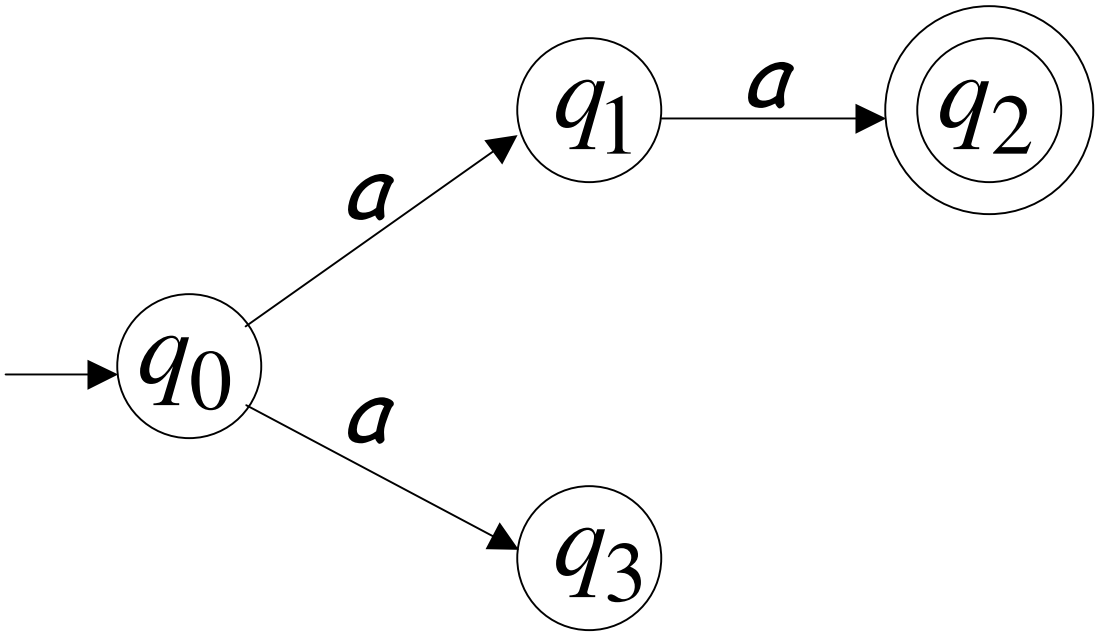
# Non Deterministic Automata

# Nondeterministic Finite Accepter (NFA)

Alphabet = $\{a\}$



# Nondeterministic Finite Accepter (NFA)

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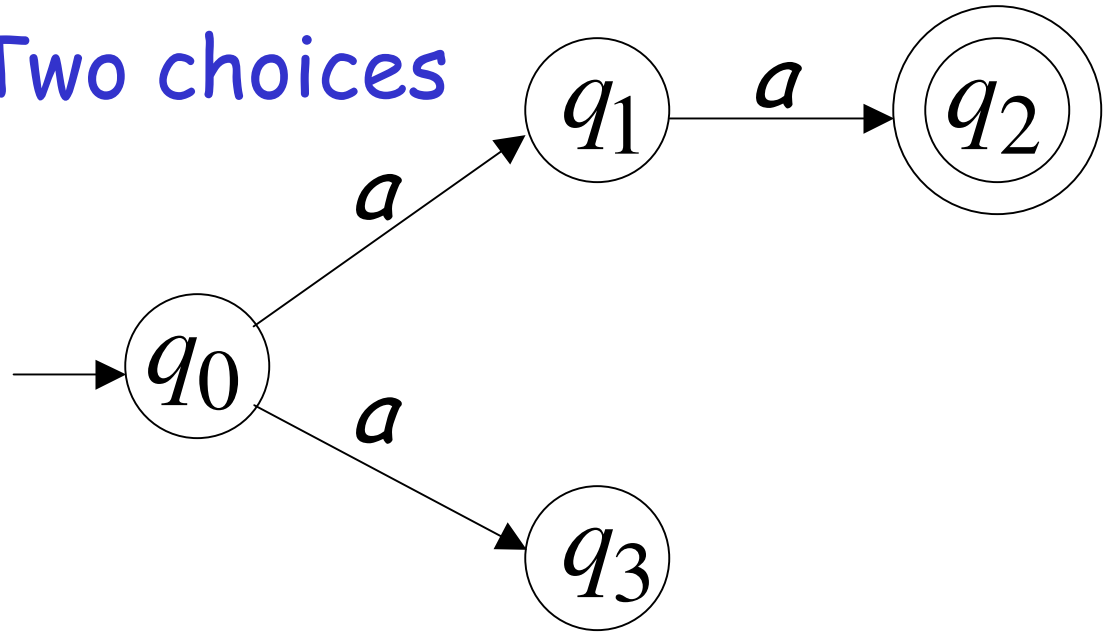
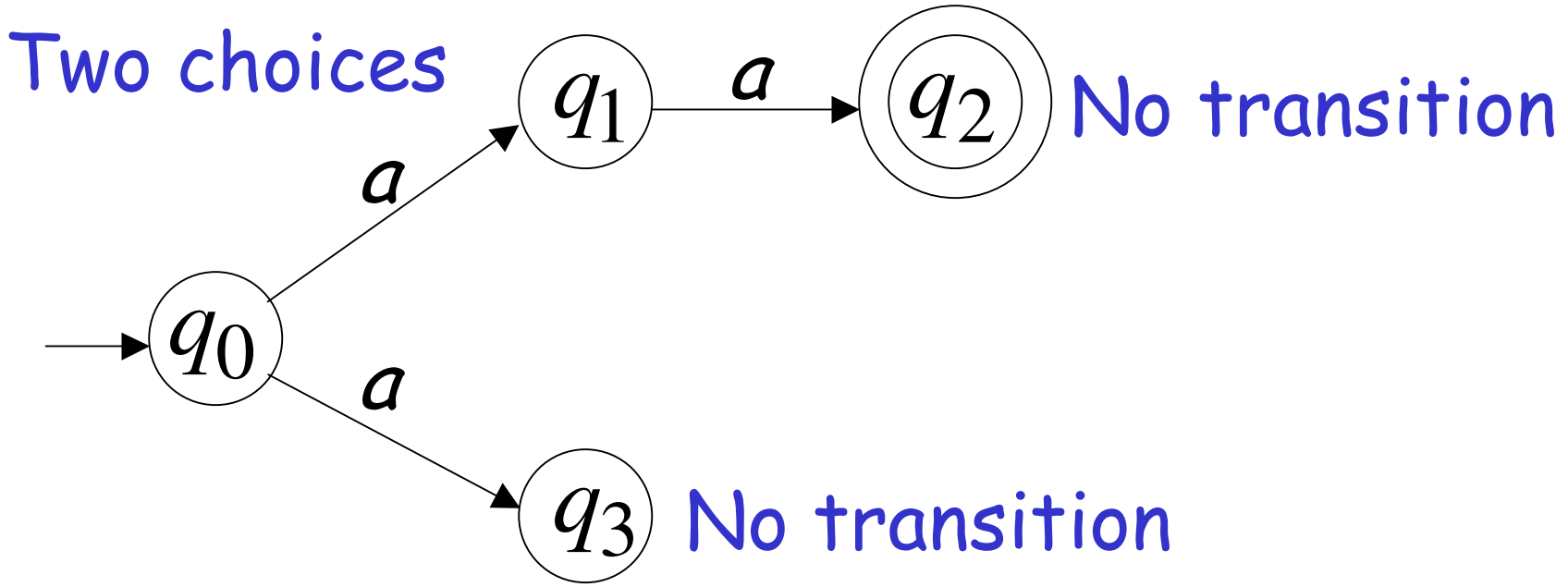
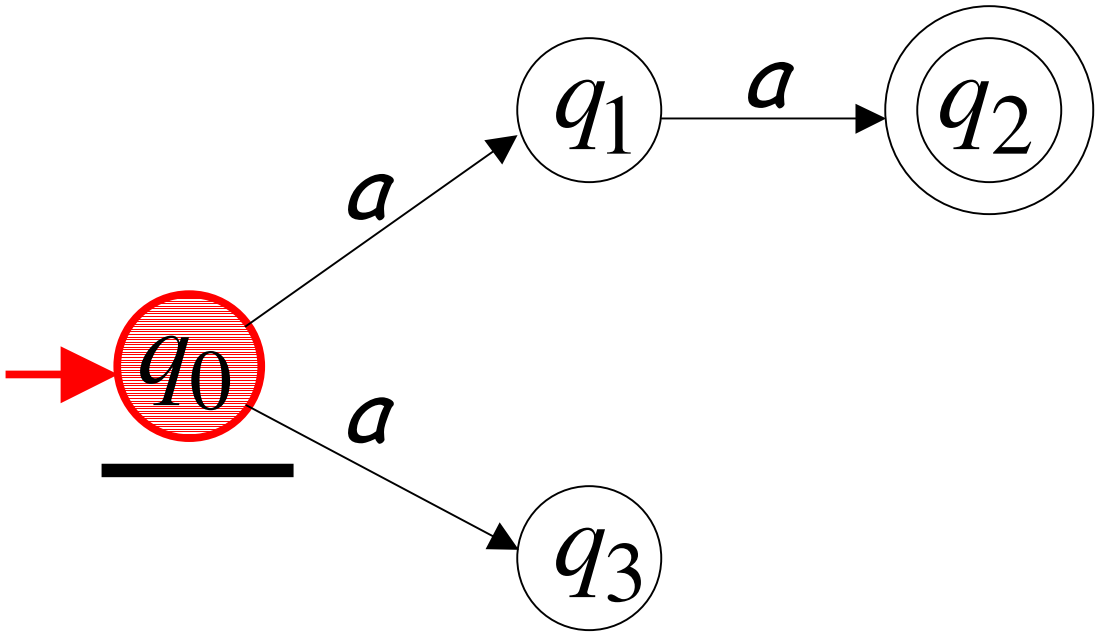


# Nondeterministic Finite Accepter (NFA)

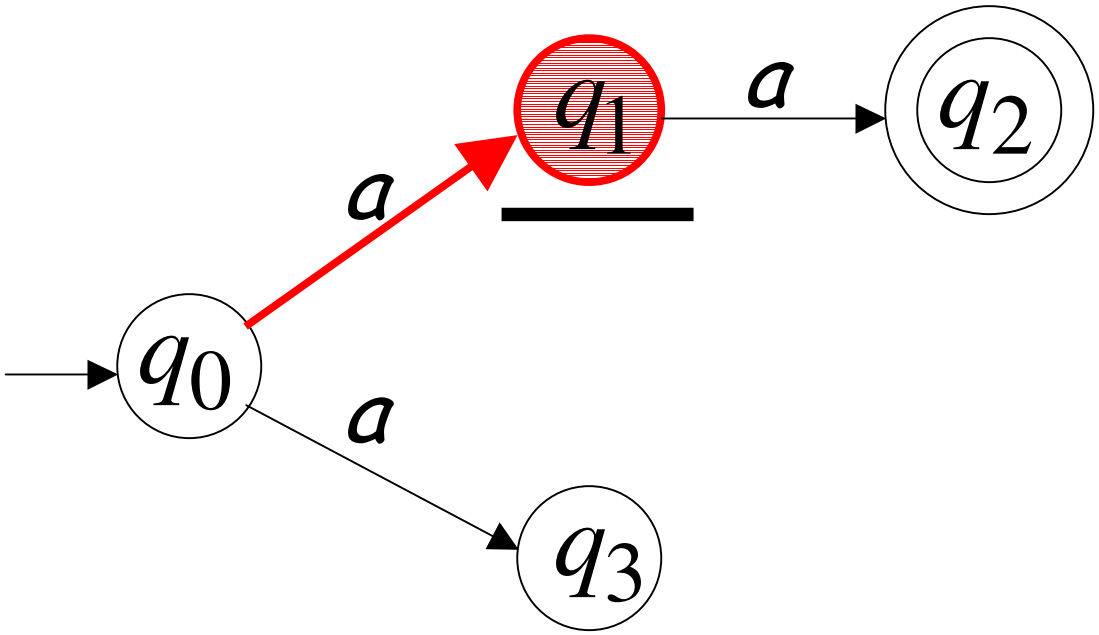
Alphabet = $\{a\}$



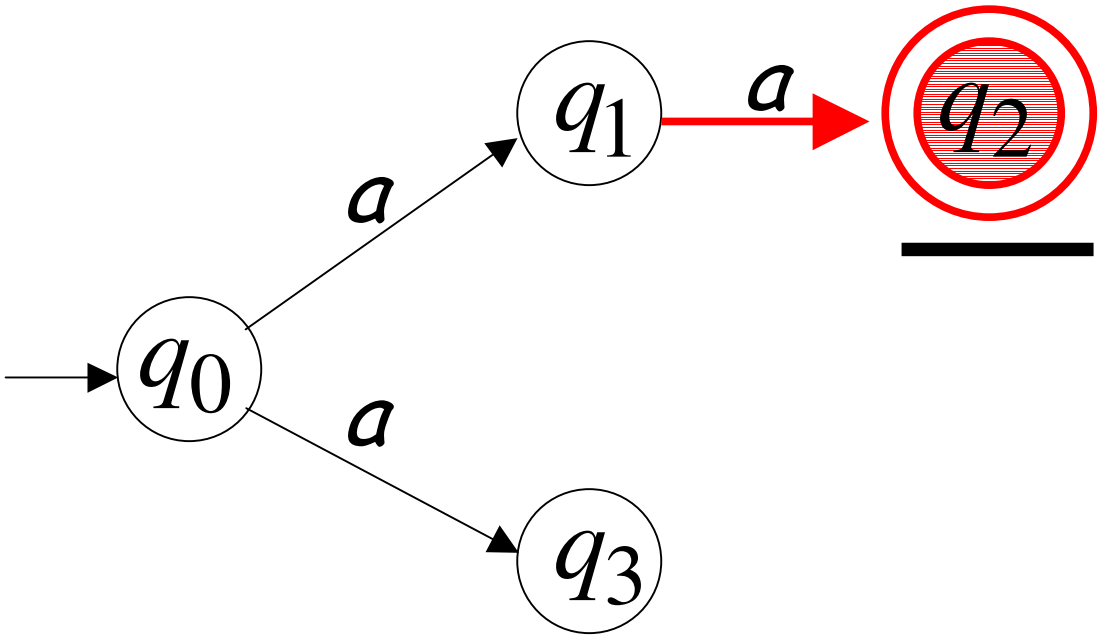
# First Choice



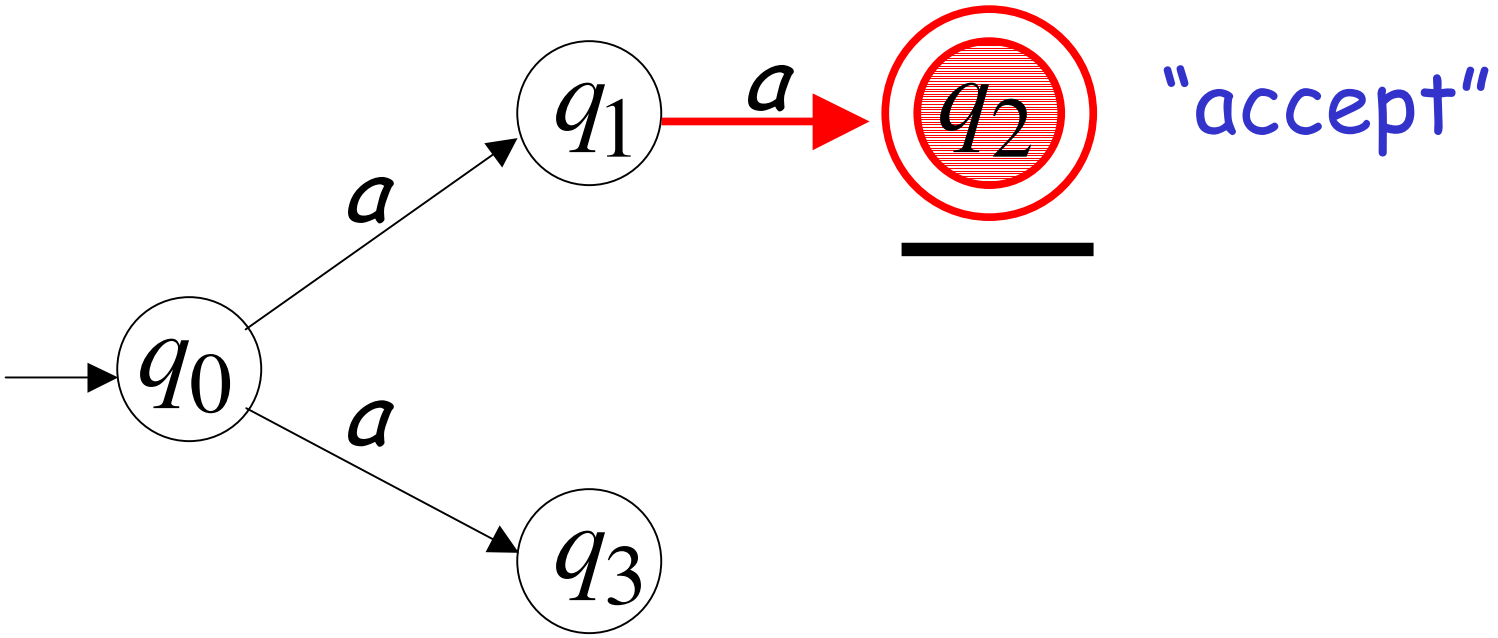
# First Choice



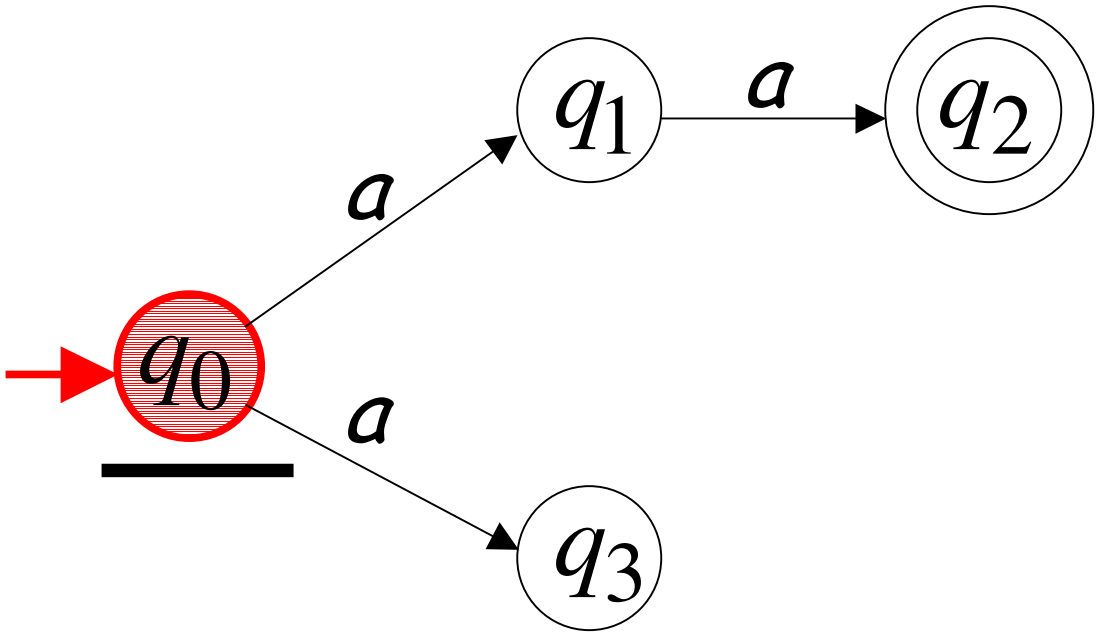
# First Choice



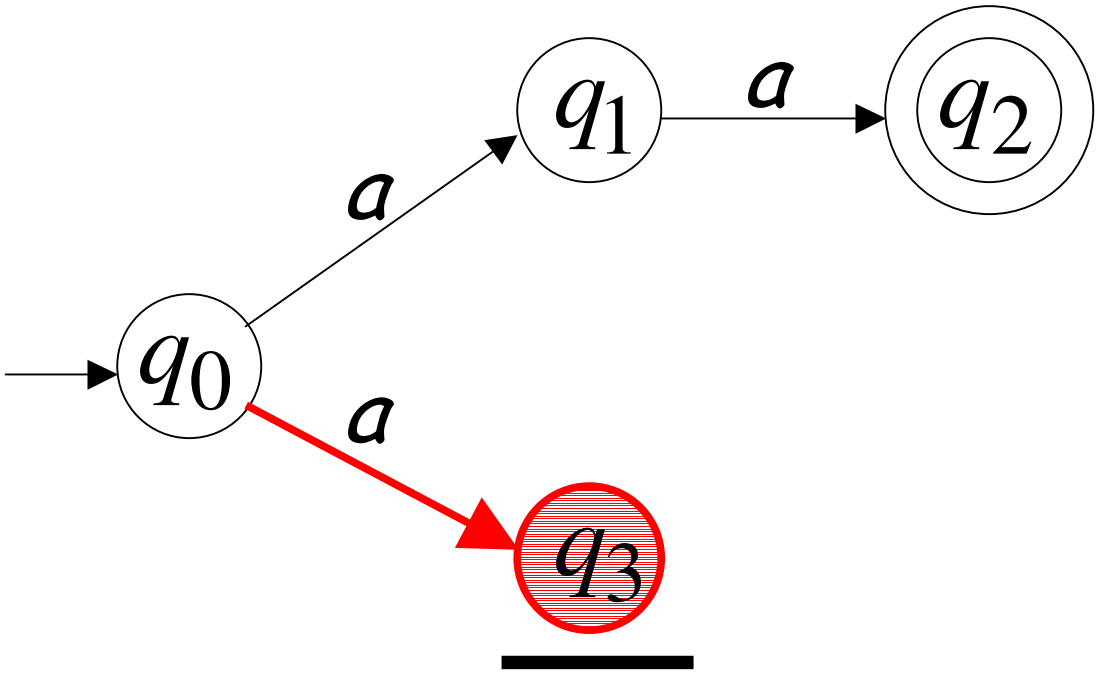
# First Choice



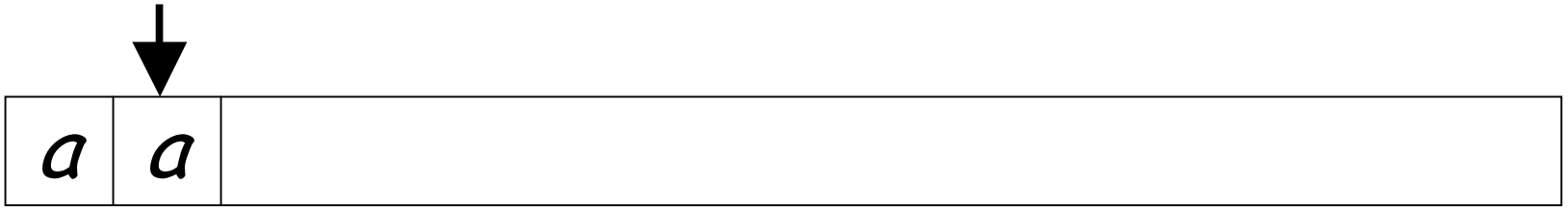
# Second Choice

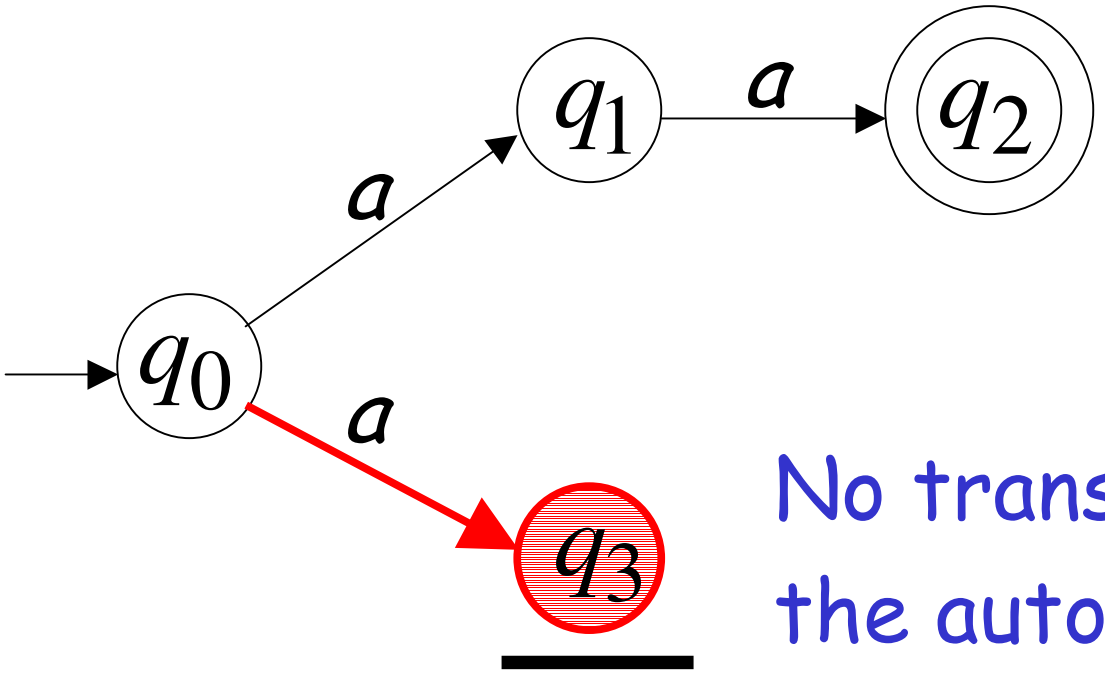


# Second Choice



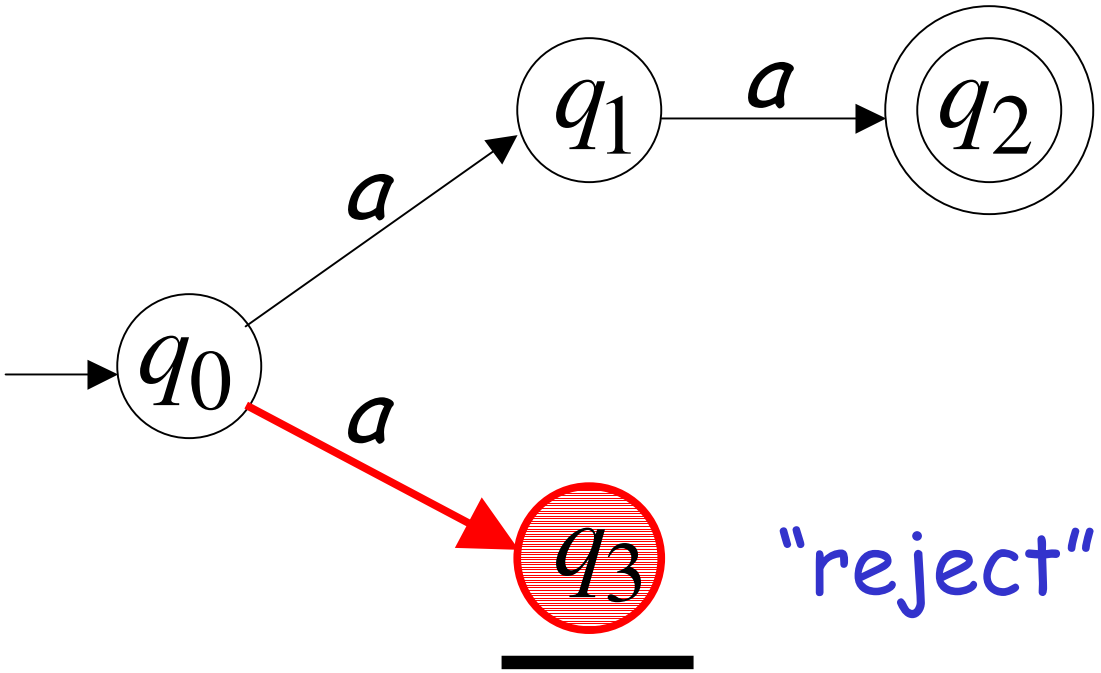
# Second Choice




No transition:
the automaton hangs

# Second Choice



# Observation

An NFA accepts a string

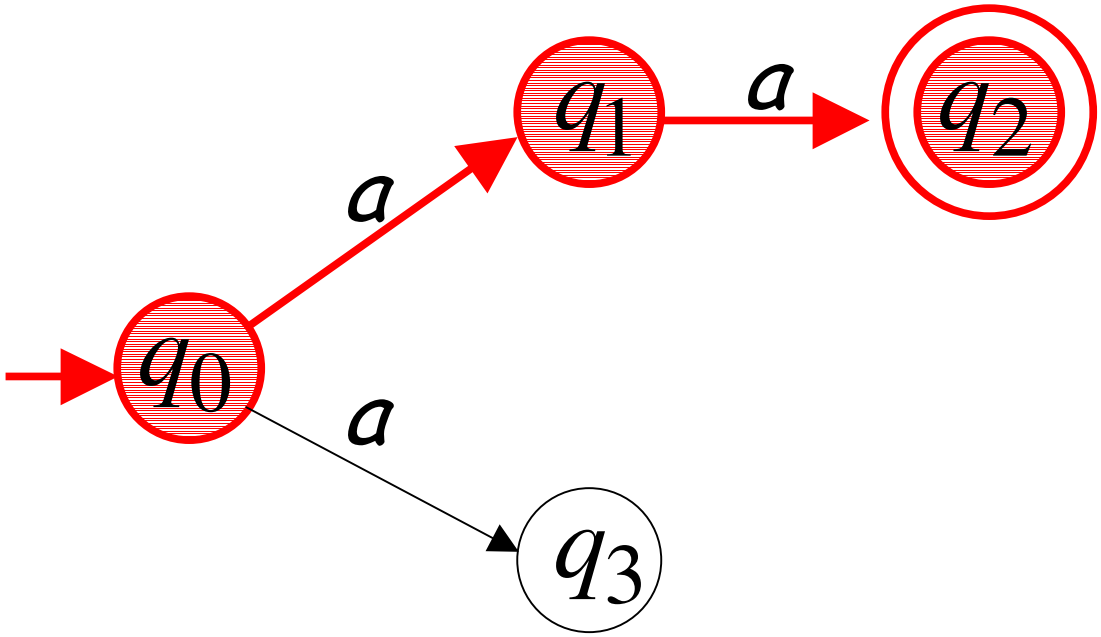
if

there is a computation of the NFA

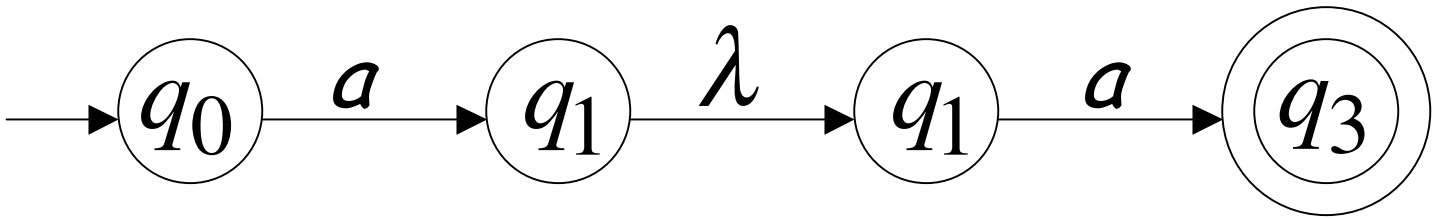
that accepts the string

# Example

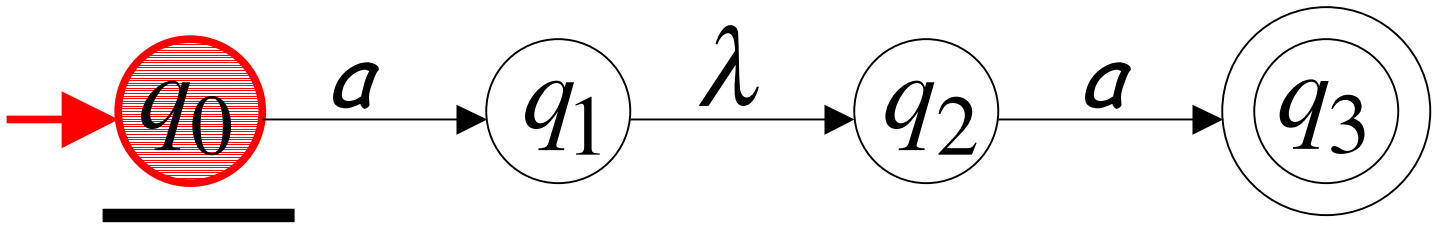
$aa$ is accepted by the NFA:

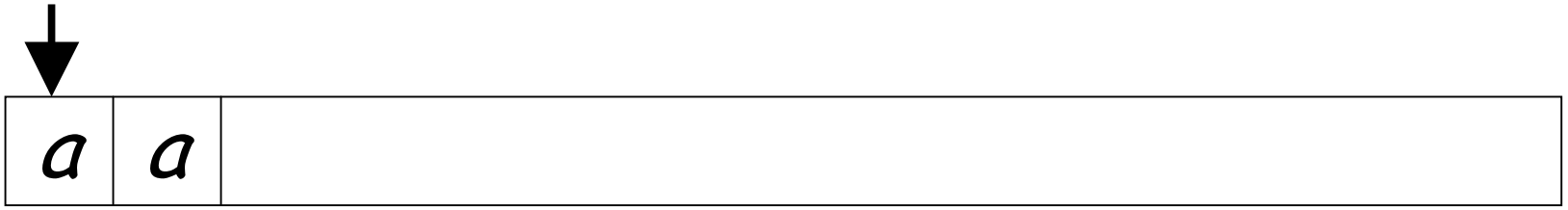


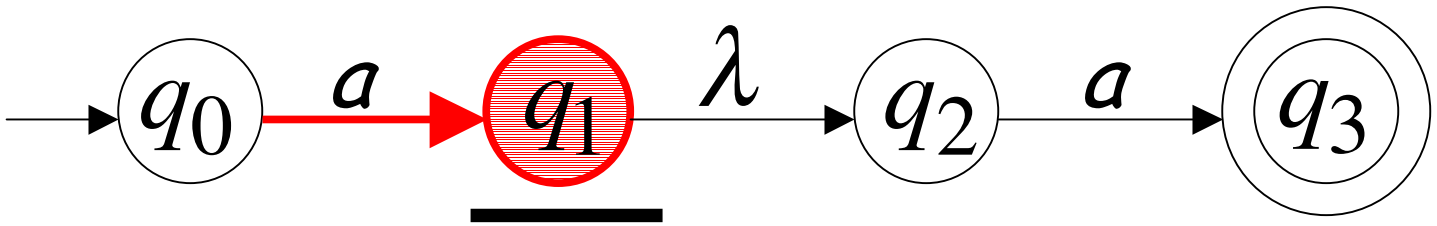
# Lambda Transitions



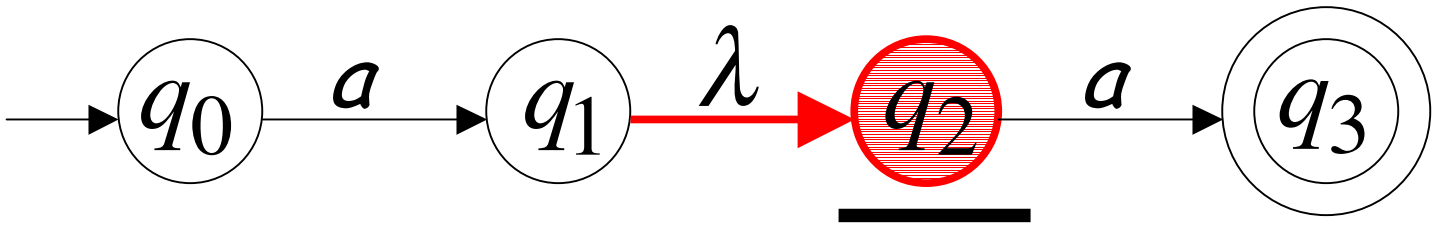
a
a


$q_0$
a
$q_1$
λ
$q_2$
a
$q_3$

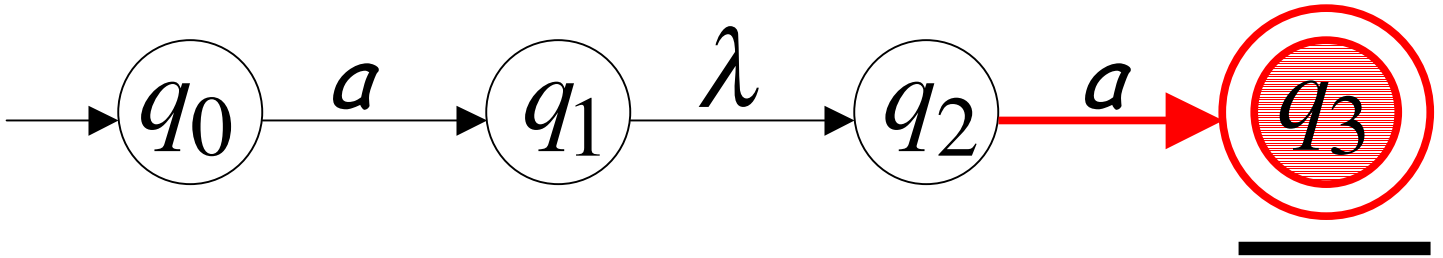

a
a

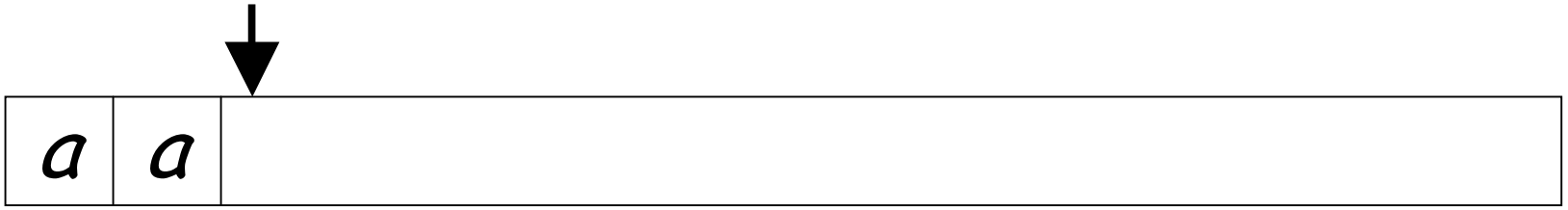

$q_0$
a
$q_1$
λ
$q_2$
a
$q_3$

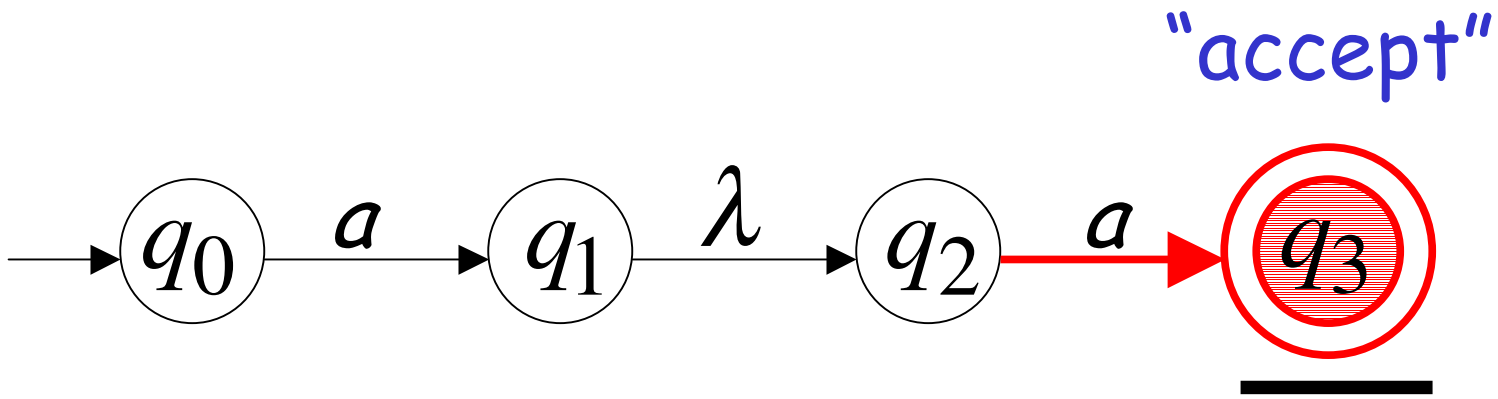
(read head doesn't move)



a
a

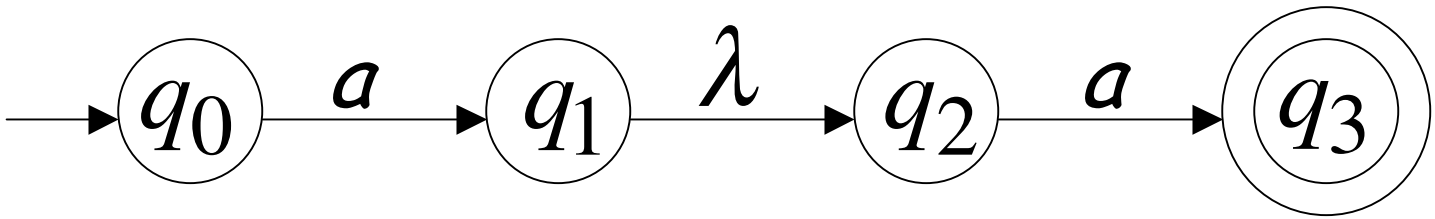

$q_0$
a
$q_1$
λ
$q_2$
a
$q_3$



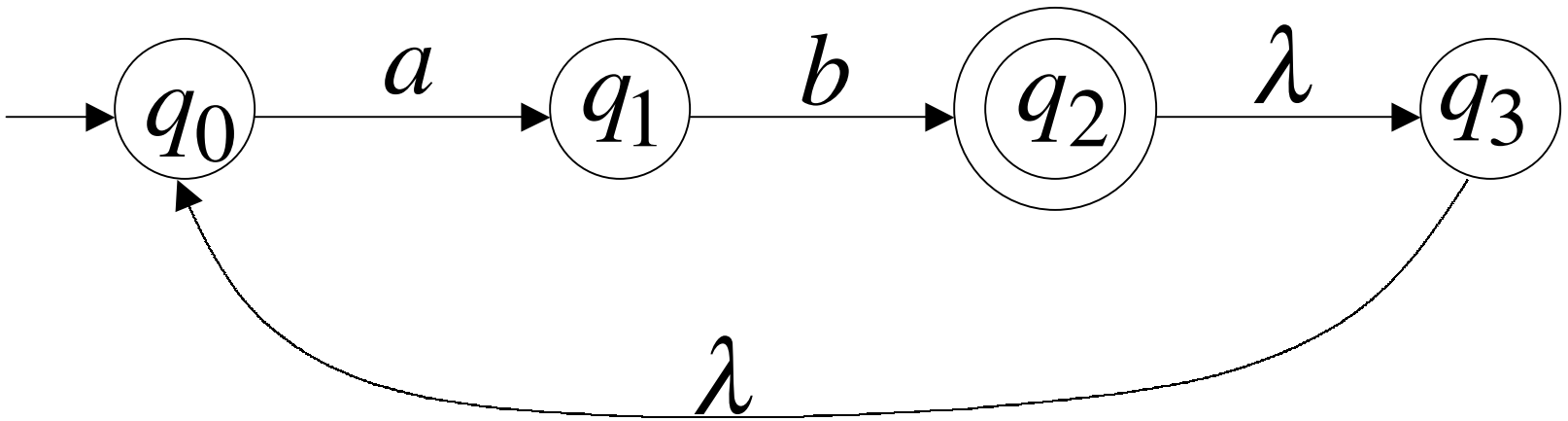



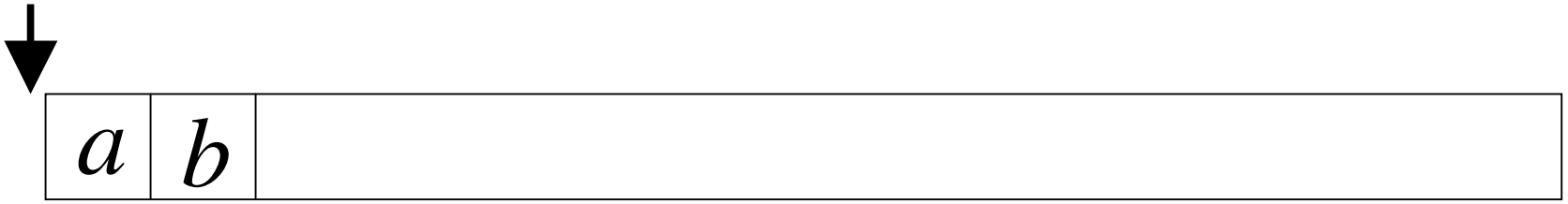

String $aa$ is accepted

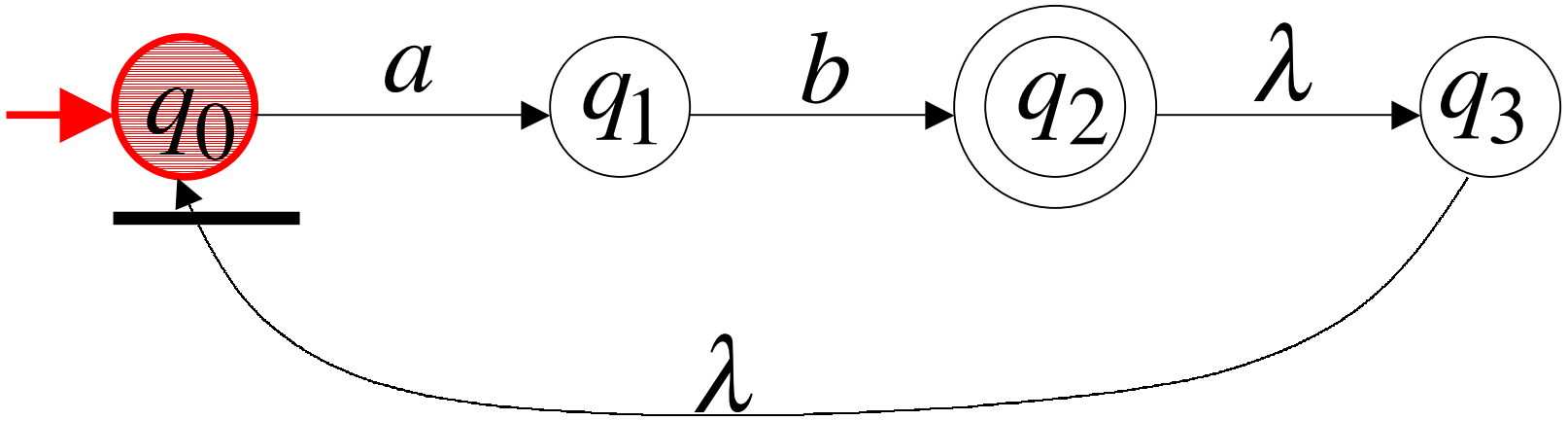
Language accepted: $L = \{aa\}$



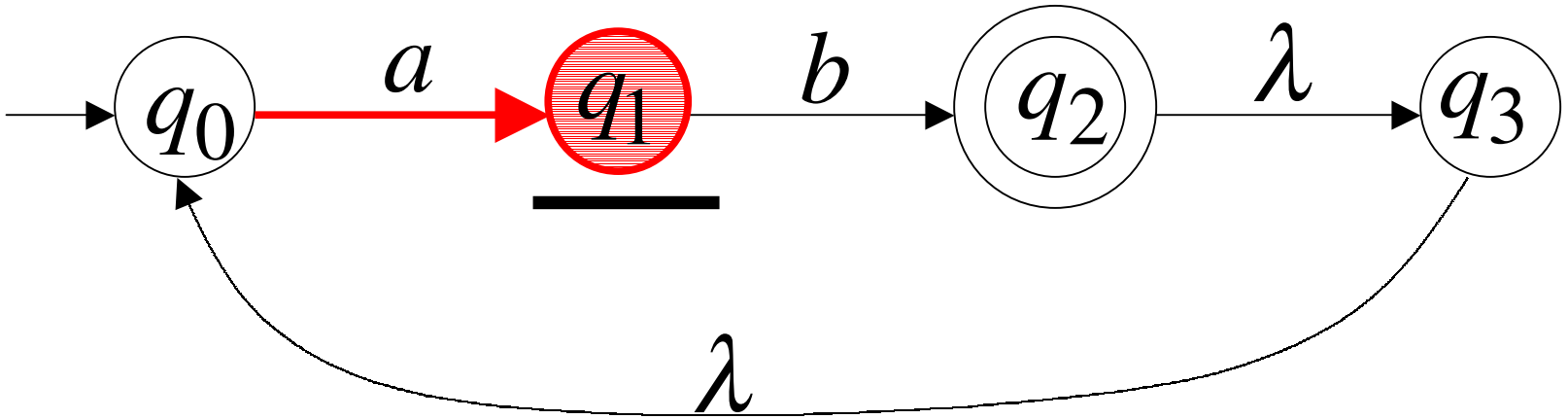
# Another NFA Example

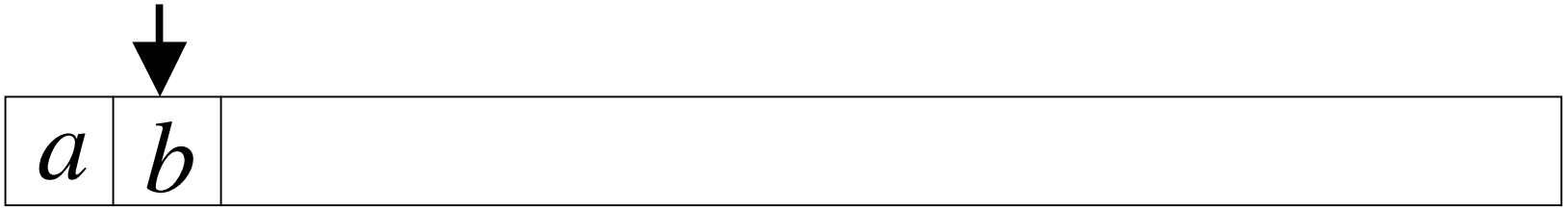


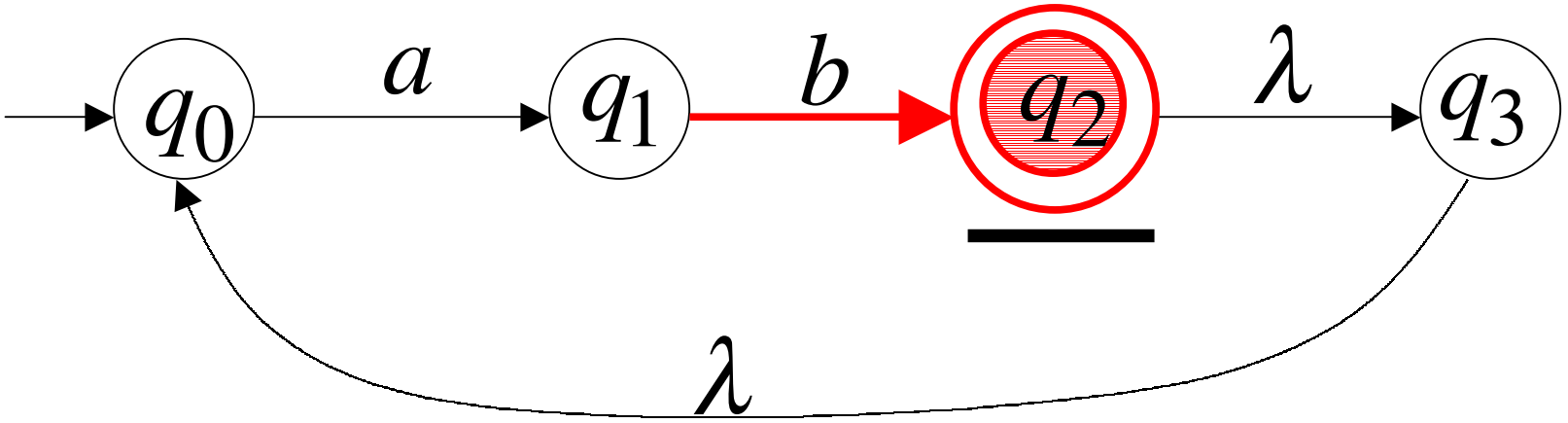

a
b


$q_0$
a
$q_1$
b
$q_2$
λ
$q_3$
λ

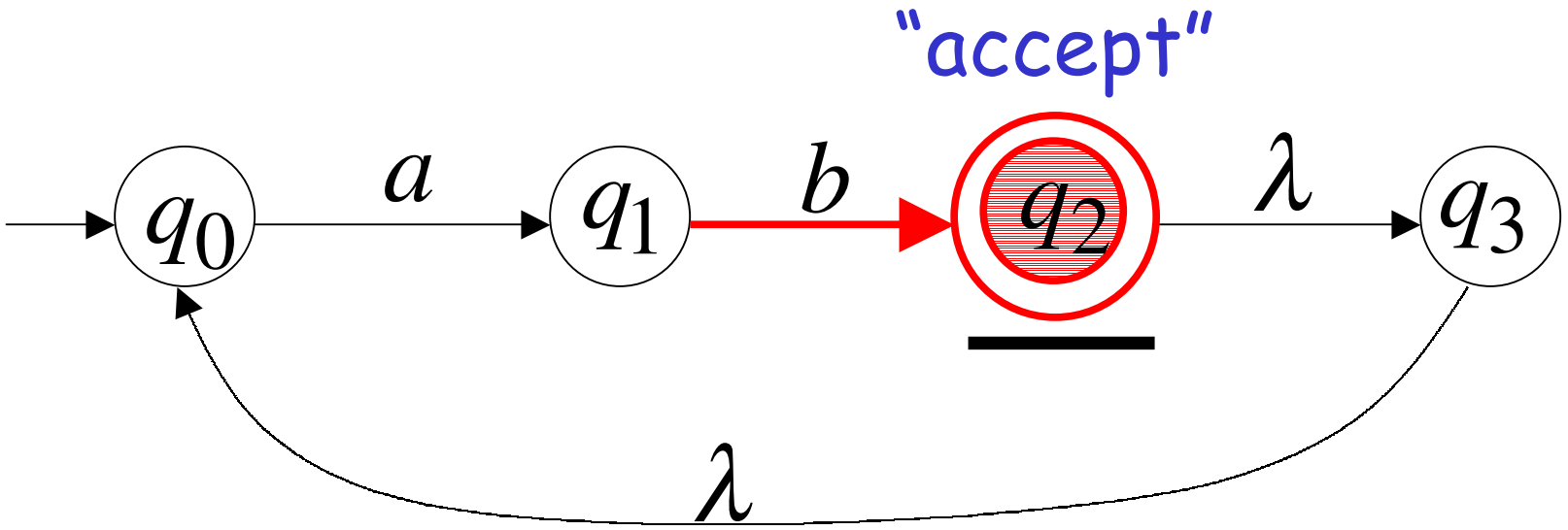
a
b


$q_0$
a
$q_1$
b
$q_2$
λ
$q_3$
λ

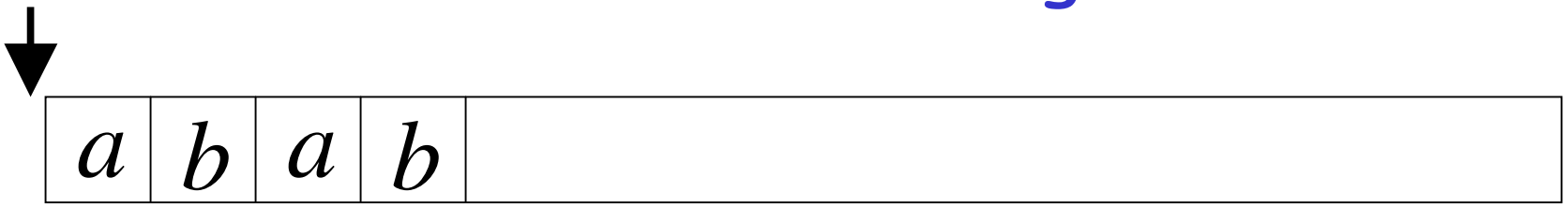

a
b

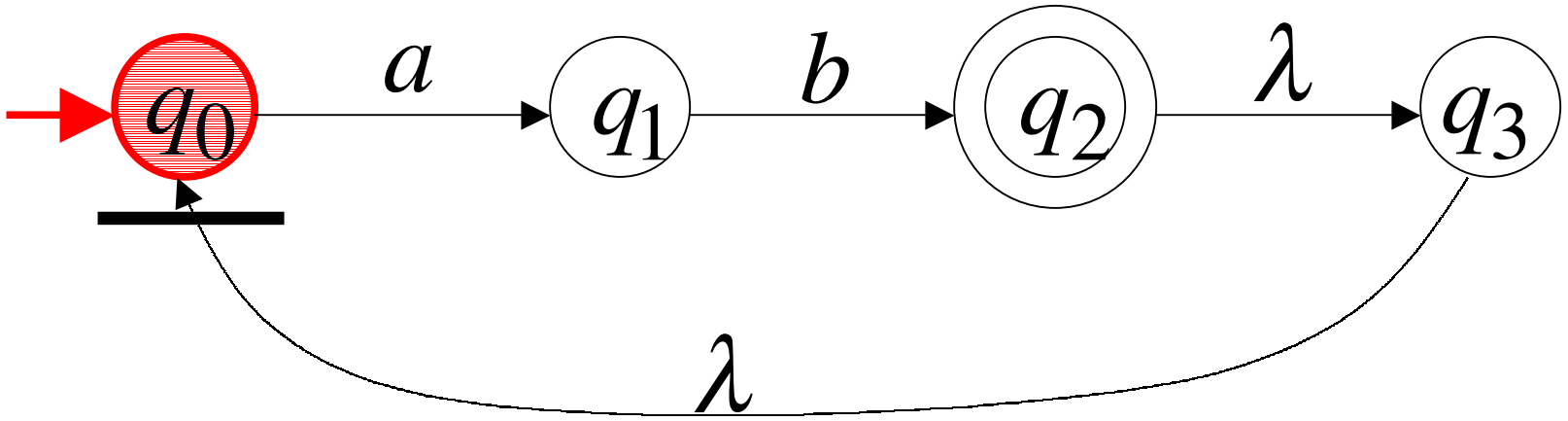

$q_0$
a
$q_1$
b
$q_2$
λ
$q_3$
λ

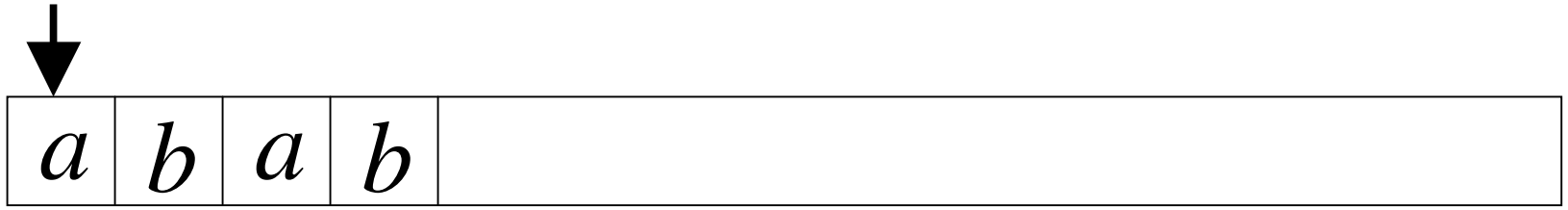
a
b

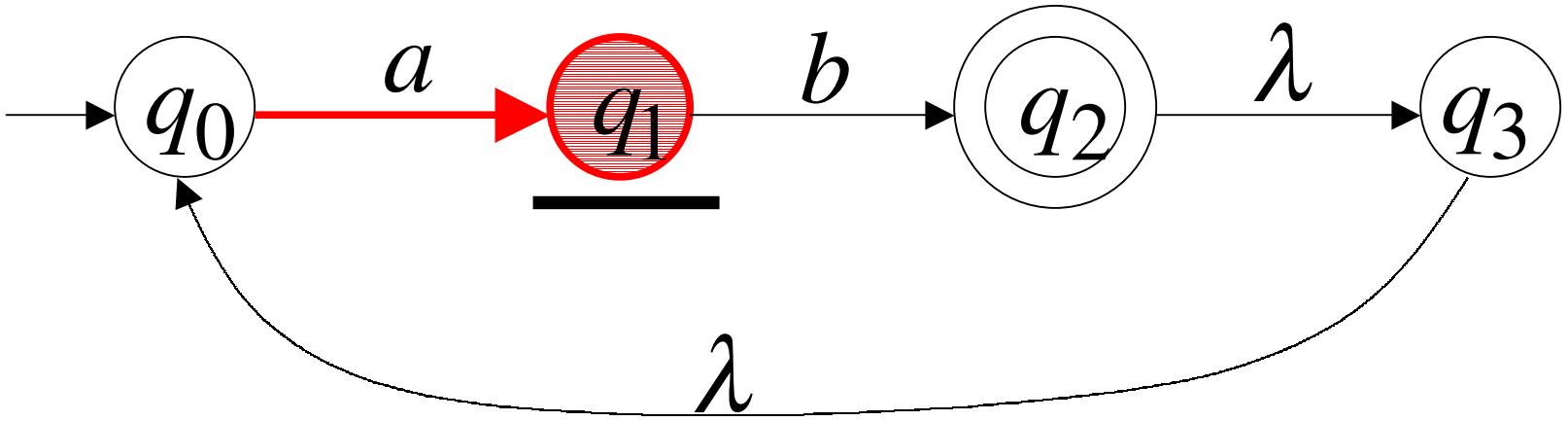

"accept"
$q_0$
$q_1$
$q_2$
$q_3$
a
b
λ
λ

# Another String

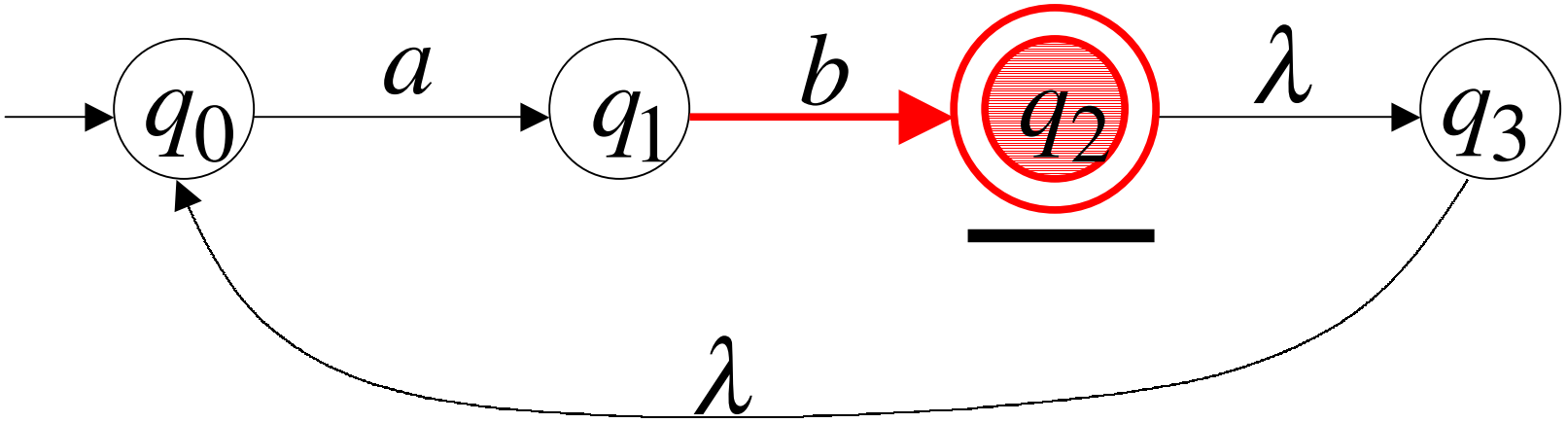


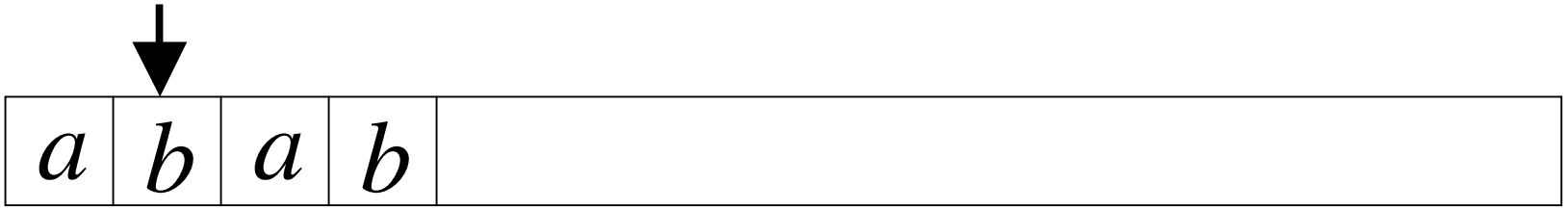



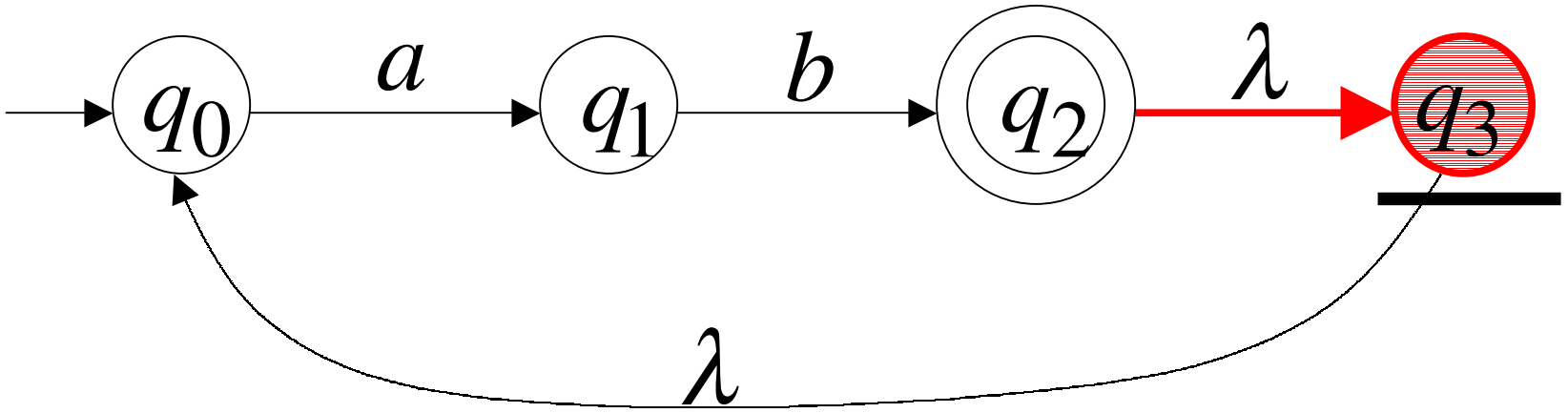

a
b
a
b

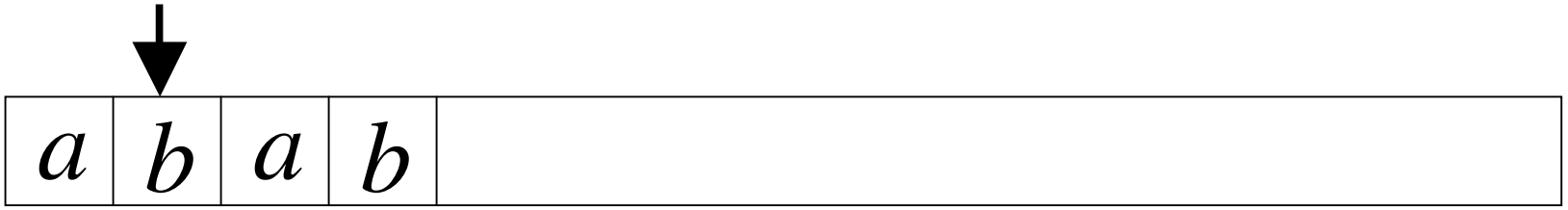

$q_0$
a
$q_1$
b
$q_2$
λ
$q_3$
λ

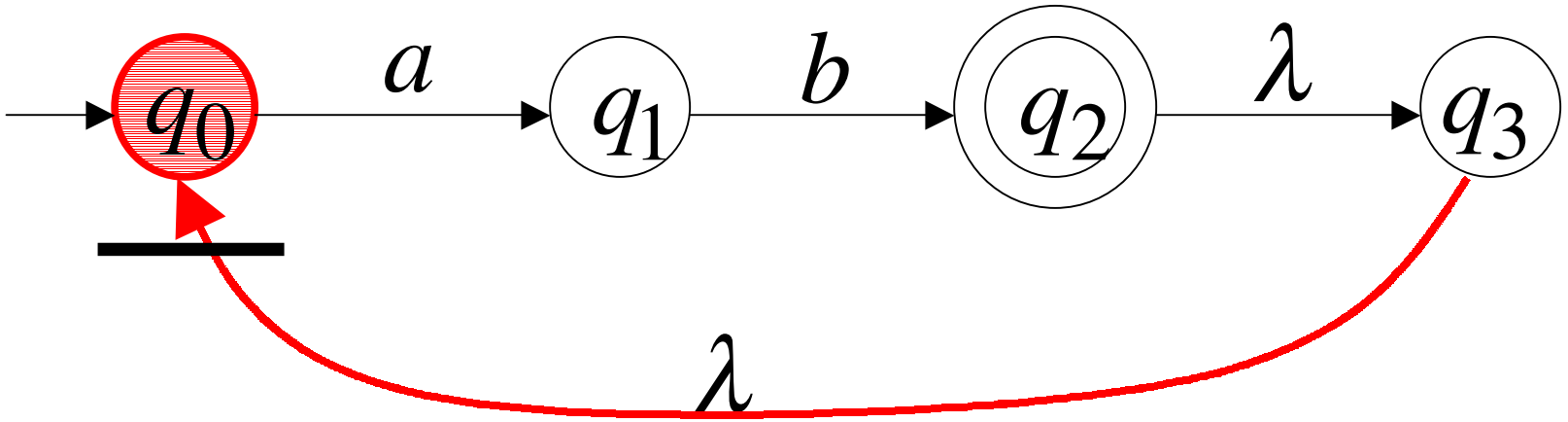
a
b
a
b


$q_0$
a
$q_1$
b
$q_2$
λ
$q_3$
λ

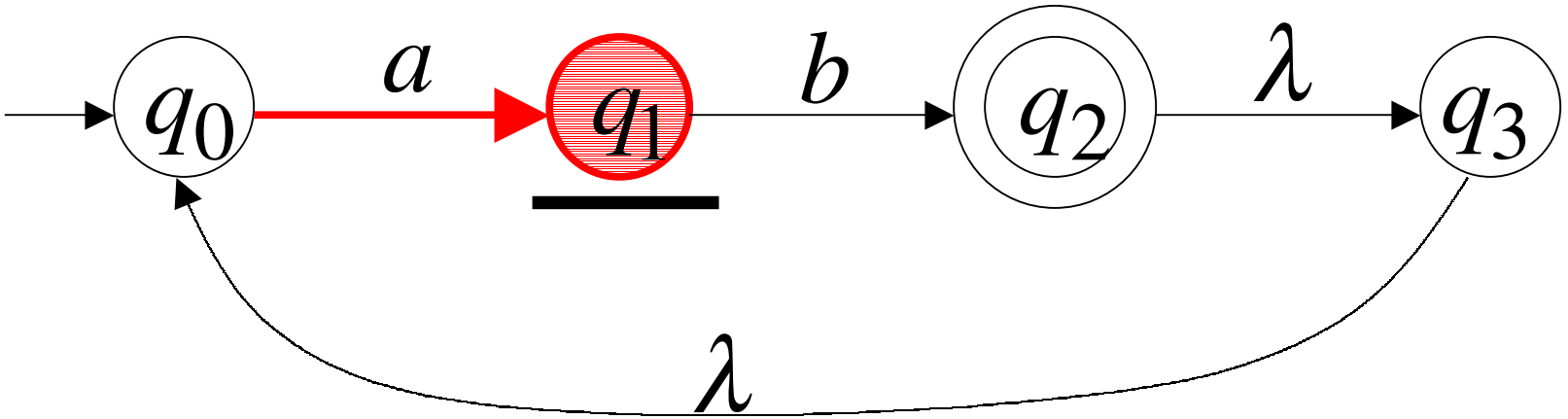

a
b
a
b

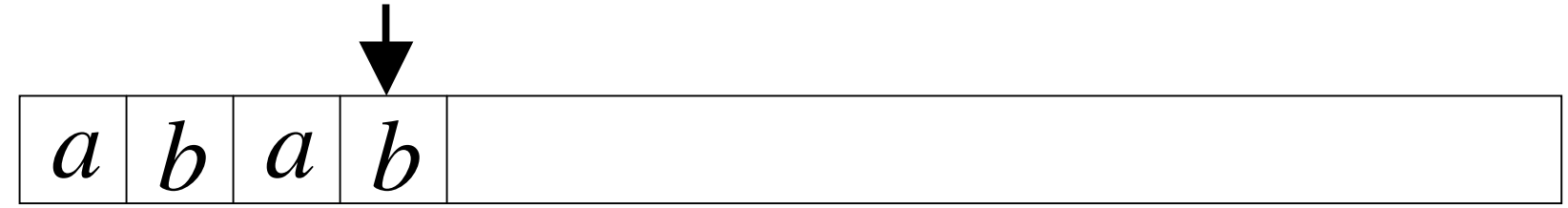

$q_0$
a
$q_1$
b
$q_2$
λ
$q_3$
λ

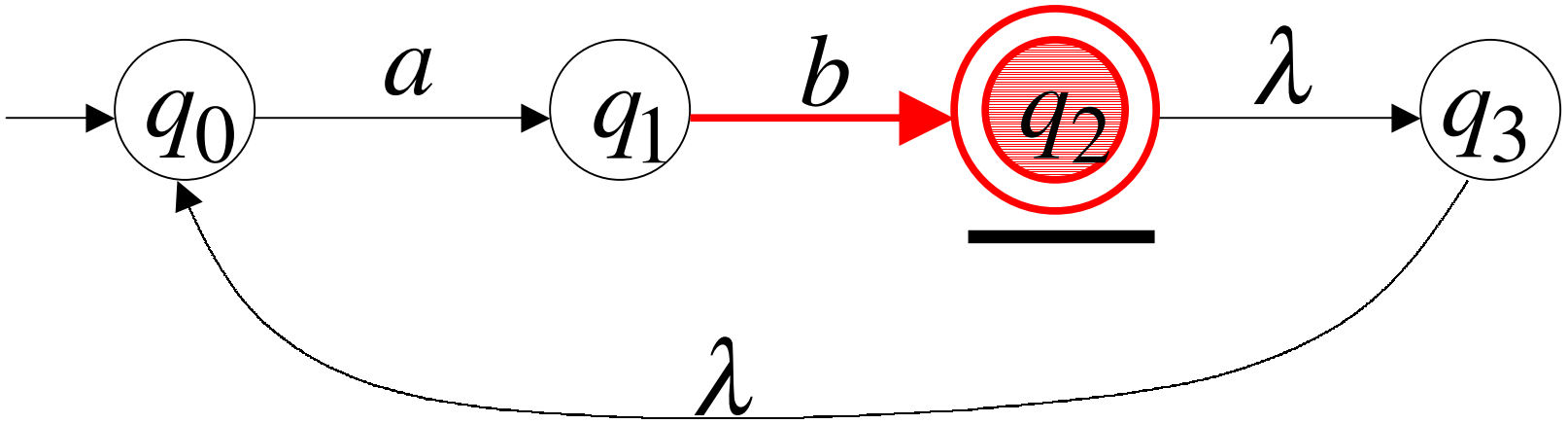

a
b
a
b

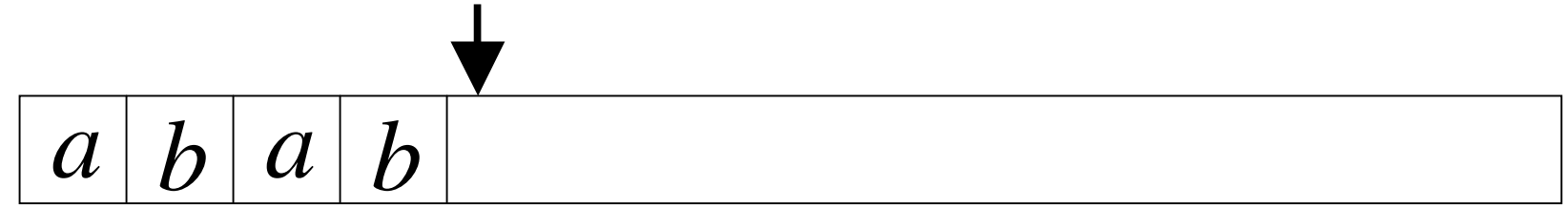

$q_0$
a
$q_1$
b
$q_2$
λ
$q_3$
λ

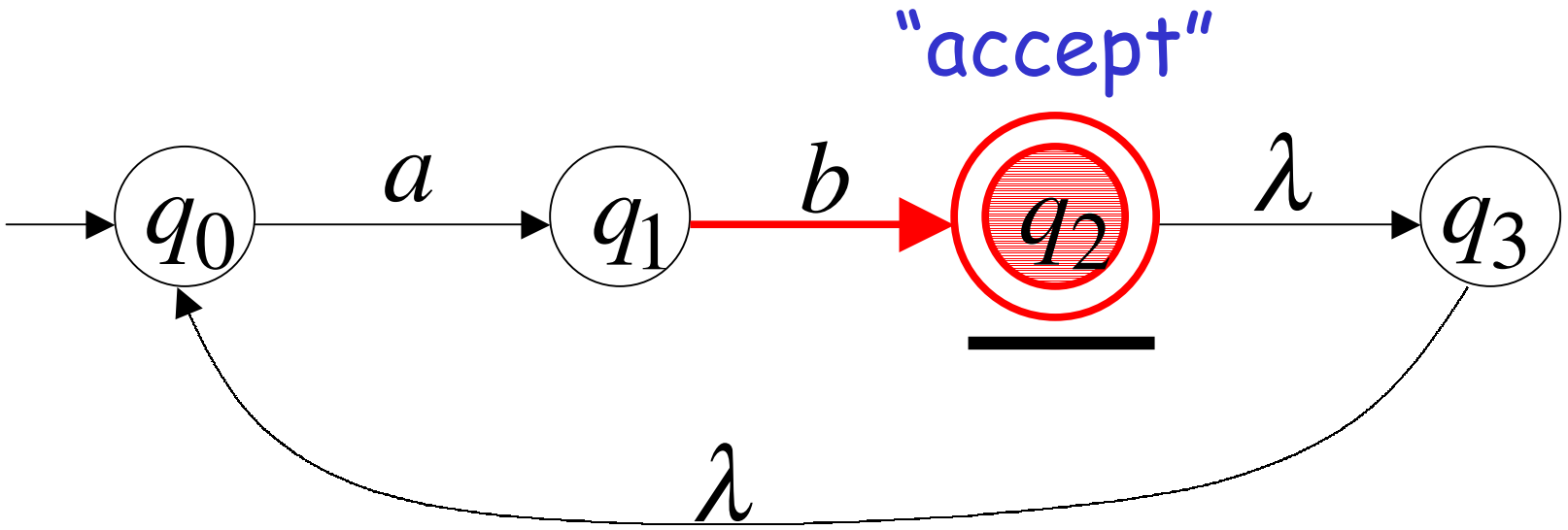
a
b
a
b


$q_0$
a
$q_1$
b
$q_2$
λ
$q_3$
λ


a
b
a
b


$q_0$
a
$q_1$
b
$q_2$
λ
$q_3$
λ

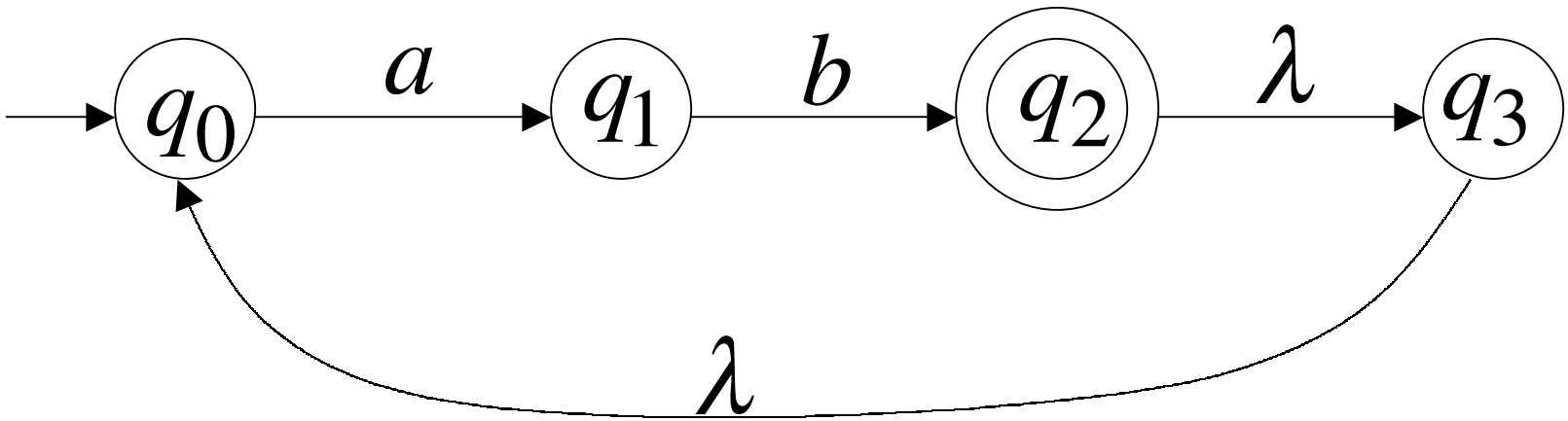

a
b
a
b


"accept"
$q_0$
$q_1$
$q_2$
$q_3$
a
b
λ
λ

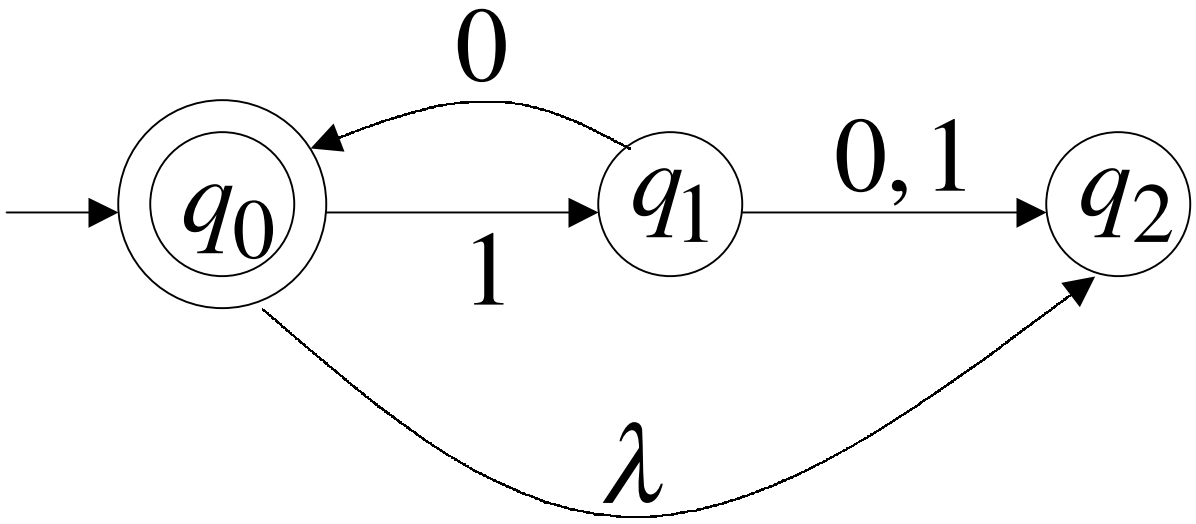
# Language accepted

$$L = \{ab,\ abab,\ ababab,\ ...\}$$

$$= \{ab\}^{+}$$

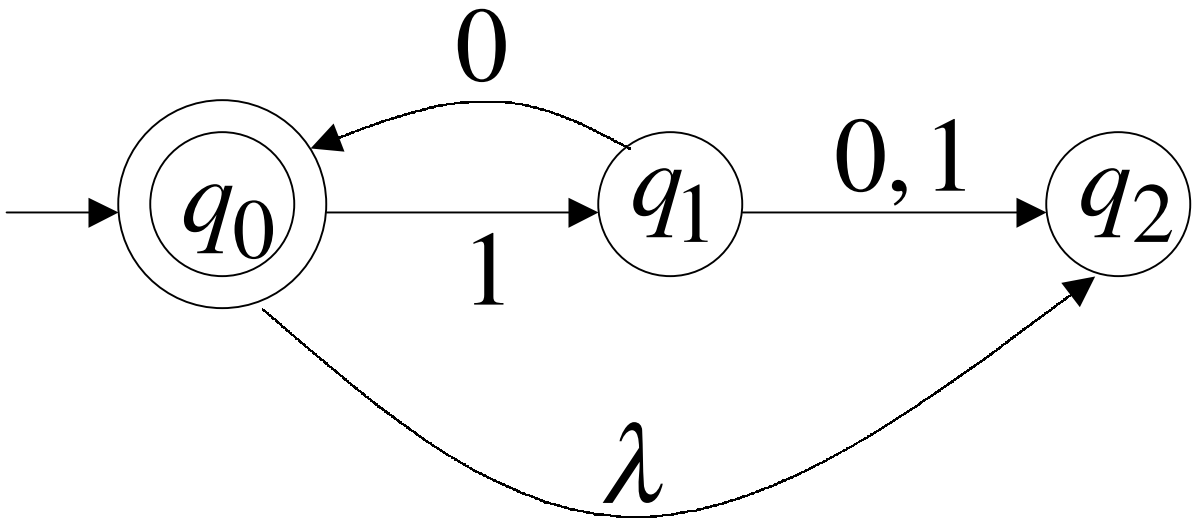


# Another NFA Example



# Language accepted

$$L = \{\lambda,\ 10,\ 1010,\ 101010,\ ...\}$$
$$= \{10\}^*$$



# Formal Definition of NFAs

$$M = (Q,\ \Sigma,\ \delta,\ q_0,\ F)$$

$Q$: Set of states, i.e. $\{q_0, q_1, q_2\}$

$\Sigma$: Input aplhabet, i.e. $\{a, b\}$

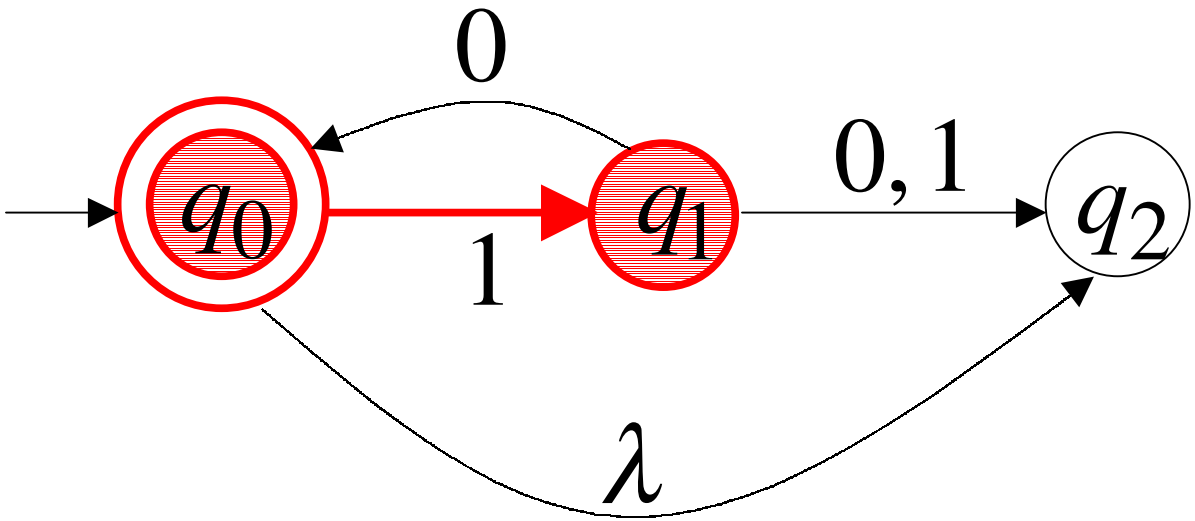
$\delta$: Transition function

$q_0$: Initial state

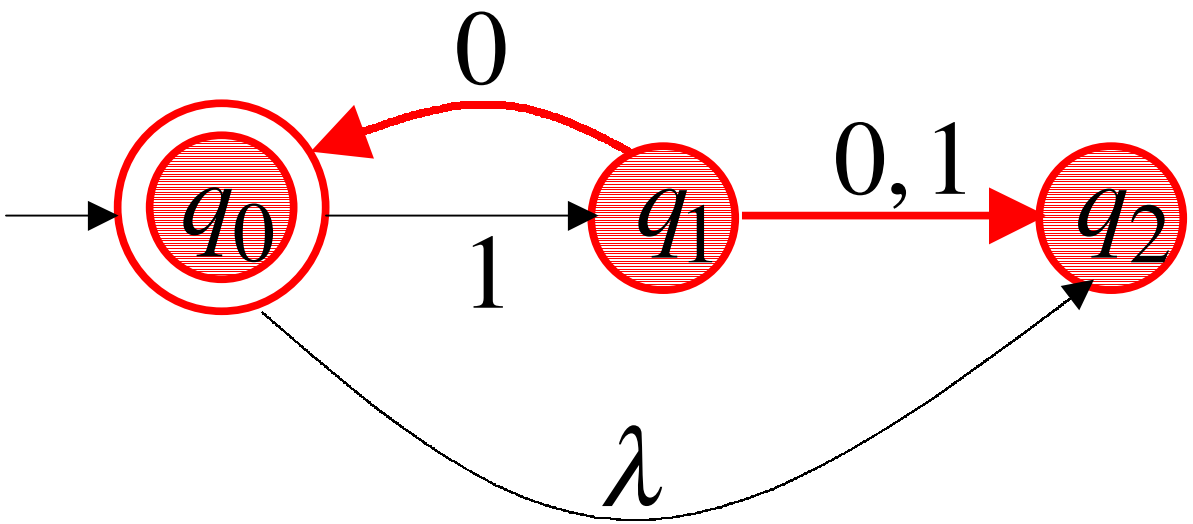
$F$: Final states

# Transition Function $\delta$

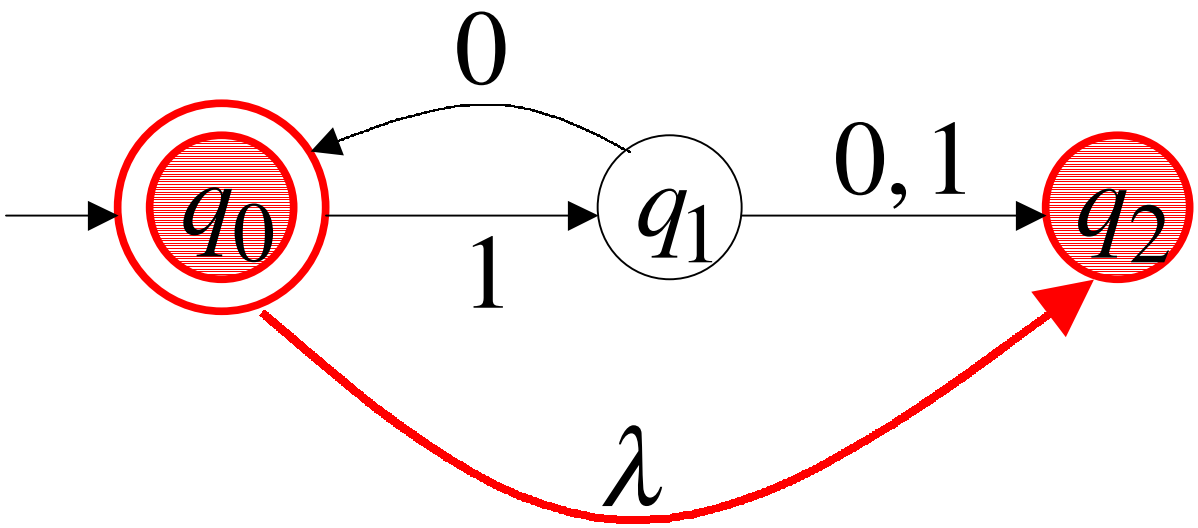
$$\delta(q_0, 1) = \{q_1\}$$



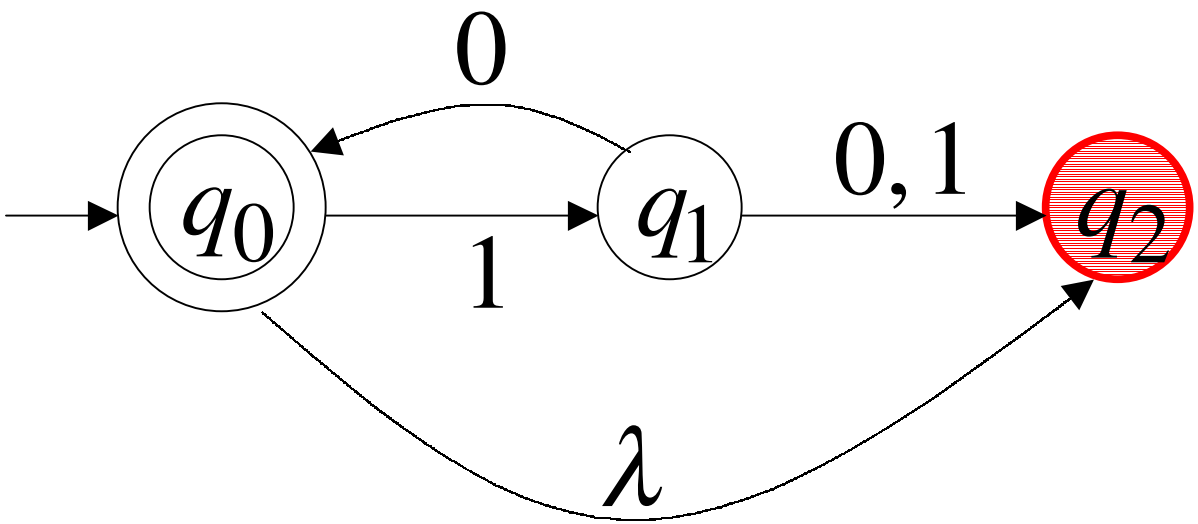
$$\delta(q_1,0)=\{q_0,q_2\}$$



$$\delta(q_0, \lambda) = \{q_0, q_2\}$$

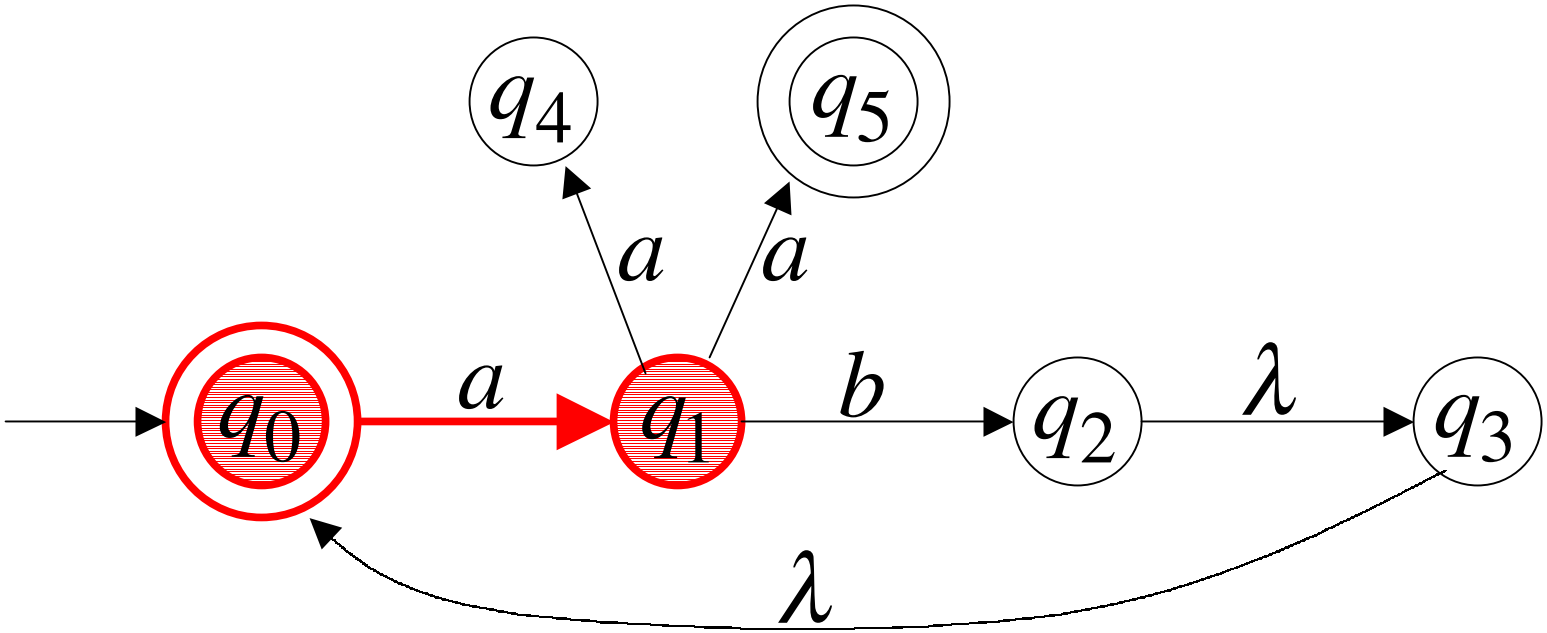


$$\delta(q_2,1)=\varnothing$$

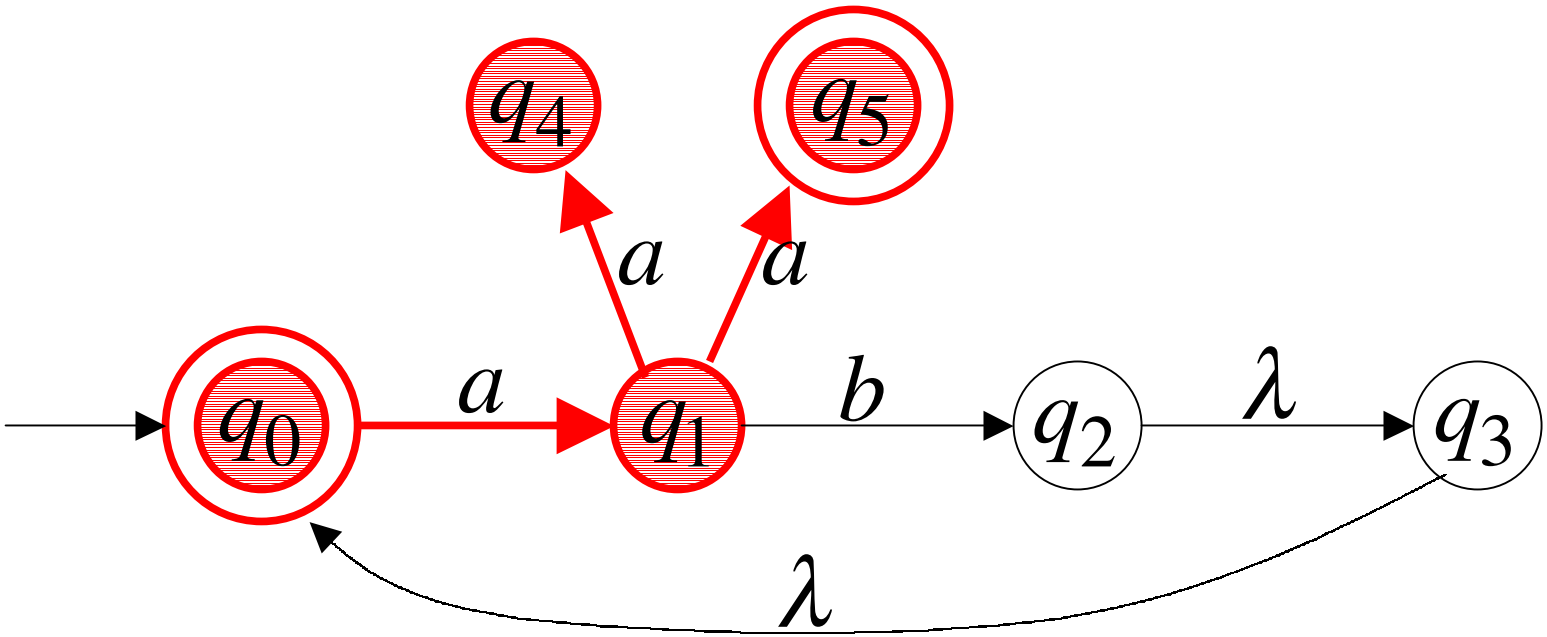


# Extended Transition Function $\delta^*$

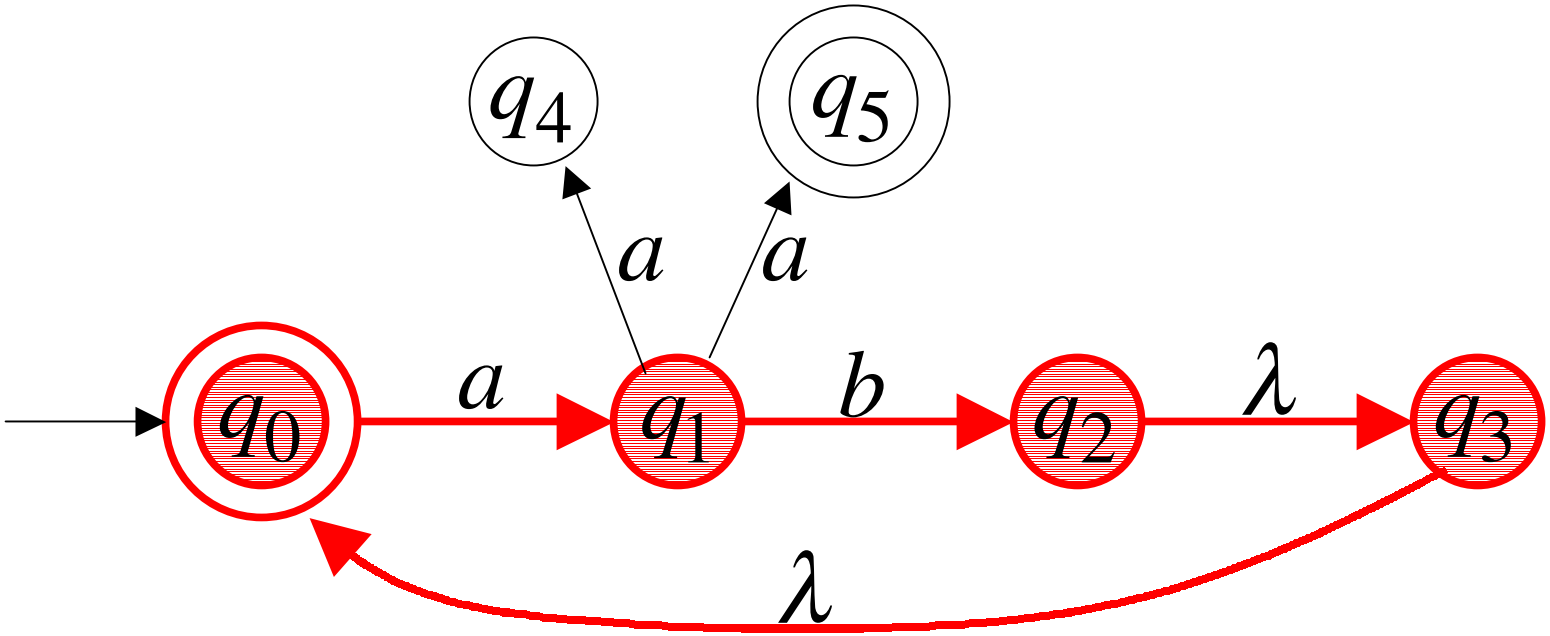
$$\delta^*(q_0, a) = \{q_1\}$$



$$\delta^*(q_0, aa) = \{q_4, q_5\}$$



$$\delta^*(q_0, ab) = \{q_2, q_3, q_0\}$$



# Formally

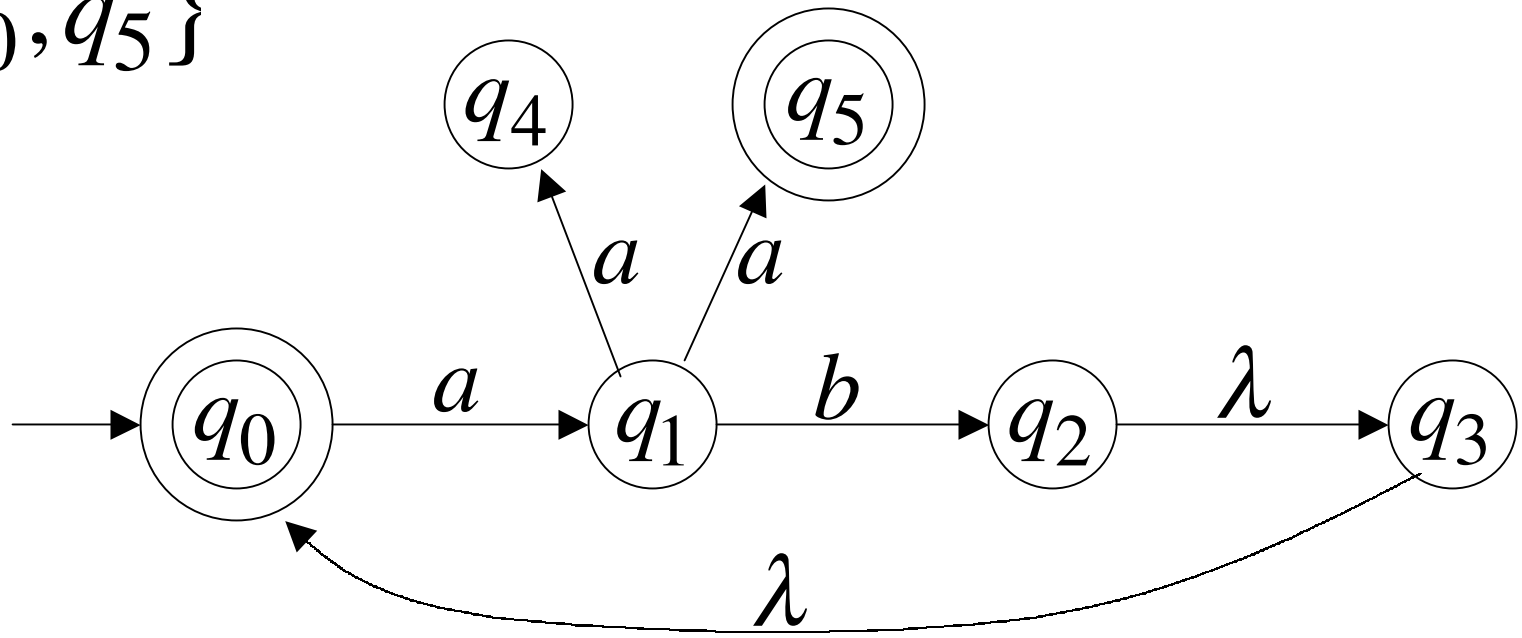
It holds $q_j \in \delta^*(q_i, w)$

if and only if

there is a walk from $q_i$ to $q_j$
with label $w$

# The Language of an NFA $M$

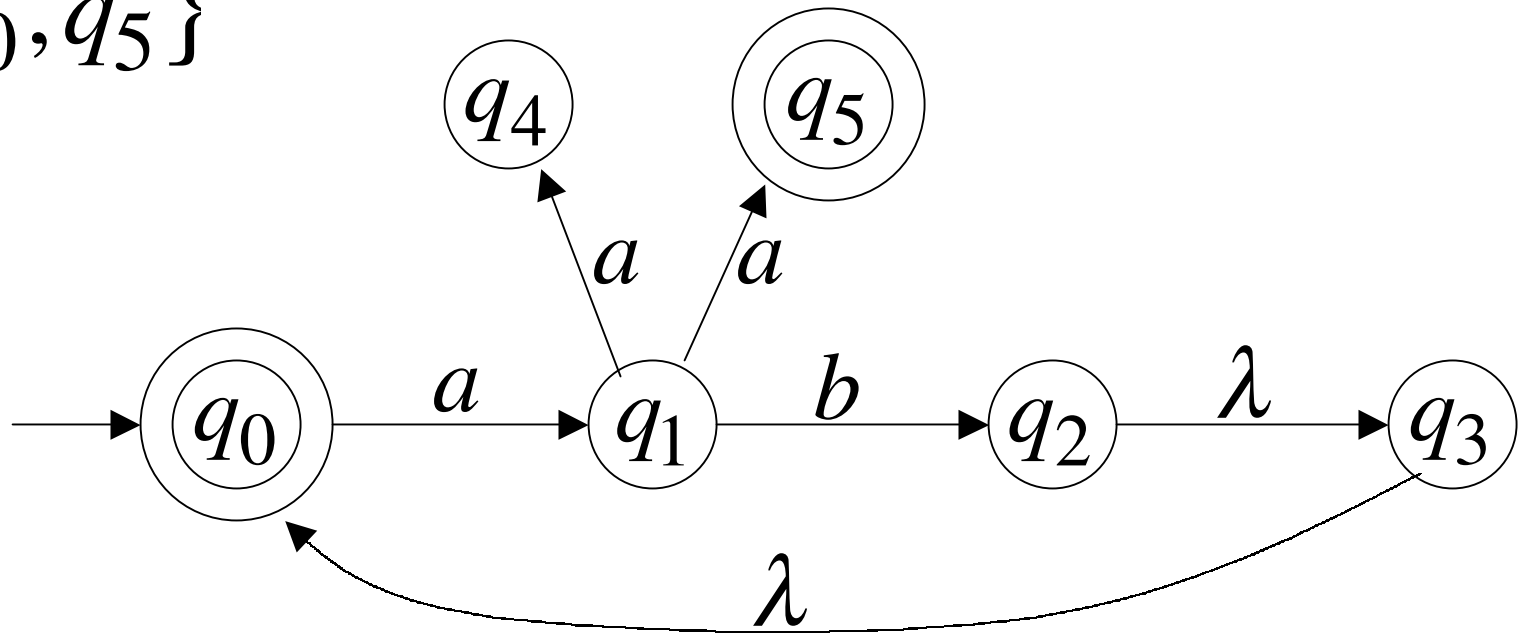
$F = \{q_0, q_5\}$




$\delta^*(q_0, aa) = \{q_4, q_5\}$

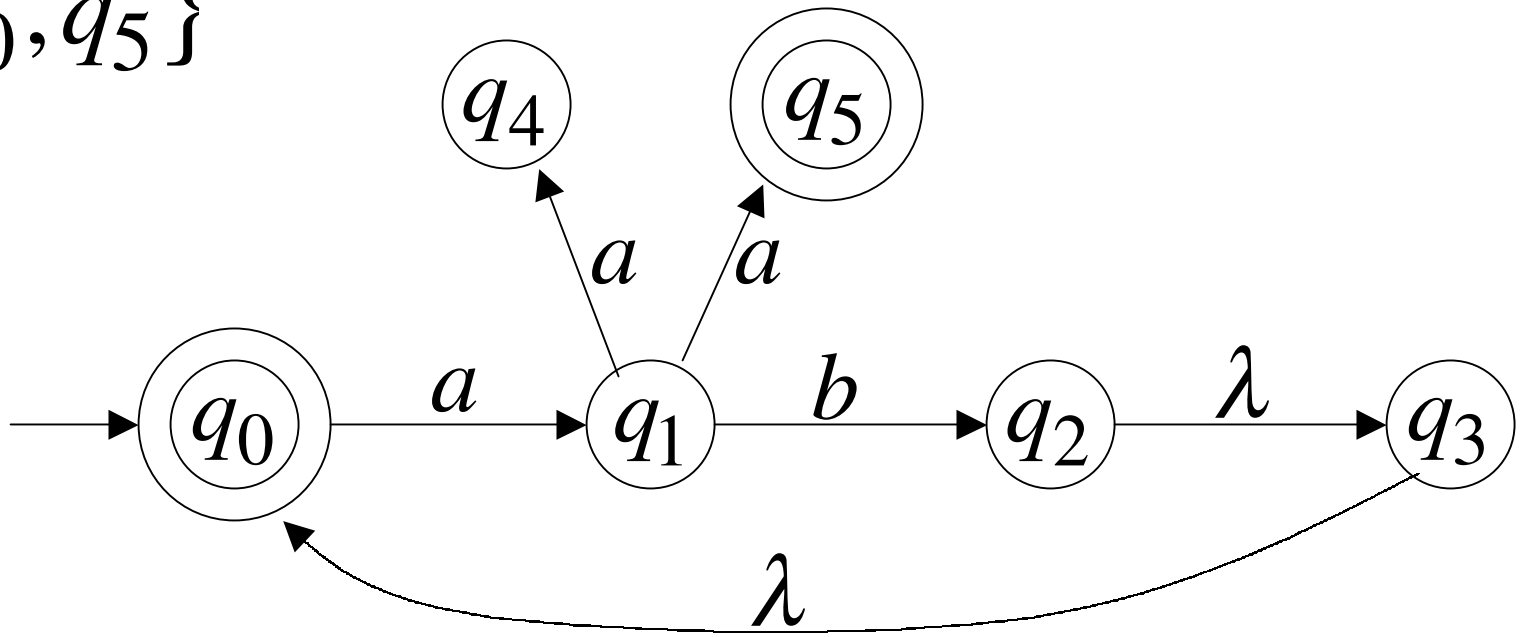
$aa \in L(M)$

$F = \{q_0, q_5\}$



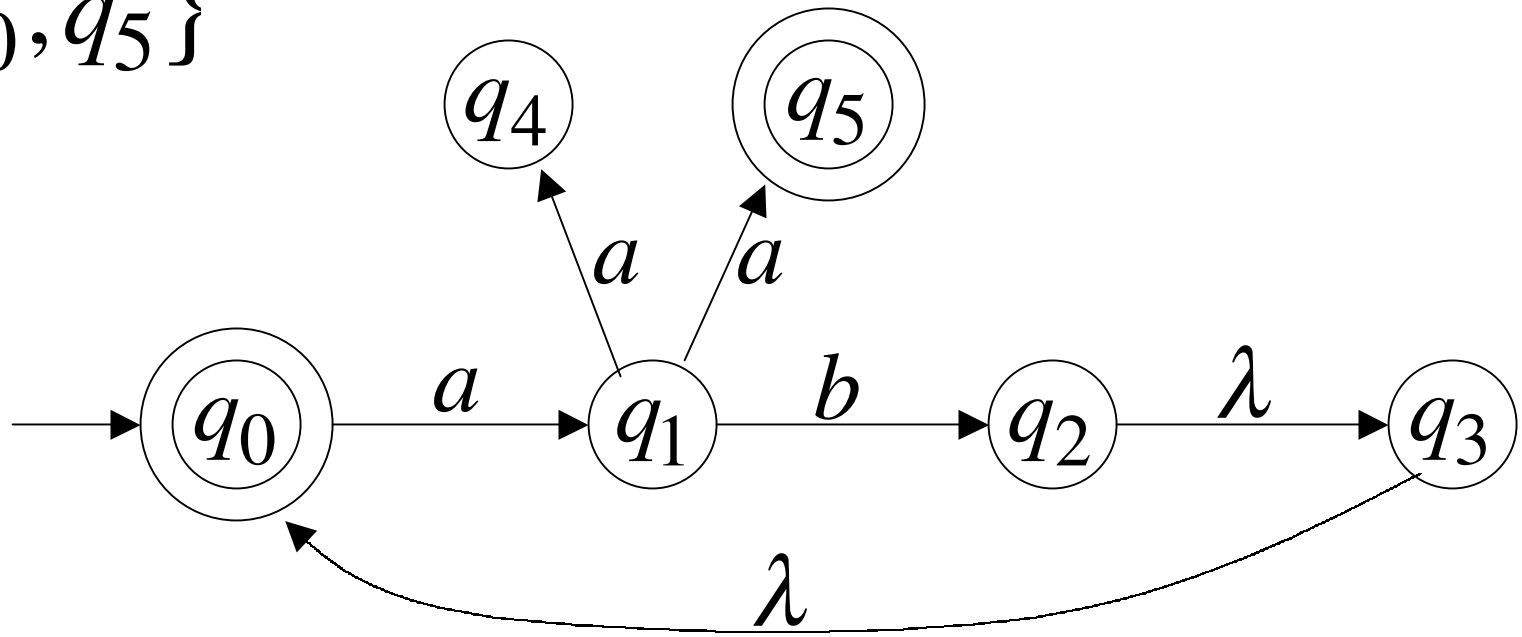

$\delta^*(q_0, ab) = \{q_2, q_3, \underline{q_0}\}$ $\qquad ab \in L(M)$

$F = \{q_0, q_5\}$

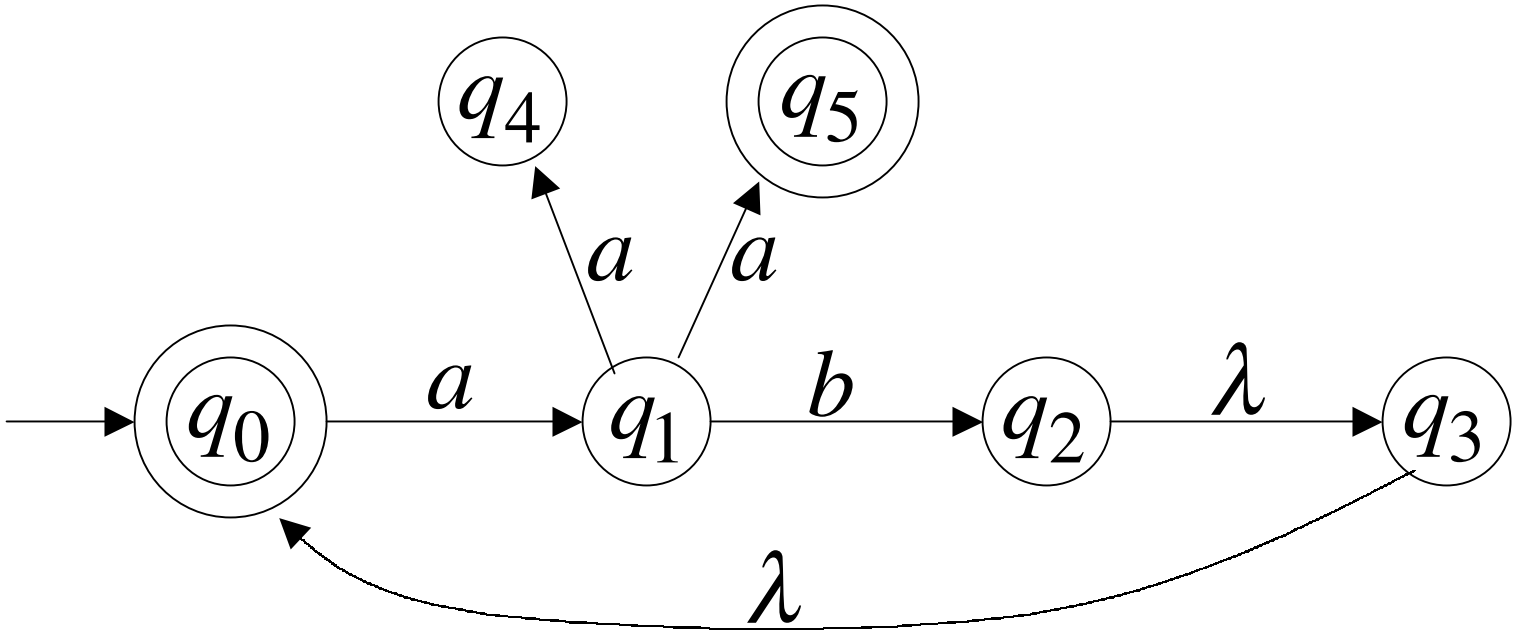



$\delta^*(q_0, abaa) = \{q_4, \underline{q_5}\}$ $\qquad$ $aaba \in L(M)$

$F = \{q_0, q_5\}$




$\delta^*(q_0, aba) = \{q_1\}$ $\qquad$ $aba \notin L(M)$




$$L(M) = \{aa\} \cup \{ab\}^* \cup \{ab\}^+\{aa\}$$

# Formally

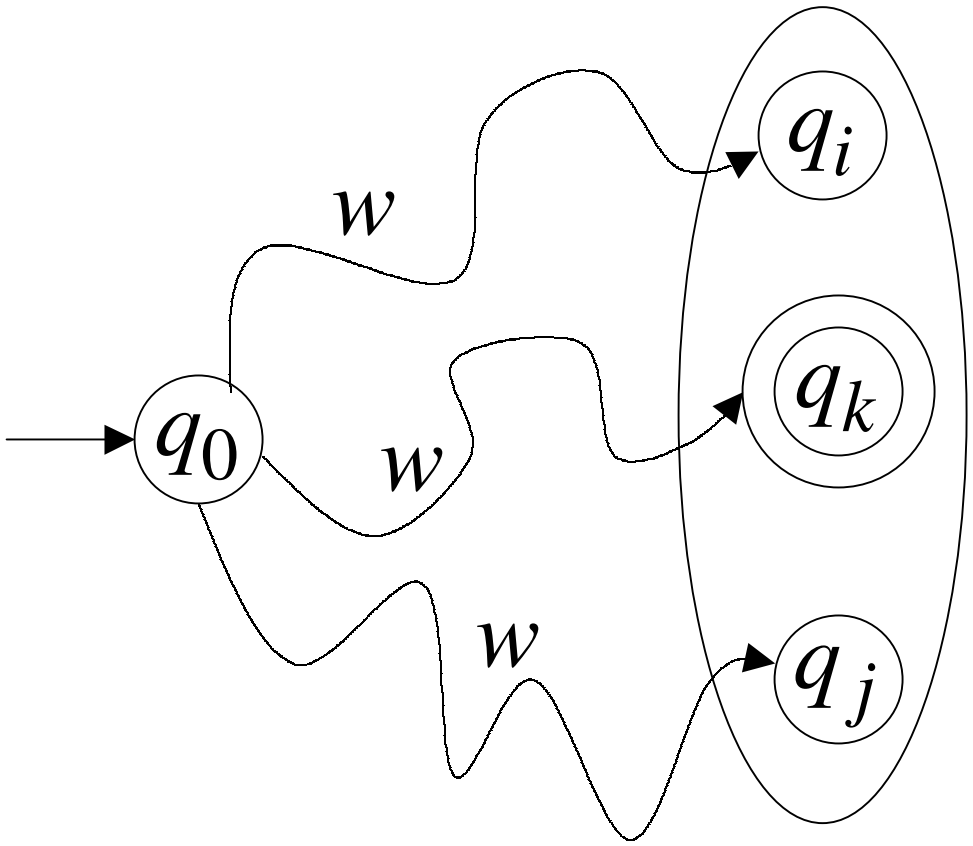
The language accepted by NFA $M$ is:

$$L(M) = \{w_1, w_2, w_3, \ldots\}$$

where $\delta^*(q_0, w_m) = \{q_i, q_j, \ldots\}$

and there is some $q_k \in F$ (final state)

$w \in L(M)$ $\delta^*(q_0, w)$




$q_k \in F$

# Equivalence of NFAs and DFAs

# Equivalence of Machines

For DFAs or NFAs:

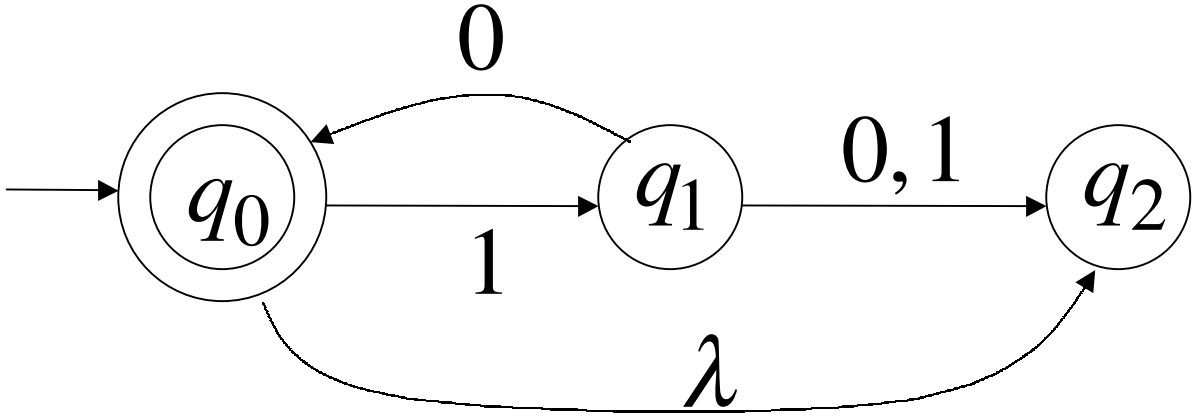
Machine $M_1$ is equivalent to machine $M_2$

if $L(M_1) = L(M_2)$

# Example

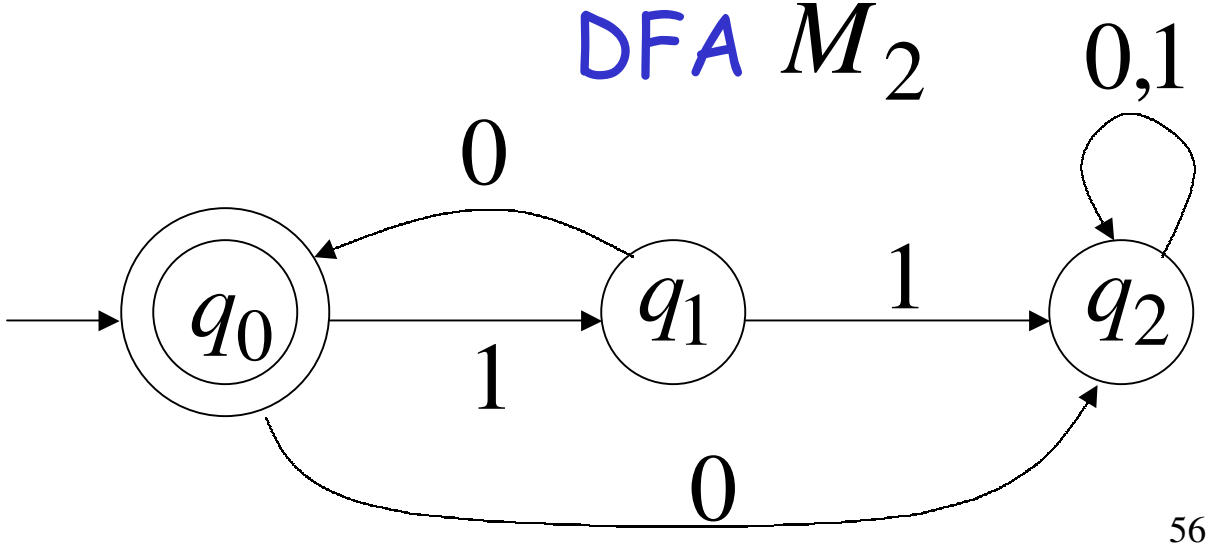
NFA $M_1$

$L(M_1) = \{10\}^*$




DFA $M_2$

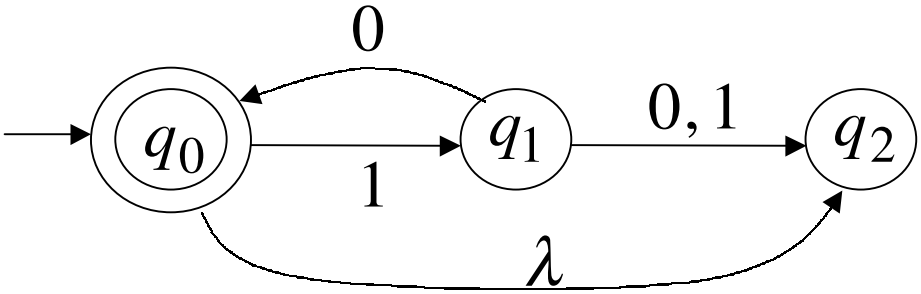
$L(M_2) = \{10\}^*$



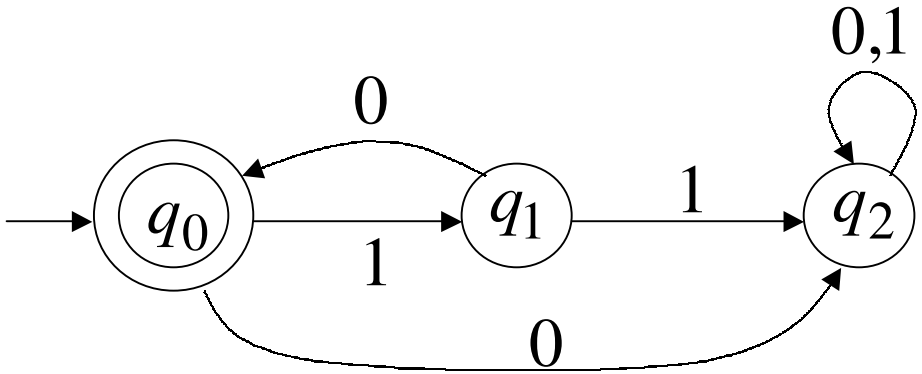
Since $L(M_1) = L(M_2) = \{10\}^*$

machines $M_1$ and $M_2$ are equivalent

NFA $M_1$




DFA $M_2$



# Equivalence of NFAs and DFAs

Question: NFAs = DFAs ?

Same power?

Accept the same languages?

# Equivalence of NFAs and DFAs

Question: NFAs = DFAs ? YES!

Same power?

Accept the same languages?

We will prove:

$$\left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by NFAs}\end{array}\right\} = \left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by DFAs}\end{array}\right\}$$

We will prove:

$$\left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by NFAs}\end{array}\right\} = \left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by DFAs}\end{array}\right\}$$

NFAs and DFAs have the same computation power

## Step 1

$$\left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by NFAs}\end{array}\right\} \supseteq \left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by DFAs}\end{array}\right\}$$

## Step 1

$$\left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by NFAs}\end{array}\right\} \supseteq \left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by DFAs}\end{array}\right\}$$

**Proof:** Every DFA is also an NFA

## Step 1

$$\left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by NFAs}\end{array}\right\} \supseteq \left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by DFAs}\end{array}\right\}$$

**Proof:** Every DFA is also an NFA

A language accepted by a DFA is also accepted by an NFA

## Step 2

$$\left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by NFAs}\end{array}\right\} \subseteq \left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by DFAs}\end{array}\right\}$$

# Step 2

$$\left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by NFAs}\end{array}\right\} \subseteq \left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by DFAs}\end{array}\right\}$$

Proof: Any NFA can be converted to an equivalent DFA

# Step 2

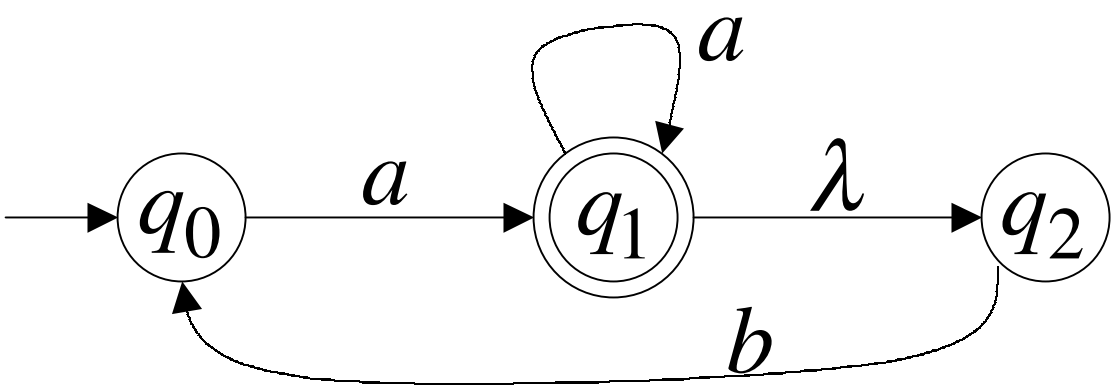
$$\left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by NFAs}\end{array}\right\} \subseteq \left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by DFAs}\end{array}\right\}$$

Proof: Any NFA can be converted to an equivalent DFA

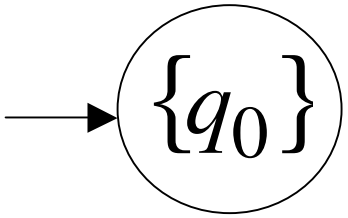
A language accepted by an NFA is also accepted by a DFA

# NFA to DFA

NFA

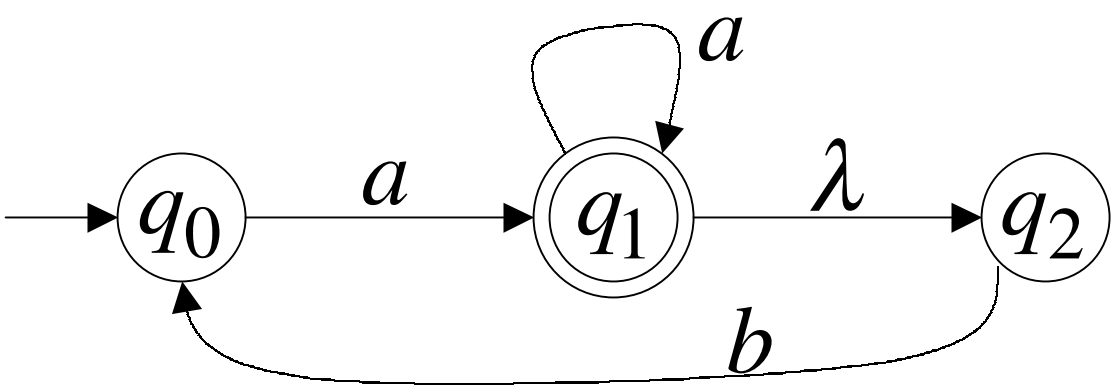



DFA

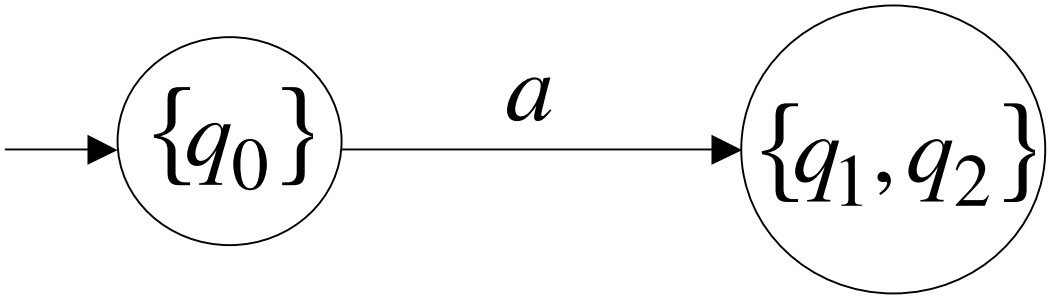


# NFA to DFA

NFA

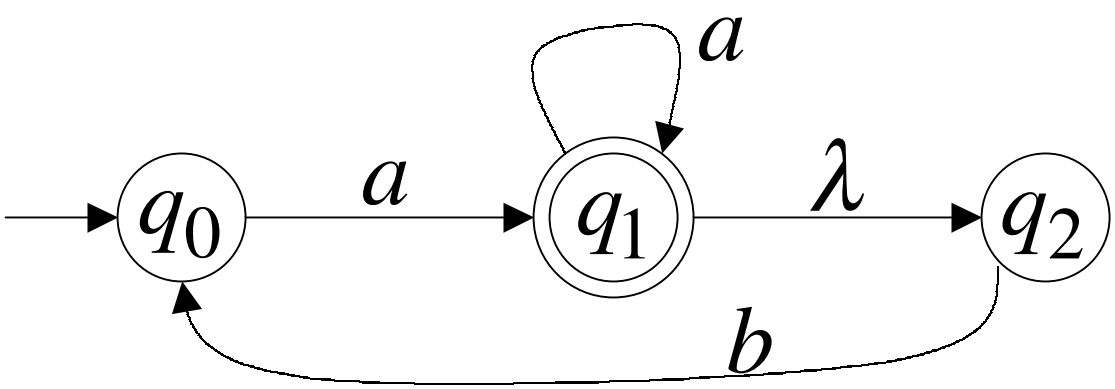



DFA

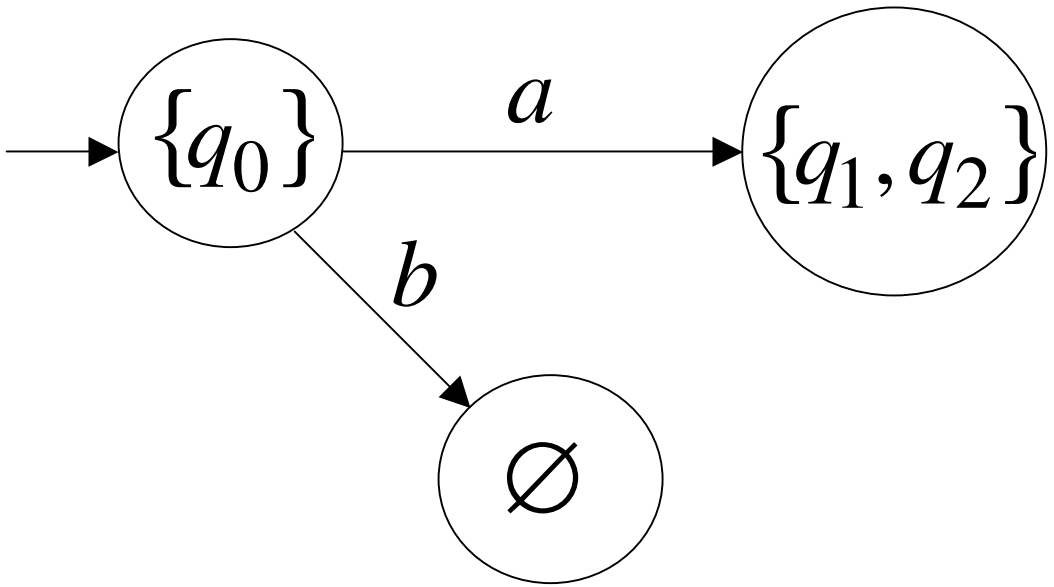


# NFA to DFA

NFA

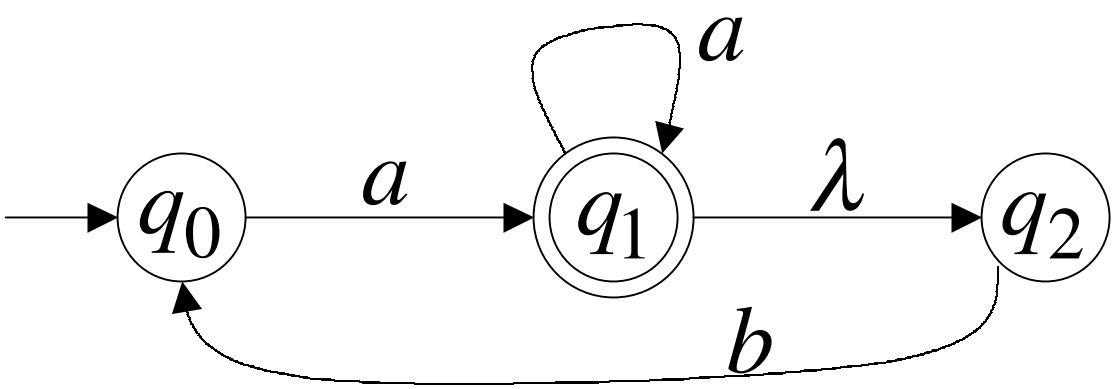



DFA

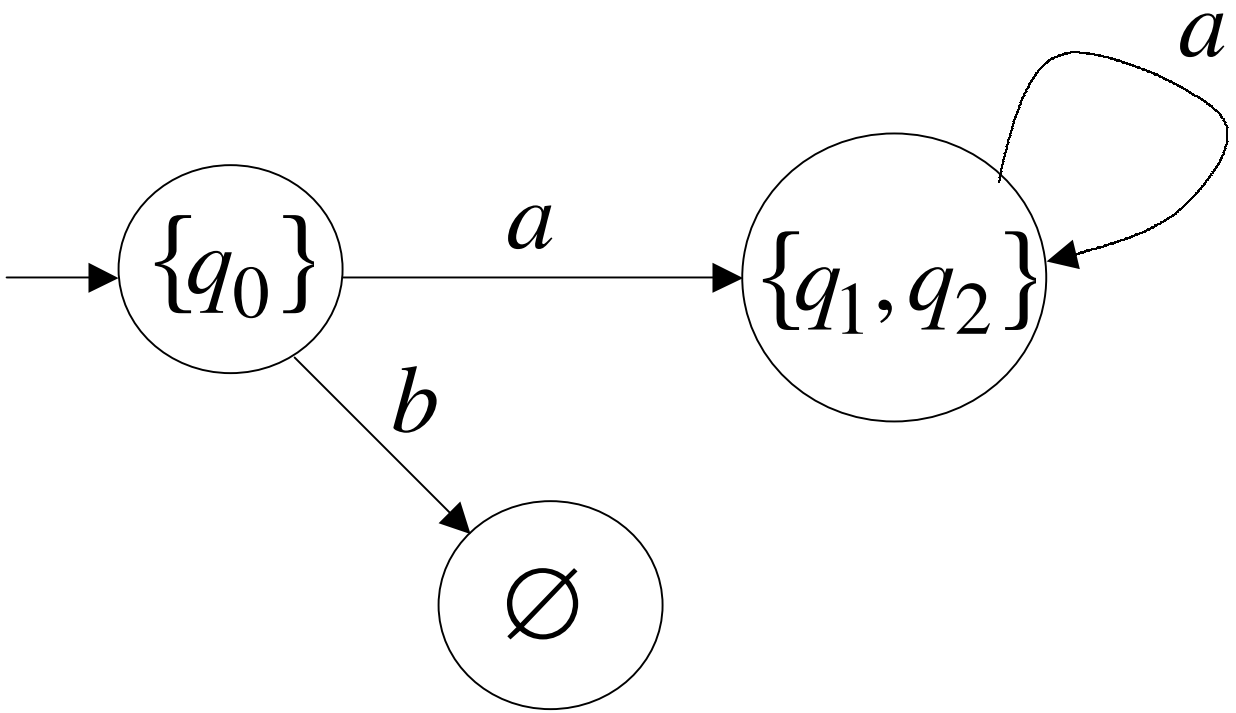


# NFA to DFA

NFA

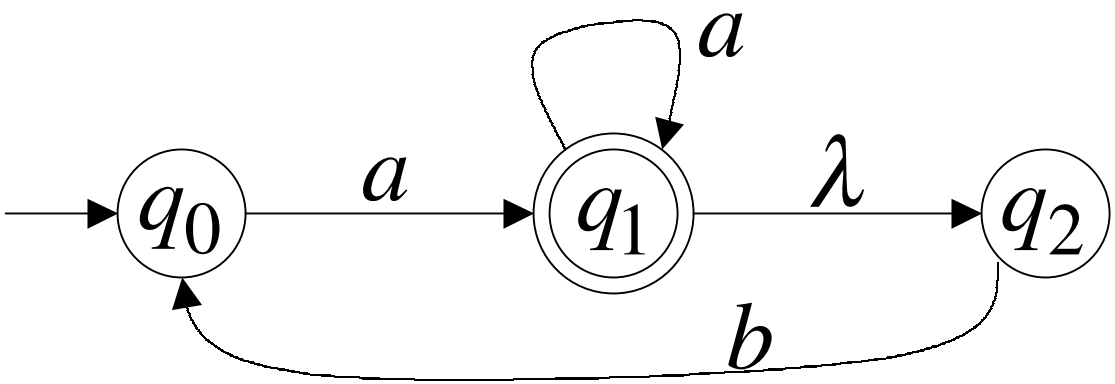



DFA



# NFA to DFA

NFA

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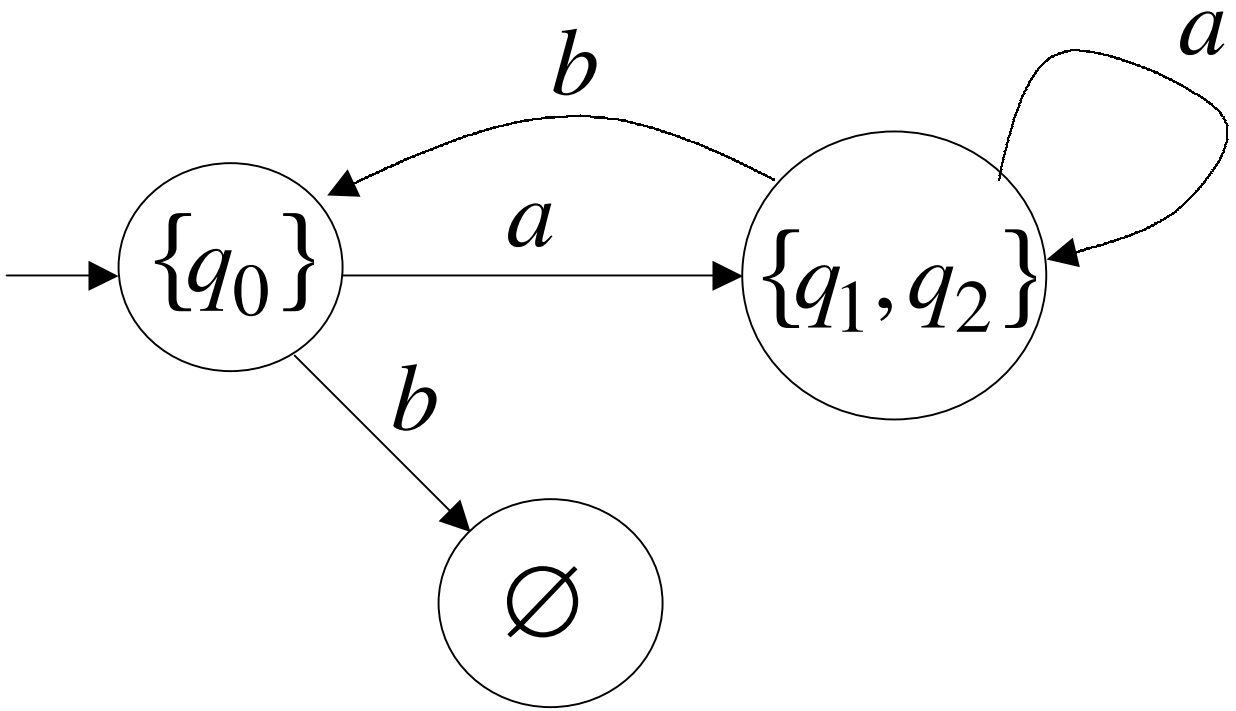
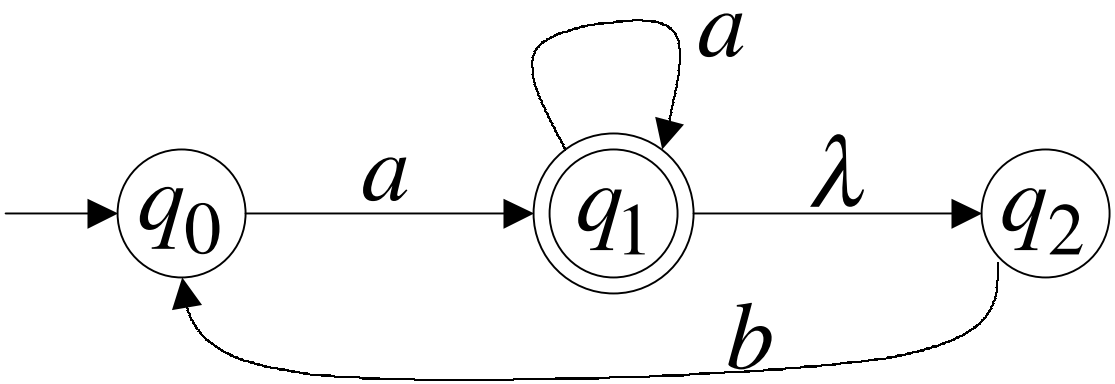


# NFA to DFA

NFA

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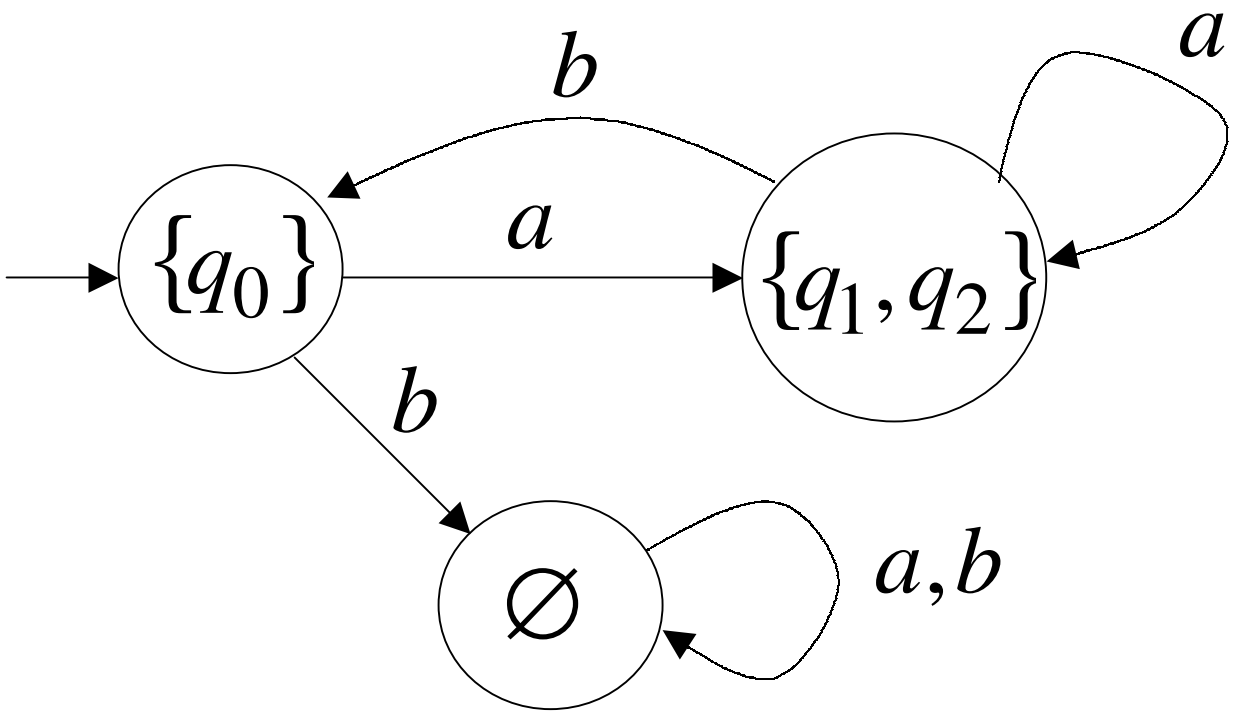
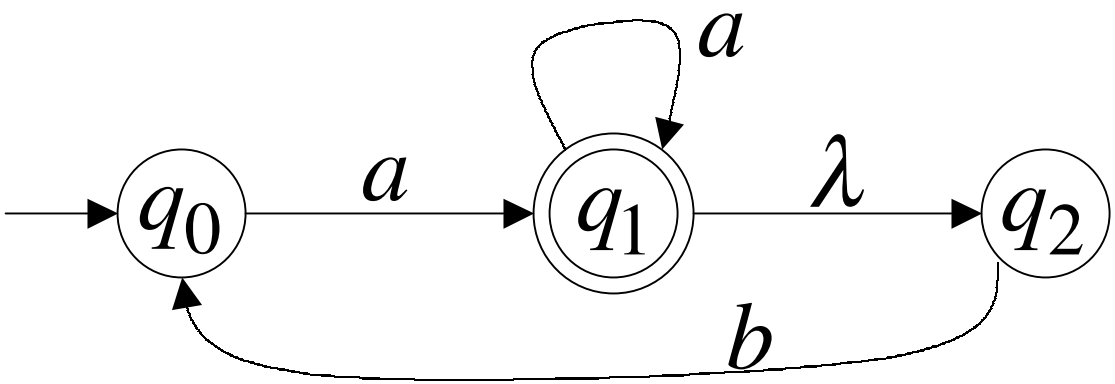
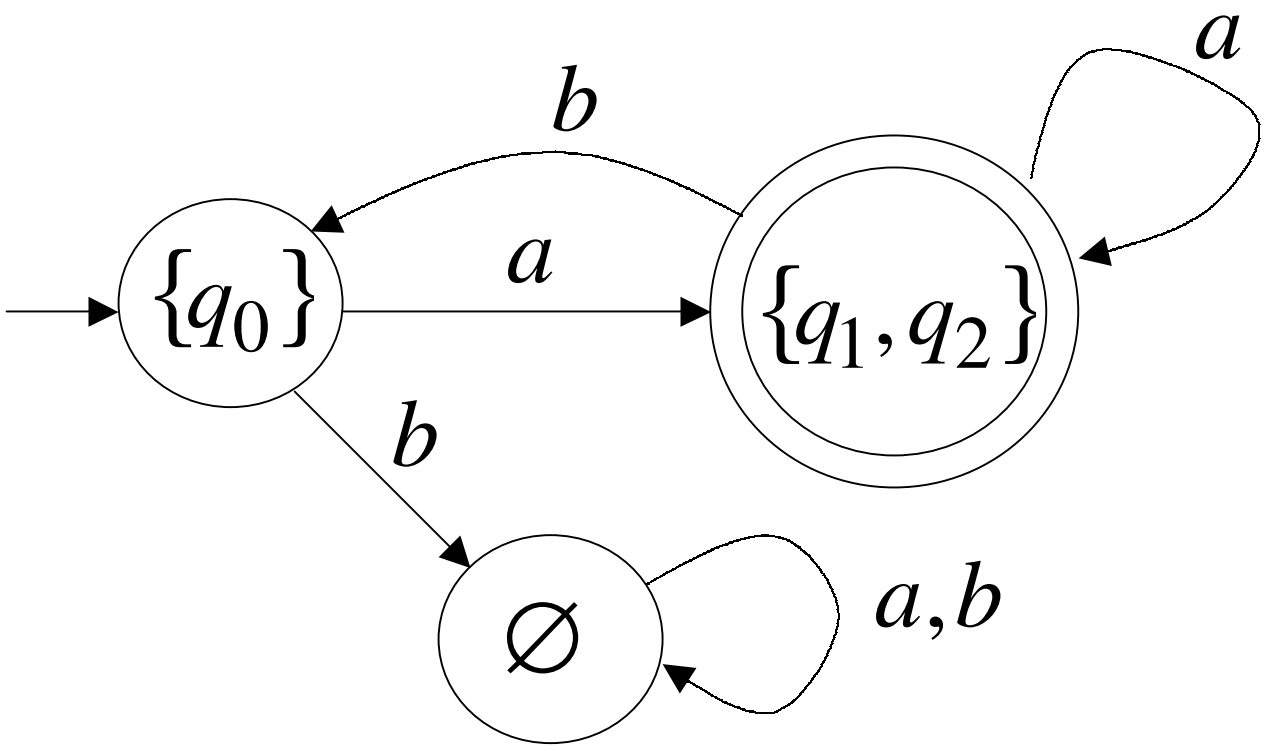


# NFA to DFA

NFA




DFA



# NFA to DFA: Remarks

We are given an NFA $M$

We want to convert it
to an equivalent DFA $M'$

With $L(M) = L(M')$

If the NFA has states

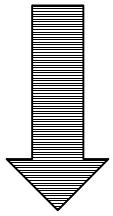
$$q_0, q_1, q_2, \ldots$$

the DFA has states in the powerset

$$\varnothing, \{q_0\}, \{q_1\}, \{q_1, q_2\}, \{q_3, q_4, q_7\}, \ldots.$$

# Procedure NFA to DFA

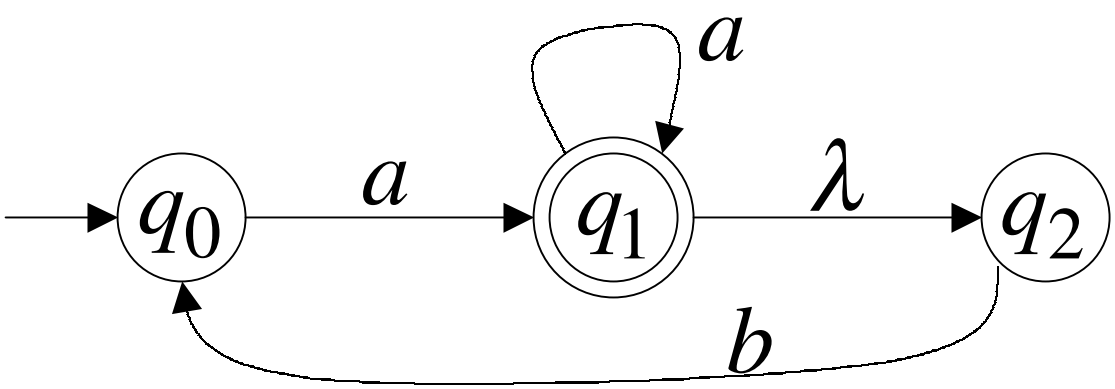
**1.** Initial state of NFA: $q_0$



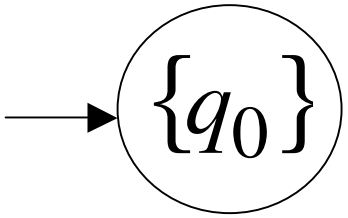
Initial state of DFA: $\{q_0\}$

# Example

NFA




DFA



# Procedure NFA to DFA

**2.** For every DFA's state $\{q_i, q_j, \ldots, q_m\}$

Compute in the NFA

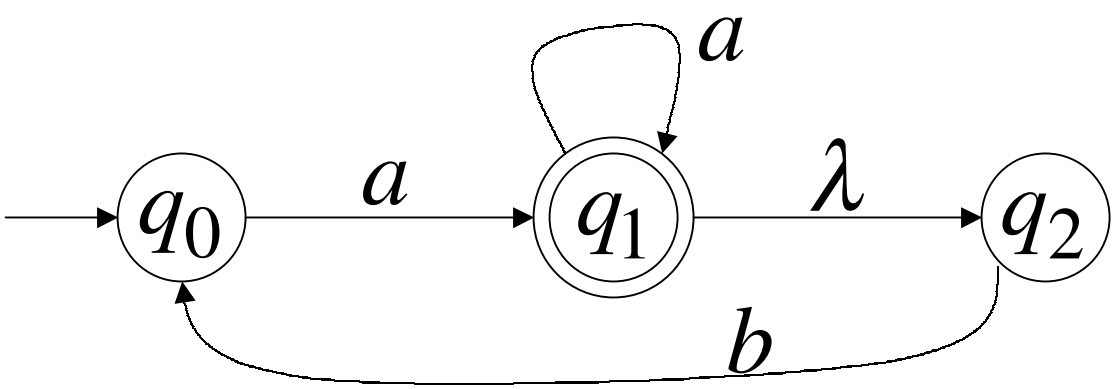
$$\left.\begin{array}{l} \delta^*(q_i, a), \\ \delta^*(q_j, a), \\ \ldots \end{array}\right\} = \{q'_i, q'_j, \ldots, q'_m\}$$

Add transition

$$\delta(\{q_i, q_j, \ldots, q_m\},\ a) = \{q'_i, q'_j, \ldots, q'_m\}$$

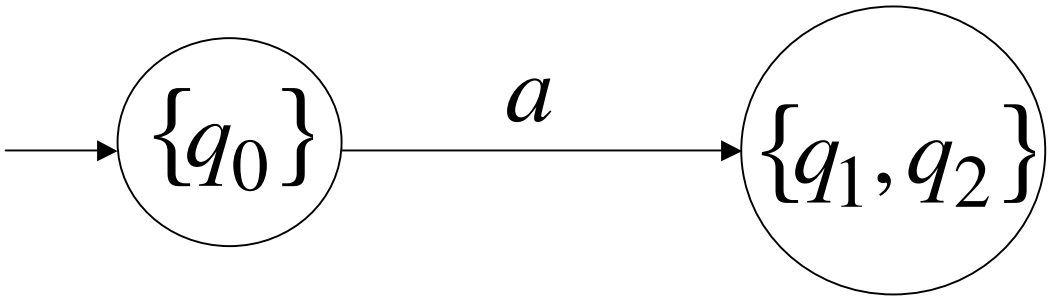
# Exampe

NFA




$$\delta^*(q_0, a) = \{q_1, q_2\}$$

DFA




$$\delta(\{q_0\}, a) = \{q_1, q_2\}$$

# Procedure NFA to DFA

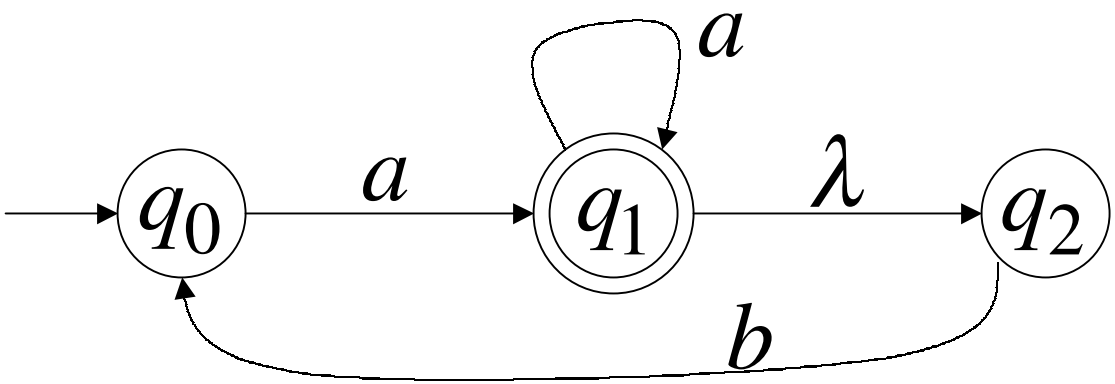
Repeat Step 2 for all letters in alphabet,

until

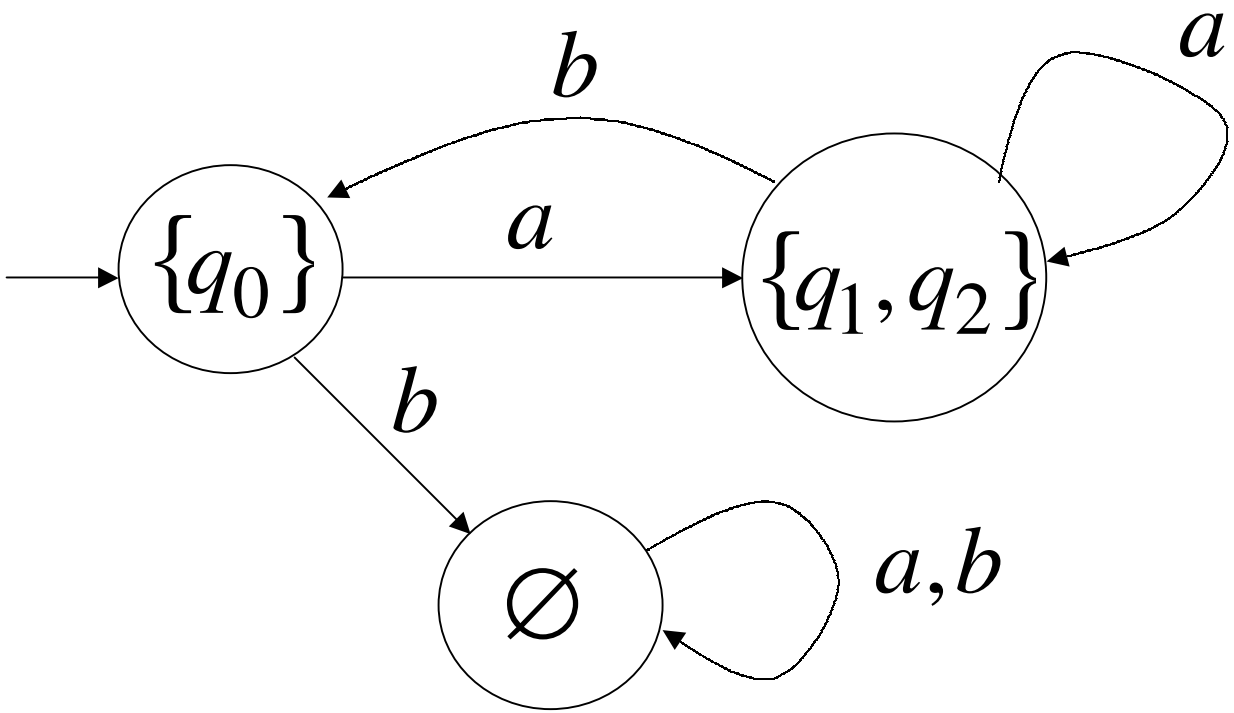
no more transitions can be added.

# Example

NFA




DFA



# Procedure NFA to DFA

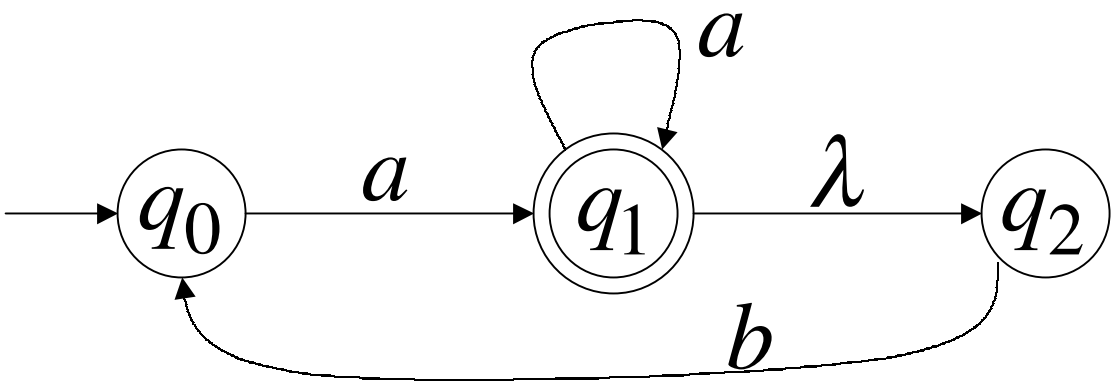
**3.** For any DFA state $\{q_i, q_j, \ldots, q_m\}$

If some $q_j$ is a final state in the NFA

Then, $\{q_i, q_j, \ldots, q_m\}$
is a final state in the DFA

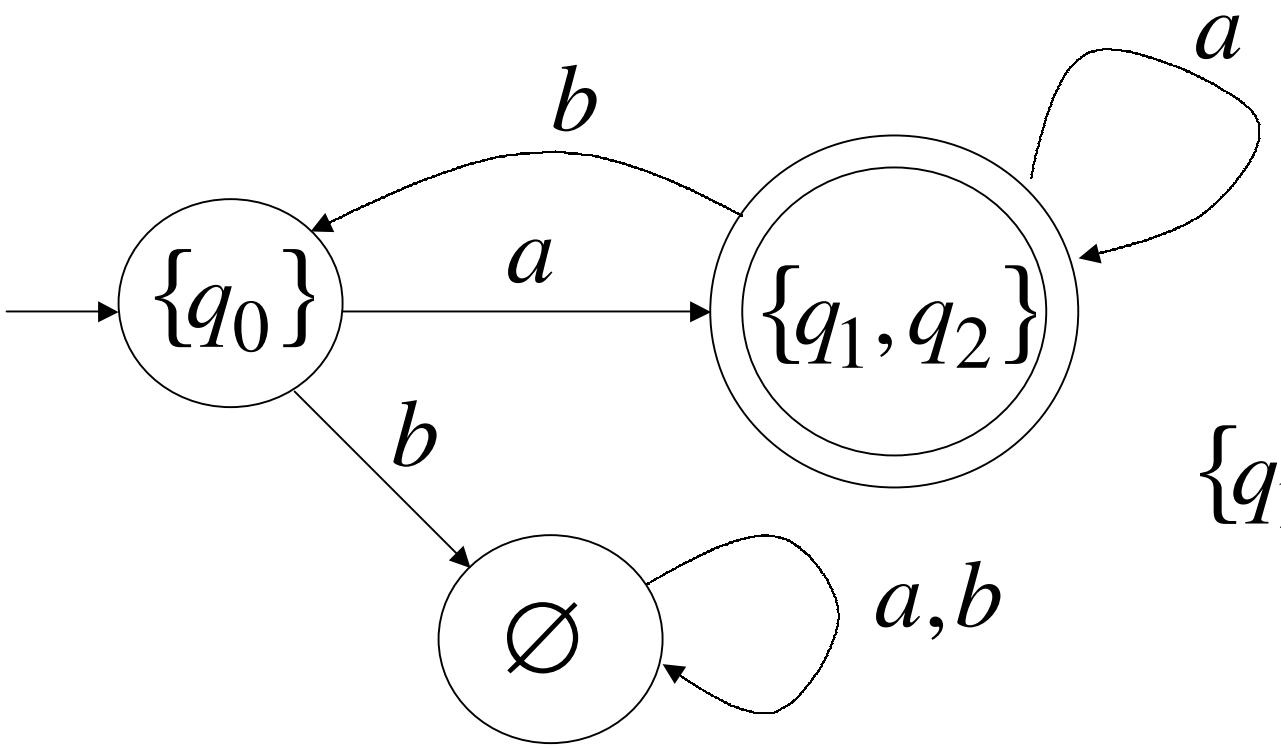
# Example

NFA




$q_1 \in F$

DFA




$\{q_1, q_2\} \in F'$

# Theorem

Take NFA $M$

Apply procedure to obtain DFA $M'$

Then $M$ and $M'$ are equivalent :

$$L(M) = L(M')$$

# Finally

We have proven

$$\left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by NFAs}\end{array}\right\} = \left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by DFAs}\end{array}\right\}$$

We have proven

$$\left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by NFAs}\end{array}\right\} = \left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by DFAs}\end{array}\right\}$$

Regular Languages

We have proven

$$\left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by NFAs}\end{array}\right\} = \left\{\begin{array}{l}\text{Languages}\\ \text{accepted}\\ \text{by DFAs}\end{array}\right\}$$

Regular Languages = Regular Languages