CSCI 2400 – Models of Computation, Section 3

Homework 8

Return by: Thursday, November 30

1. Consider the *traveling salesperson problem:*

We have a map with n cities $c_1, c_2, \ldots, c_n$. For every pair of cities c_i, c_j there is a road that connects the two cities and this road has length d_{ij}. The various road lengths may be different. The problem is to find the shortest route, starting from city c_1, that goes through all the cities, and returns back to city c_1. This is the route that the salesperson will follow.

(a) Describe an algorithm for a nondeterministic Turing machine that finds the shortest route in polynonial number of time steps (with respect to n). (Each time step corresponds to a transition of the machine.)

(b) Describe an algorithm for a deterministic Turing machine that finds the shortest route. Is the time of your algorithm polynomial?

2. (a) Give the algorithm of a Turing Machine that enumerates the strings of the following language in proper order.

$$L = \{a^n b^n : n \geq 1\}$$

(b) For this language, is the proper order the same with the alphabetical order? Explain your answer.

3. (a) Prove that the union of two countable sets is a countable set.

(b) Use the result of (a) to prove that the set of non recursively enumerable languages is not countable.

4. Show that if a language is not recursively enumerable then its complement cannot be recursive.

5. Suppose that language L is such that there is a Turing machine that enumerates the elements of L in proper order. Show that this means that L is recursive.