

1. (10 points) Show that the following inference rule is sound (using the definition of soundness)

$$\frac{\begin{array}{l} A \Leftrightarrow \neg B \\ C \Rightarrow A \wedge B \end{array}}{\neg C}$$

2. (30 points) For the given (Horn) knowledge base  $\Delta$  and query  $w$  shown below:

$$\Delta = \left\{ \begin{array}{l} A \\ B \\ A \wedge B \Rightarrow L \\ A \wedge P \Rightarrow L \\ B \wedge L \Rightarrow M \\ L \wedge M \Rightarrow P \\ P \Rightarrow Q \end{array} \right.$$

$$w = Q$$

- (a) Prove  $w$  using forward chaining. Show and explain your work.  
 (b) Prove  $w$  using backward chaining. Show and explain your work.
3. (12 points) Transform the following sentences into Conjunctive Normal Form and simplify them as much as possible.
- (a)  $(\neg W \wedge X) \vee (A \Rightarrow \neg B)$   
 (b)  $\neg(P \Rightarrow (Q \vee R)) \wedge S$   
 (c)  $(A \vee (C \wedge B)) \Rightarrow F$   
 (d)  $(V \Rightarrow W) \Rightarrow (X \Rightarrow \neg W)$
4. (18 points) Translate the following problem statements into Propositional logic:

On Saturdays in the summer, it is either sunny or rainy but not both. If it is sunny, then Alice goes on a picnic with her friends and plays softball. If it is rainy, they go to the gym and play basketball. If Alice plays basketball or softball, then she gets exercise.

Use the following predicates:

- S = sunny  
 R = rainy  
 P = Alice goes on a picnic  
 G = Alice goes to the gym  
 F = Alice plays softball  
 B = Alice plays basketball  
 E = Alice gets exercise

Show that Alice gets exercise by doing a refutation proof using resolution.