may be implemented efficiently in terms of vectors and lists, although vectors are preferable.
    - All stack operations are $O(1)$
  - **Queues** allow insertion at one end, called the *back* and removal from the other end, called the *front*
    - There is no access to values in the middle of a queue.
    - Queues may be implemented efficiently in terms of a list. Using vectors for queues is also possible, but requires more work to get right.
    - All queue operations are $O(1)$

22.2 Suggested Exercises: Tree Traversal using a Stack and Queue

Given a pointer to the root node in a binary tree, write an iterative (non-recursive) function that:

- Uses an STL `stack` to print the elements with a pre-order traversal ordering. *This is straightforward.*
- Uses an STL `stack` to print the elements with an in-order traversal ordering. *This is more complicated.*
- Uses an STL `queue` to print the elements with a breadth-first traversal ordering.

22.3 What's a Priority Queue?

- Priority queues are used in prioritizing operations. Examples include a personal "to do" list, what order to do homework assignments, jobs on a shop floor, packet routing in a network, scheduling in an operating system, or events in a simulation.
- Among the data structures we have studied, their interface is most similar to a queue, including the idea of a `front` or `top` and a `tail` or a `back`.
- Each item is stored in a priority queue using an associated "priority" and therefore, the `top` item is the one with the lowest value of the priority score. The `tail` or `back` is never accessed through the public interface to a priority queue.
- The main operations are `push` (a.k.a. `insert`), and `pop` (a.k.a. `delete_min`).

22.4 Some Data Structure Options for Implementing a Priority Queue

- Vector or list, either sorted or unsorted
  - At least one of the operations, `push` or `pop`, will cost linear time, at least if we think of the container as a linear structure.
- Binary search trees
  - If we use the priority as a `key`, then we can use a combination of finding the minimum key and erase to implement `pop`. An ordinary binary-search-tree insert may be used to implement `push`.
  - This costs logarithmic time in the average case (and in the worst case as well if balancing is used).
- The latter is the better solution, but we would like to improve upon it — for example, it might be more natural if the minimum priority value were stored at the root.
  - We will achieve this with binary *heap*, giving up the complete ordering imposed in the binary *search tree*.

22.5 Definition: Binary Heaps

- A binary heap is a complete binary tree such that at each internal node, p, the value stored is less than the value stored at either of p's children.
  - A complete binary tree is one that is completely filled, except perhaps at the lowest level, and at the lowest level all leaf nodes are as far to the left as possible.
- Binary heaps will be drawn as binary trees, but implemented **using vectors**!
- Alternatively, the heap could be organized such that the value stored at each internal node is greater than the values at its children.

22.6 Exercise: Drawing Binary Heaps

Draw two different binary heaps with these values: 52 13 48 7 32 40 18 25 4

Draw several other trees with these values that *not* binary heaps.

22.7 Implementing Pop (a.k.a. Delete Min)

- The value at the top (root) of the tree is replaced by the value stored in the last leaf node.
  This has echoes of the erase function in binary search trees.
- The last leaf node is removed.
  QUESTION: But how do we find the last leaf? Ignore this for now... we'll answer this later in the lecture.
- The value now at the root likely breaks the heap property. We use the `percolate_down` function to restore the heap property. This function is written here in terms of tree nodes with child pointers (and the priority stored as a `value`), but later it will be re-written in terms of vector subscripts.

```
percolate_down(TreeNode<T> * p) {
  while (p->left) {
    TreeNode<T>* child;
    //  Choose the child to compare against
    if (p->right && p->right->value < p->left->value)
      child = p->right;
    else
      child = p->left;
    if (child->value < p->value) {
      swap(child, p); // value and other non-pointer member vars
      p = child;
    }
    else
      break;
  }
}
```

22.8 Implementing Push (a.k.a. Insert)

- To add a value to the heap, a new last leaf node in the tree is created to store that value.
- Then the `percolate_up` function is run. It assumes each node has a pointer to its parent.

```
percolate_up(TreeNode<T> * p) {
  while (p->parent)
    if (p->value < p->parent->value) {
      swap(p, parent);  // value and other non-pointer member vars
      p = p->parent;
    }
    else
      break;
}
```

22.9 Push (Insert) and Pop (Delete-Min) Usage Exercise

Suppose the following operations are applied to an initially empty binary heap of integers. Show the resulting heap after each `delete_min` operation. (Remember, the tree must be **complete**!)

```
push 5, push 3, push 8, push 10, push 1, push 6,
pop,
push 14, push 2, push 4, push 7,
pop,
pop,
pop
```

22.10 Implementing a Heap with a Vector (instead of Nodes & Pointers)

- In the vector implementation, the tree is never explicitly constructed. Instead the heap is stored as a vector, and the child and parent "pointers" can be implicitly calculated.
- To do this, number the nodes in the tree starting with 0 first by level (top to bottom) and then scanning across each row (left to right). These are the vector indices. Place the values in a vector in this order.
- As a result, for each subscript, i,
  - The parent, if it exists, is at location $\lfloor (i-1)/2 \rfloor$.
  - The left child, if it exists, is at location $2i+1$.
  - The right child, if it exists, is at location $2i+2$.
- For a binary heap containing n values, the last leaf is at location $n-1$ in the vector and the last internal (non-leaf) node is at location $\lfloor (n-2)/2 \rfloor$.
- The standard library (STL) `priority_queue` is implemented as a binary heap.

22.11 Heap as a Vector Exercises

- Draw a binary heap with values: 52 13 48 7 32 40 18 25 4, first as a tree of nodes & pointers, then in vector representation.

- Starting with an initially empty heap, show the vector contents for the binary heap after each `delete_min` operation.

```
push 8, push 12, push 7, push 5, push 17, push 1,
pop,
push 6, push 22, push 14, push 9,
pop,
pop,
```

22.12 Heap Operations Time Complexity Analysis

- Both `percolate_down` and `percolate_up` are $O(\log n)$ in the worst-case. Why?

- But, `percolate_up` (and as a result `push`) is $O(1)$ in the average case. Why?

22.13 Building A Heap ... starting with all of your data in an unorganized vector

- In order to build a heap from a unorganized vector of values, for each index from $\lfloor (n-1)/2 \rfloor$ down to 0, run `percolate_down`. Show that this fully organizes the data as a heap and requires at most $O(n)$ operations.

- If instead, we ran `percolate_up` from each index starting at index 0 through index n-1, we would get properly organized heap data, but incur a $O(n \log n)$ cost in the worst case. Why?

22.14 Heap Sort

- Heap Sort is a simple algorithm to sort a vector of values: Build a heap and then run n consecutive `pop` operations, storing each "popped" value in a new vector.
- It is straightforward to show that this requires $O(n \log n)$ time.
- **Exercise:** Implement an *in-place* heap sort. An in-place algorithm uses only the memory holding the input data – a separate large temporary vector is not needed. Side goal: Keep the number of element copy/swap operations to a minimum!

22.15 Summary Notes about Vector-Based Priority Queues

- Priority queues are conceptually similar to queues, but the order in which values / entries are removed ("popped") depends on a priority.
- Heaps, which are simply drawn with a binary tree but are implemented in a vector, are the data structure of choice for a priority queue.
- In some applications, the priority of an entry may change while the entry is in the priority queue. This requires that there be "hooks" (usually in the form of indices) into the internal structure of the priority queue. This is an implementation detail we have not discussed.