CSCI 2400 – Models of Computation, Section 3

# Solutions to Homework 7

**Problem 1.** Problem 9, page 245. Give the transition diagram.

**Problem 2.** Problem 11, page 246. Give the transition diagram.

**Problem 3.**

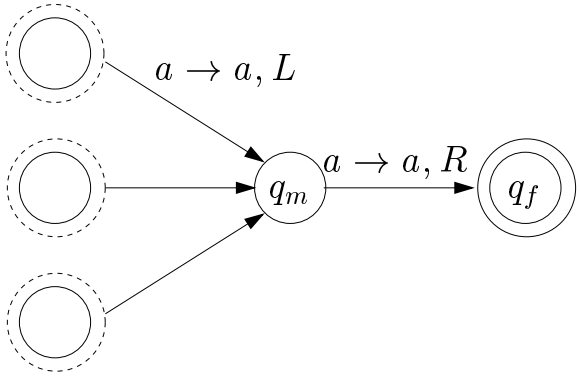


Figure 1: Conversion to a single final state

Yes, it is generally true that for every Turing Machine there exists an equivalent Turing machine with a single final state.

As shown in Figure 1, we can add the new state $q_f$ which is the single final state. We add transitions from the old final states to the new final state. To keep the head on the same position as in the old final states, we add an intermediate state $q_m$. With the transition to state $q_m$ the head moves left, and with the transition to state $q_f$ the head moves right. (We add similar transitions for all possible symbols $a$.)

**Problem 4.** Problem 3, a,b,c, page 251.

**Problem 5.**
**(a)** We only need to define the $\delta$ function, which is the only thing different from the standard model. We add the stay option to the machine, so there are three possible moves $L, R, S$, where $S$ means that the head stays in the same place. Therefore we have:

$$\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R, S\}$$

The transitions are divided in two categories:

(i) Transitions that only change the symbol on the tape:

$$\delta(q, a) = (p, b, S)$$

(ii) Transitions that move the head:

$$\delta(q, a) = (p, a, L), \quad \text{or}$$
$$\delta(q, a) = (p, a, R).$$

**(b)** For convenience, we will call the new kind of machines as MOS machines (Move Or Stay machines). We will prove that the MOS machines have the same power with standard Turing Machines. In particular, we will prove that MOS machines can simulate standard Turing machines, and vice versa.

- **MOS machines simulate standard Turing machines**.

  Every transition of a standard Turing machine can be simulated with two transitions in the MOS machine:

| Standard Turing machine | MOS machine |
|---|---|
| $\delta(q, a) \to (p, b, L)$ | $\delta(q, a) \to (q', b, S)$<br>$\delta(q', b) \to (p, b, L)$ |
| $\delta(q, a) \to (p, b, R)$ | $\delta(q, a) \to (q', b, S)$<br>$\delta(q', b) \to (p, b, R)$ |

  State $q'$ is an intermediate auxiliary state.

- **Standard Turing Machines simulate MOS machines**.

  Every transition of an MOS machine can be simulated by a standard Turing machine. Transitions of type (ii) of the MOS machine are translated immediately to the same transitions in the standard Turing machine. The only interesting case are transitions of type (i). Every transition of type (i) can be simulated with two transitions in the standard Turing machine:

| MOS machine | Standard Turing machine |
|---|---|
| $\delta(q, a) \to (p, b, S)$ | $\delta(q, a) \to (q', b, L)$<br>$\delta(q', x) \to (p, x, R)$ |

  State $q'$ is an intermediate auxiliary state, and $x$ can be any symbol (we repeat the transitions for all possible symbols).