CSCI 2400 – Models of Computation

Solution for Homework #2

1. Give regular expressions for the following languages on $\Sigma = \{a, b, c\}$.

   (a) all strings containing exactly one a.
   Solution
   $(b + c)^*a(b + c)^*$

   (b) all strings containing no more than three a's.
   Solution
   $(b + c)^*(a + \lambda)(b + c)^*(a + \lambda)(b + c)^*(a + \lambda)(b + c)^*$

   (d) all strings that contain no run of a's of length greater than two.
   Solution
   $(b + c)^* + (b + c)^*((a + aa)(b + c)^+)^*(a + aa)(b + c)^*$

   (e) all strings in which all runs of a's have lengths that are multiples of three.
   Solution
   $(b + c)^*((aaa)(b + c)^*)^*$

2. Find a regular grammar that generates the language $L(aa^*(ab + a)^*)$.

   Solution
   $G = (V, T, S, P)$, where
   $V = \{S, A, B\}$,
   $T = \{a, b\}$,
   $P = \{S \to aA,\ A \to aA|aB|\lambda,\ B \to bA\}$

   The derivation of a string $aaaababa$:
   $S \Rightarrow aA \Rightarrow aaA \Rightarrow aaaA \Rightarrow aaaaB \Rightarrow aaaabA \Rightarrow aaaabaB$
   $\Rightarrow aaaababA \Rightarrow aaaababaA \Rightarrow aaaababa$.

3. Find a regular grammar that generates the language on $\Sigma = \{a, b\}$ consisting of all strings with no more than three a's.

Solution
$G = (V, T, S, P)$, where
$V = \{S, A, B\}$,
$T = \{a, b\}$,
$P = \{S \to bS|aA|\lambda,\ A \to bA|aB|\lambda,\ B \to bB|aC|\lambda,\ C \to bC|\lambda\}$

The derivation of a string $babbaab$:
$S \Rightarrow bS \Rightarrow baA \Rightarrow babA \Rightarrow babbA \Rightarrow\Rightarrow babbaB$
$\Rightarrow babbaaC \Rightarrow babbaabC \Rightarrow babbaab.$

4. Find regular grammar for the following languages on $\{a, b\}$.
L=$\{w : (n_a(w) - n_b(w)) \bmod 3 = 1\}$

Solution
$G = (V, T, S, P)$, where
$V = \{S, A, B\}$,
$T = \{a, b\}$,
$P = \{S \to aA|bB,\ A \to aB|bS|\lambda,\ B \to aS|bA\}$

The derivation of a string $abaaaaba$:
$S \Rightarrow aA \Rightarrow abS \Rightarrow abaA \Rightarrow abaaB \Rightarrow abaaaS$
$\Rightarrow abaaaaA \Rightarrow abaaaabS \Rightarrow abaaaabaA \Rightarrow abaaaaba.$

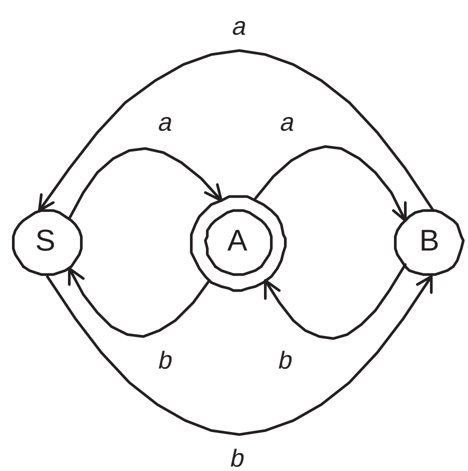

Figure 1: finite automaton accepting L=$\{w : (n_a(w) - n_b(w)) \bmod 3 = 1\}$

5. The *min* of a language L is defined as

$$min(L) = \{w \in L : \text{there is no } u \in L,\ v \in \Sigma^+, \text{ such that } w = uv\}.$$

Solution
Take the transition graph of a DFA for L and delete all edges going out of any final vertex. Note that this works only if we start with a DFA!