CSCI-1200 Data Structures — Fall 2013
Lecture 18 – Trees, Part III

Review from Lecture 17

- Drawing pictures of binary search trees vs. drawing *abstract* representations of the data stored in an STL set or map.
- Implementing `ds_set` operations: destroy, insert, erase
- Finding the in order successor to a node: add parent pointers *or* add a list/vector/stack of pointers to the iterator.

Today's Lecture

- Tree height, longest-shortest paths, breadth-first search
- Erase with parent pointers, increment operation on iterators
- Limitations of our `ds_set` implementation, brief intro to red-black trees

18.1 Height and Height Calculation Algorithm

- The *height* of a node in a tree is the length of the longest path down the tree from that node to a leaf node. The height of a leaf is 1. We will think of the height of a null pointer as 0.
- The height of the tree is the height of the root node, and therefore if the tree is empty the height will be 0.

  Exercise: Write a simple recursive algorithm to calculate the height of a tree.

- What is the best/average/worst-case running time of this algorithm? What is the best/average/worst-case memory usage of this algorithm? Give a specific example tree that illustrates each case.

18.2 Shortest Paths to Leaf Node

- Now let's write a function to instead calculate the *shortest* path to a NULL child pointer.

- What is the running time of this algorithm? Can we do better? *Hint: How does a breadth-first vs. depth-first algorithm for this problem compare?*

18.3 Tree Iterator Increment/Decrement - Implementation Choices

- The increment operator should change the iterator's pointer to point to the next TreeNode in an in-order traversal — the "in-order successor" — while the decrement operator should change the iterator's pointer to point to the "in-order predecessor".
- Unlike the situation with lists and vectors, these predecessors and successors are not necessarily "nearby" (either in physical memory or by following a link) in the tree, as examples we draw in class will illustrate.
- There are two common solution approaches:
  - Each node stores a parent pointer. Only the root node has a null parent pointer. [method 1]
  - Each iterator maintains a stack of pointers representing the path down the tree to the current node. [method 2]
- If we choose the parent pointer method, we'll need to rewrite the `insert` and `erase` member functions to correctly adjust parent pointers.
- Although iterator increment looks expensive in the worst case for a single application of `operator++`, it is fairly easy to show that iterating through a tree storing n nodes requires $O(n)$ operations overall.

Exercise: [method 1] Write a fragment of code that given a node, finds the in-order successor using parent pointers. Be sure to draw a picture to help you understand!

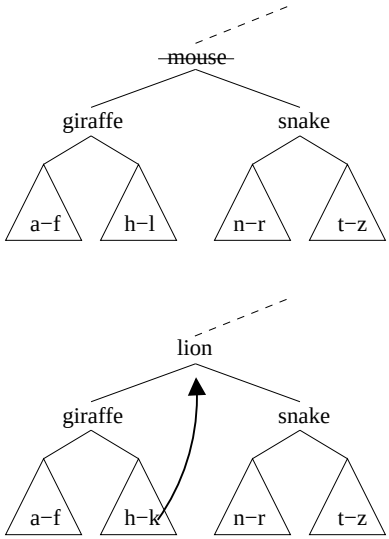
Exercise: [method 2] Write a fragment of code that given a tree iterator containing a pointer to the node *and* a stack of pointers representing path from root to node, finds the in-order successor (without using parent pointers).

Either version can be extended to complete the implementation of increment/decrement for the `ds_set` *tree iterators.*

Exercise: What are the advantages & disadvantages of each method?

18.4 Erase (now with parent pointers)

- If we choose to use parent pointers, we need to add to the Node representation, and re-implement several functions.
- Let's re-implement erase. Remember that first we need to find the node to remove. Once it is found, the actual removal is easy if the node has no children or only one child. It is harder if there are two children:
  - Find the node with the greatest value in the left subtree or the node with the smallest value in the right subtree.
  - The value in this node may be safely moved into the current node because of the tree ordering.
  - Then we recursively apply erase to remove that node — which is guaranteed to have at most one child.



Exercise: Rewrite erase, now with parent pointers.

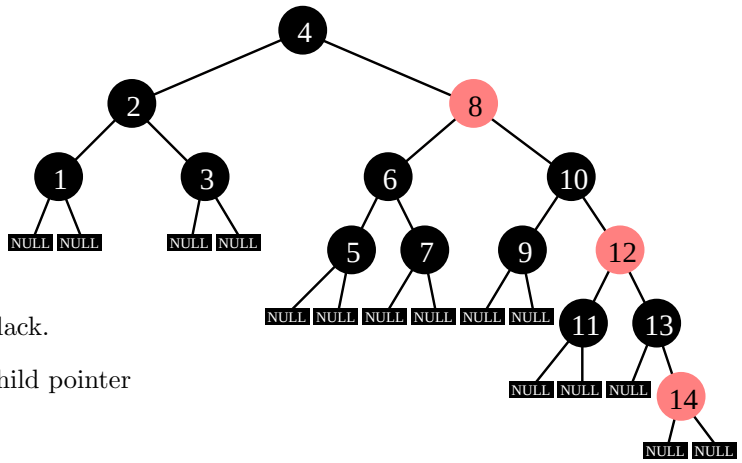
18.5 Limitations of Our BST Implementation

- The efficiency of the main insert, find and erase algorithms depends on the height of the tree.
- The best-case and average-case heights of a binary search tree storing n nodes are both $O(\log n)$. The worst-case, which often can happen in practice, is $O(n)$.
- Developing more sophisticated algorithms to avoid the worst-case behavior will be covered in Introduction to Algorithms. One elegant extension to binary search tree is described below...

18.6 Red-Black Trees

In addition to the binary search tree properties, the following red-black tree properties are maintained throughout all modifications to the data structure:

1. Each node is either red or black.
2. The NULL child pointers are black.
3. Both children of every red node are black.
   Thus, the parent of a red node must also be black.
4. All paths from a particular node to a NULL child pointer contain the same number of black nodes.



What tree does our `ds_set` implementation produce if we insert the numbers 1-14 *in order*?
The tree at the right is the result using a red-black tree. Notice how the tree is still quite balanced.
Visit these links for an animation of the sequential insertion and re-balancing:

http://www.ibr.cs.tu-bs.de/courses/ss98/audii/applets/BST/RedBlackTree-Example.html

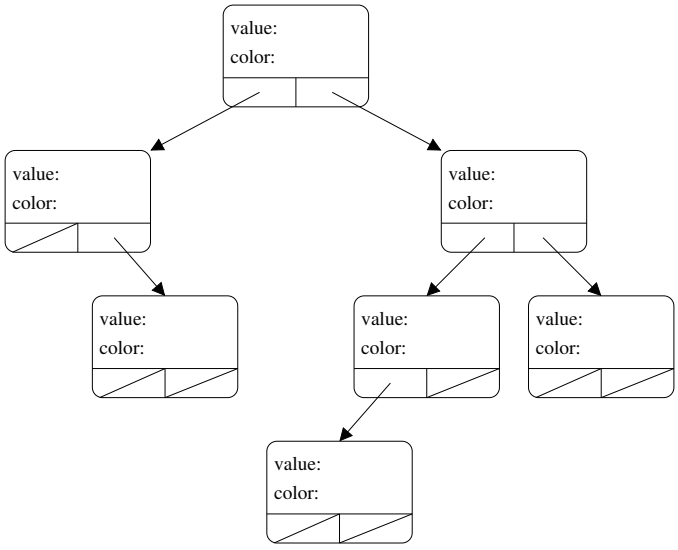
http://www.youtube.com/watch?v=vDHFF4wjWYU&noredirect=1

- What is the best/average/worst case height of a red-black tree with n nodes?

- What is the best/average/worst case shortest-path from root to leaf node in a red-black tree with n nodes?

18.7 Exercise [/6]

Fill in the tree on the right with the integers 1-7 to make a binary search tree. Also, color each node "red" or "black" so that the tree also fulfills the requirements of a Red-Black tree.



Draw two other red-black binary search trees with the values 1-7.

```
// ---------------------------------------------------------------------
// TREE NODE CLASS
template <class T>
class TreeNode {
public:
  TreeNode() : left(NULL), right(NULL), parent(NULL) {}
  TreeNode(const T& init) : value(init), left(NULL), right(NULL), parent(NULL) {}
  T value;
  TreeNode* left;
  TreeNode* right;
  TreeNode* parent; // to allow implementation of iterator increment & decrement
};

// ---------------------------------------------------------------------
// TREE NODE ITERATOR CLASS
template <class T>
class tree_iterator {
public:
  tree_iterator() : ptr_(NULL) {}
  tree_iterator(TreeNode<T>* p) : ptr_(p) {}
  tree_iterator(const tree_iterator& old) : ptr_(old.ptr_) {}
  ~tree_iterator() {}
  tree_iterator& operator=(const tree_iterator& old) { ptr_ = old.ptr_;  return *this; }
  // operator* gives constant access to the value at the pointer
  const T& operator*() const { return ptr_->value; }
  // comparions operators are straightforward
  friend bool operator== (const tree_iterator& lft, const tree_iterator& rgt)
  { return lft.ptr_ == rgt.ptr_; }
  friend bool operator!= (const tree_iterator& lft, const tree_iterator& rgt)
  { return lft.ptr_ != rgt.ptr_; }
  // increment & decrement operators
  tree_iterator<T> & operator++() { /* implemented in Lecture 18 */









    return *this;
  }
  tree_iterator<T> operator++(int) {  tree_iterator<T> temp(*this);  ++(*this);  return temp;  }
  tree_iterator<T> & operator--() { /* implementation omitted */ }
  tree_iterator<T> operator--(int) {  tree_iterator<T> temp(*this);  --(*this);  return temp;  }
private:
  // representation
  TreeNode<T>* ptr_;
};

// ---------------------------------------------------------------------
// DS_ SET CLASS
template <class T>
class ds_set {
public:
  ds_set() : root_(NULL), size_(0) {}
  ds_set(const ds_set<T>& old) : size_(old.size_) { root_ = this->copy_tree(old.root_,NULL); }
  ~ds_set() { this->destroy_tree(root_); root_ = NULL; }
  ds_set& operator=(const ds_set<T>& old) { /* implementation omitted */ }

  typedef tree_iterator<T> iterator;
  int size() const { return size_; }
  bool operator==(const ds_set<T>& old) const { return (old.root_ == this->root_); }

  iterator find(const T& key_value) { return find(key_value, root_); }
  std::pair< iterator, bool > insert(T const& key_value) { return insert(key_value, root_, NULL); }
  int erase(T const& key_value) { return erase(key_value, root_); }
```

```
  // MAKE SURE THE DATA STRUCTURE'S CHILD & PARENT POINTERS ARE CONSISTENT
  bool sanity_check() const {
    if (root_ == NULL) return true;
    if (root_->parent != NULL) return false;
    return sanity_check(root_);
  }
  iterator begin() const {
    if (!root_) return iterator(NULL);
    TreeNode<T>* p = root_;
    while (p->left) p = p->left;
    return iterator(p);
  }
  iterator end() const { return iterator(NULL); }

private:
  // REPRESENTATION
  TreeNode<T>* root_;
  int size_;
  // now with parent pointers, need to pass the current node as the parent pointer for the child
  TreeNode<T>*  copy_tree(TreeNode<T>* old_root, TreeNode<T>* the_parent) {
    if (old_root == NULL) return NULL;
    TreeNode<T> *answer = new TreeNode<T>();
    answer->value = old_root->value;
    answer->left = copy_tree(old_root->left,answer);
    answer->right = copy_tree(old_root->right,answer);
    answer->parent = the_parent; // link up the parent
    return answer;
  }
  void destroy_tree(TreeNode<T>* p) { /* implementation omitted */ }
  iterator find(const T& key_value, TreeNode<T>* p) { /* implementation omitted */ }
  // now with parent pointers, need to pass the current node as the parent pointer for the child
  std::pair<iterator,bool> insert(const T& key_value, TreeNode<T>*& p, TreeNode<T>* the_parent) {
    if (!p) {
      p = new TreeNode<T>(key_value);
      p->parent = the_parent; // link up the parent
      this->size_++;
      return std::pair<iterator,bool>(iterator(p), true);
    }
    else if (key_value < p->value)
      return insert(key_value, p->left, p);
    else if (key_value > p->value)
      return insert(key_value, p->right, p);
    else
      return std::pair<iterator,bool>(iterator(p), false);
  }

  int erase(T const& key_value, TreeNode<T>* &p) { /* Implemented in Lecture 18 */









  }

  bool sanity_check(TreeNode<T>* p) const {
    if (p == NULL) return true;
    if (p->left != NULL && p->left->parent != p) { return false; }
    if (p->right != NULL && p->right->parent != p) { return false; }
    return sanity_check(p->left) && sanity_check(p->right);
  }
};
```