CSCI 2400 – Models of Computation

Solution for Homework #5

1. Show language $L = \{a^n b^j c^k : k = jn\}$ is not context-free.

   Solution

   Assume that L is a context free language.
   Let $w = a^m b^m c^{m^2}$, $w \in L$. By the Pumping Lemma, w can be decomposed as $w = uvxyz$ with $|vxz| \leq m$ and $|vy| \geq 1$ such that $uv^i xy^i z \in L$, $i \leq 0$.

case 1 $\underbrace{a\ldots a}_{uvxy}\underbrace{b\ldots bc\ldots c}_{z}$

If $i = 0$, $uv^0xy^0z = a^{m-|vy|}b^m c^{m^2} \notin L$.

case 2 $\underbrace{a\ldots}_{uv}\underbrace{ab}_{x}\underbrace{\ldots}_{y}\underbrace{bc\ldots}_{z}$

If $i = 0$, $uv^0xy^0z = a^{m-|v|}b^{m-|y|}c^{m^2} \notin L$.

case 3 $\underbrace{a\ldots ab}_{u}\underbrace{\ldots}_{vxy}\underbrace{bc\ldots c}_{z}$

If $i = 0$, $uv^0xy^0z = a^m b^{m-|vy|}c^{m^2} \notin L$.

case 4 $\underbrace{a\ldots ab}_{u}\underbrace{\ldots}_{v}\underbrace{bc}_{x}\underbrace{\ldots c}_{yz}$

If $i = 0$, $uv^0xy^0z = a^m b^{m-|v|}c^{m^2-|y|} \notin L$.

case 5 $\underbrace{a\ldots ab\ldots bc}_{u}\underbrace{\ldots c}_{vxyz}$

If $i = 0$, $uv^0xy^0z = a^m b^m c^{m^2-|vy|} \notin L$.

case 6 v or y containing ab or bc

If $i > 1$, uv^ixy^iz would be $a\ldots ab\ldots ba\ldots ab\ldots b \;\ldots\; c\ldots c$ or $a\ldots ab\ldots bc\ldots cb \;\ldots\; c$.

This is contradictory to the assumption that language L is context free. Therefore L is not context free.

2. Show language $L = \{w \in \{a,b,c\}^* : n_a(w) + n_b(w) = 2n_c(w)\}$ is not context-free.

3. Show language $L = \{ww^R a^{|w|} : w \in \{a,b\}^*\}$ is not context-free.

   Solution

   Assume that L is a context free language. Let $w = a^m b^m$. Then $ww^R a^{|w|} = a^m b^m b^m a^m a^{2m} \in L$. By the Pumping Lemma, $ww^R a^{|w|}$ can be decomposed as $ww^R a^{|w|} = uvxyz$ with $|vxz| \leq m$ and $|vy| \geq 1$ such that $uv^ixy^iz \in L$, $i \leq 0$.

case 1 $\underbrace{a\ldots ab\ldots}_{uvxy}\underbrace{bb\ldots ba\ldots aa\ldots a}_{z}$

If $i = 0$, $|w| = 2m - |vy|$ is less than $|w^R|$. So $uv^0xy^0z \notin L$.

case 2 $\underbrace{a\ldots ab\ldots bb}_{u}\underbrace{\ldots baa\ldots}_{vxy}\underbrace{aa\ldots a}_{z}$

If $i = 0$, $|w^R| = 2m - |vy|$ is less than $|w|$. So $uv^0xy^0z \notin L$.

case 3 $\underbrace{a\ldots ab\ldots bb\ldots ba\ldots aa}_{u}\underbrace{\ldots a}_{vxyz}$

If $i=0$, $|a^{|w|}| = 2m - |vx|$ is less than $|w|$. So $uv^0xy^0z \notin L$.

case 4 $\underbrace{a\ldots ab}_{u}\underbrace{\ldots bb\ldots}_{vxy}\underbrace{ba\ldots aa\ldots a}_{z}$

If $i=0$, $|ww^R| = 4m - |vy|$ is less than $2 * |a^{|w|}| = 4m$. So $uv^0xy^0z \notin L$.

case 5 $\underbrace{a\ldots ab\ldots bb\ldots ba}_{u}\underbrace{\ldots aa}_{vxy}\underbrace{\ldots a}_{z}$

If $i=0$, $|w^Ra^{|w|}| = 4m - |vy|$ is less than $2 * |w| = 4m$. So $uv^0xy^0z \notin L$.

This is contradictory to the assumption that language L is context free. Therefore L is not context free.

4. Construct a Turing machine that will accepts language $L = \{a^nb^m : n \geq 1, n \neq m\}$.

Solution

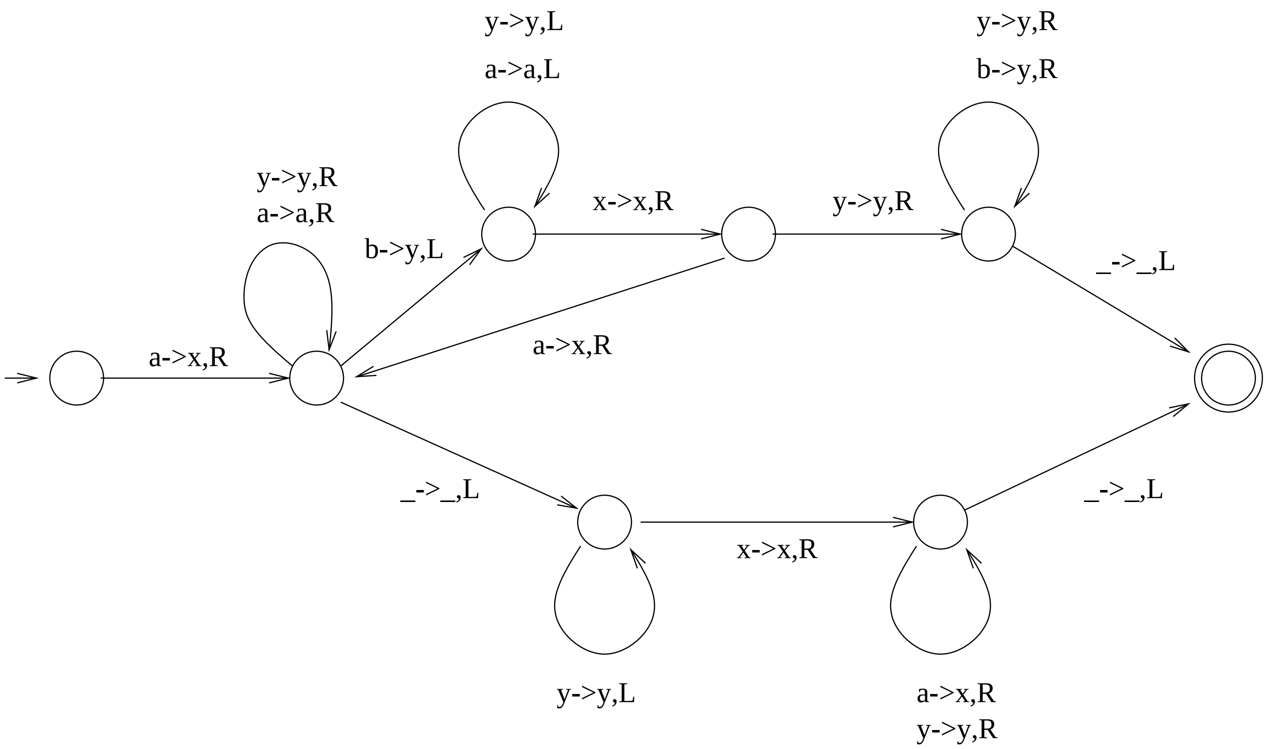

Figure 1: Turing Machine that accepts $L = \{a^nb^m : n \geq 1, n \neq m\}$

5. Construct a Turing machine to compute the function

$$f(w) = w^R$$

where $w \in \{0,1\}^+$.

Solution

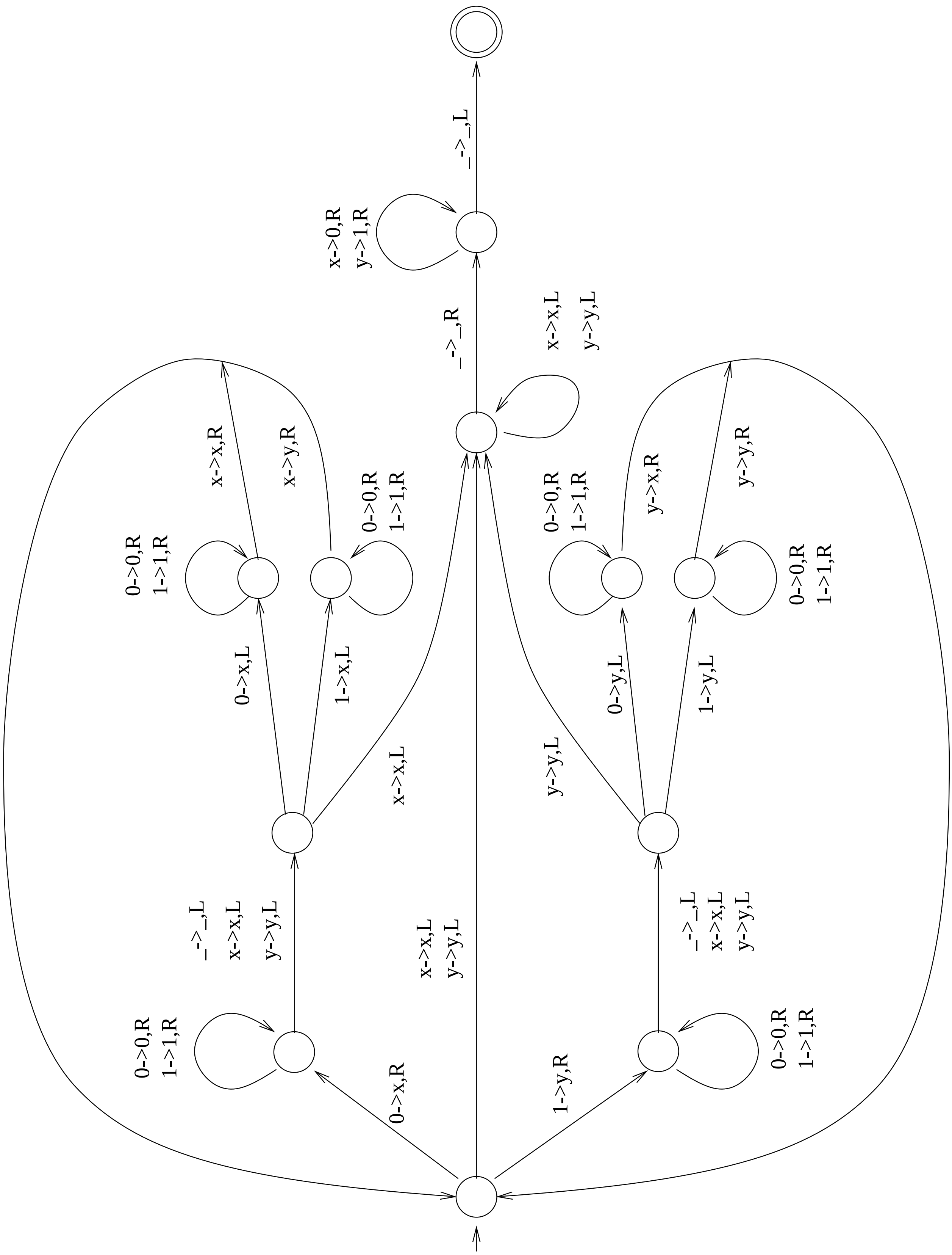

Figure 2: Turing Machine to compute $f(w) = w^R$