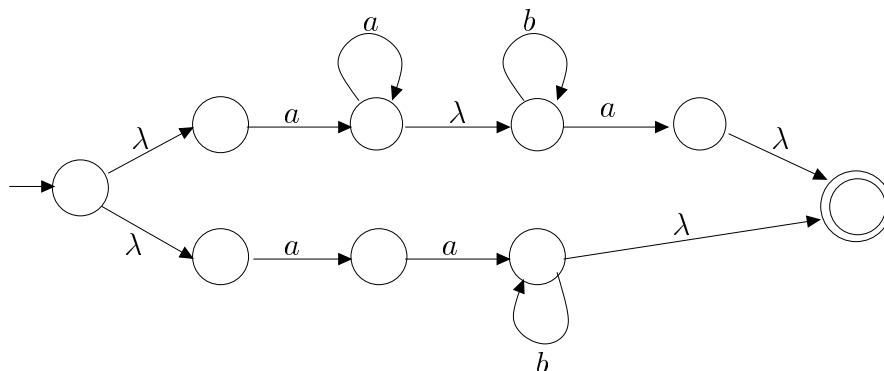


## Solutions to the Practice Midterm Exam

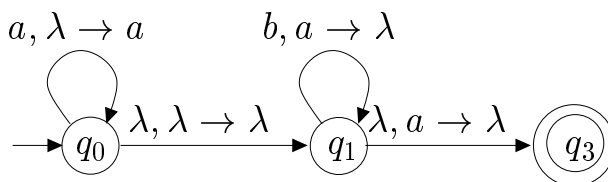
1.



2.  $1(1 + 0)^*11(1 + 0)^* + 11(1 + 0)^*$

3. Let's assume for contradiction that  $L$  is a regular language. We apply the pumping lemma to  $L$ . Let  $m$  be the parameter of the pumping lemma. We choose to pump the string  $a^m b^5 c^m$  which is in the language  $L$ . Since  $xyz = a^m b^5 c^m$  and  $|xy| \leq m$  we have that the string  $y$  is a substring of the first  $a^m$ . Therefore, the string  $y$  has the form  $y = a^p$ , for some integer  $p$ ,  $1 \leq p \leq m$  (since  $|y| \geq 1$ ). Now, we pump up  $y$  once and we obtain the string  $a^{m+p} b^5 c^m$ . By the pumping lemma, we have that  $a^{m+p} b^5 c^m$  is in the language  $L$ . However,  $a^{m+p} b^5 c^m$  is not in the language  $L$  since  $m + p \neq m$ . Therefore, we have a contradiction, and thus the language  $L$  is not regular.

4.



The initial stack symbol is  $\$$ . State  $q_0$  reads the  $a$ 's and pushes them into the stack. State  $q_1$  reads the  $b$ 's and pops an  $a$  from the stack for each input

*b*. Finally, state  $q_3$  is the accept state which the automaton enters only if there is an *a* in the stack, which means that the numbers of *a*'s was more than the number of *b*'s.

5.

(a)

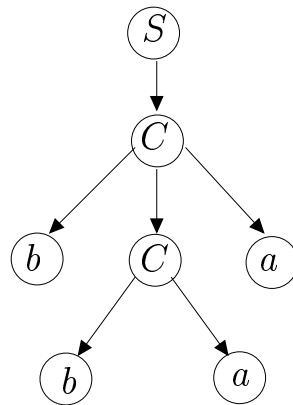
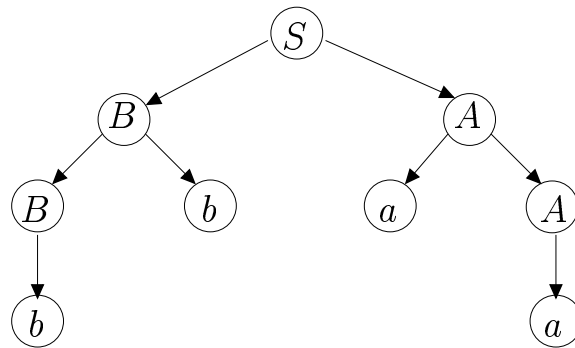
$$S \rightarrow aSa|bSb|A$$

$$A \rightarrow aAb|\lambda$$

(b)

$$S \Rightarrow aSa \Rightarrow abSba \Rightarrow abAba \Rightarrow abaAbba \Rightarrow abaaAbbba \Rightarrow abaabbba$$

6. Yes, the grammar is ambiguous. The reason is that there is string generated by the grammar that has two different derivation trees. This string is *bbaa*. The two derivation trees are:



7.

$$S \rightarrow AV_1$$

$$V_1 \rightarrow T_b V_2$$

$$V_2 \rightarrow BT_a$$

$$A \rightarrow AV_3$$

$$V_3 \rightarrow BT_a$$

$$A \rightarrow a$$

$$B \rightarrow BV_4$$

$$V_4 \rightarrow T_a A$$

$$B \rightarrow b$$

$$T_a \rightarrow a$$

$$T_b \rightarrow b$$