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Analysis of Opinion Evolution in a Multi-cultural Student Social Network

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Abstract

The spread of opinions in social networks is dependent on structural properties of the network and the individual characteristics of its nodes. To capture this dependence, several abstract models of such spread were proposed. First, we model the difference between the dynamics of opinion spread in communities with a static social network versus a dynamic social network. Here we use the theoretical model of spread of opinions called the Binary Agreement Model based on the naming game. Using this model, we study potential mechanisms for the dependencies observed in the data by matching model generated evolution of opinion with the empirically observed evolution in the data. Second, we examine the unique set of behavioral network data (based on electronic logs of dyadic contact via smartphones) collected at the University of Notre Dame. The participants are a sample of members of the entering class of freshmen in the fall of 2011 whose opinions on a wide variety of political and social issues have been regularly recorded—at the beginning and end of each semester—for the last three years. Using this data set, we measure the evolution of participants' opinions and ascertain how much this evolution depends on the cultural traits of individuals and the structural properties of social networks that they form. Our analysis of our empirical dataset shows that ties among people who are more likely to share opinions (e.g. same race, gender, or socioeconomic class) decay at a slower rate than ties among persons who are likely to have different opinions. The analysis also indicates that the partner selection of individuals is associated sharing a (political) opinion. These results offer an assessment of the level of impact of culture and social network dynamics on the evolution of opinions in multi-cultural social networks.

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1. Introduction

Recent work across the social and computational sciences shows that the web of ties linking people to one another and the relational events that constitute those ties can be profitably modeled using the tools of social network analysis (SNA) and network science (NS) [1,2]. From this perspective, sets of ties linking people to one another constitute a social network. Classical network analysis focused on the static modeling of patterns of social ties connecting people to one another [2], whereas more recent work has begun to conceive of social networks as dynamic systems subject to complex patterns of evolution and change [3].

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Social networks are important because they serve as the primary conduits through which opinions, ideas, behaviors, objects, practices, and even biological agents spread from person to person [4,5]. Persons are more likely to be persuaded to adopt a new idea or practice when it is transmitted to them by a trusted source or contact than when they are exposed to it via impersonal mechanisms such as the mass or electronic media [6,7,8]. Because opinion spread depends on network topology, whether a given opinion ends up capturing the minds of a large segment of the population or stays confined within a small social group is contingent on the exact configuration of

requires us to understand the processes by which people transmit cultural knowledge through social ties [10,11]. The traditional study of opinion spread, opinion evolution, and opinion transmission via social network ties has assumed a dynamic opinion landscape coupled to a large, static network structure [7,8]. More recently, the availability of temporally fine-grained data on social interactions has brought with it a renewed focus on modeling networks as complex, dynamic systems [12,13,14,15]. From this perspective, we can best think of social networks as generated by temporal changes in both relational states (tie dissolution and formation) and relational events (number of communications from one person to another) [16].

social ties at a given time [7,8,9]. This means that studying the evolution of opinion spread and opinion change

The topology of the network at any one time is constituted by both the relational states relevant to the system and the relational events that are involved in generating changes in relational states. For instance, two people may be linked by a durable social tie or share a common contact (a state configuration) while engaging in a number of interactions within a given time window (an aggregation of events). State and event dynamics occur at different time scales, ranging from single interactions taking up a few seconds, to longer-term processes (e.g., friendship) of tie formation, tie maintenance, and tie decay taking months or even years to play out [17,18].

For instance, matching processes on personal traits, which are subject to change and transmission (e.g. opinions), can be shown to account for gross similarity in fixed (and thus incapable of being transmitted) higher-level traits such as race and ethnicity [19]. The dynamic evolution of relational states in networks (the maintenance and dissolution of ties), is related to the likelihood that two individuals exhibit a match in pre-existing opinions that themselves may "flow" via network ties. From this perspective, there should exist an empirical coupling between the opinion structure of a given social network (the distribution of opinions across persons) and the social structure (the distribution of ties linking persons to one another) [20].

Modeling the spread of opinions in social networks thus requires that we specify generative mechanisms that account for both the evolution of relational states and events and the processes via which persons change (or keep) their opinions [7,8,11,12]. Failing to do this, researchers are bound to attribute observed matching across behaviors and traits to "contagion" processes, when they could be attributed to an assortment of other mechanisms that are correlated with the observation of a tie between two individuals. Studies have attempted to correct for the fact that, at any one time, the mechanisms that generate matching across individuals are also correlated with the probability of observing a tie between them (selection effects) , and these studies have generally found that contagion effects are reduced by as much as 70% [6,21].

In order for an opinion to diffuse across social ties, they must be durable (i.e. relatively stable in relation to the trait) network connections between persons dissimilar on the trait in question (or be a priori possible). Social influence occurs as dissimilar agents become more similar because of their social interactions. Relational events (e.g. calling, having dinner, going out, and messaging) premised on the stability of the relational state (being friends) drives people toward increasing similarity at the trait level. Personal traits align along the ties that persist over time [22].

However, there is also the possibility that agents decide not to change and instead decide to change either the pace of relational events (e.g. decrease the rate of interaction) or categorically change the relevant relational state (stop being friends). Personal opinions may remain unchanged while the network changes, as when people decide to terminate a tie because there is a difference in opinion [21]. In order to understand which opinions are more likely to diffuse we need, to know which behavioral differences are more likely to lead to tie dissolution and therefore less diffusion, and which behaviors are more likely to change as a result of social interaction through tie persistence and interpersonal interaction.

Methodologically, coupled dynamic systems pose difficult modeling challenges for inferring causality. The basic issue is how to isolate the different generative processes that lead to the observed network—trait matching (homophily), which is also referred to as "birds of a feather flock together." The problem is that there are a number of possibilities all of which are capable of generating the basic empirical result. Clustering on some trait (smoking, obesity, mental health, political attitudes) could be the result of (1) social influence (dissimilar people become more similar through their social interaction), (2) tie formation preferences and self-selection (people with certain predispositions seek out and form ties with similar others), and (3) differential tie persistence (dissimilar social ties are more likely to decay). Models of opinion and network evolution as coupled dynamic systems must, therefore, not only include mechanisms through which behaviors can diffuse through social interaction, but also mechanisms specifying how (and when) social networks evolve through tie formation and decay.

Finally, the spread of opinion is of significant interest to the U.S. military and its allies because opinions can impact the military strategy and the effectiveness of military operations. The U.S. Army states opinion can "…directly affect operations in a theater of operations halfway around the world and vice-versa" [23, p. 9]. Opinion dynamics are situated in a global information environment. Consequently, understanding and predicting the mechanisms for opinion spread across and within cultures can help inform military strategy and thus improve the efficacy of military operations.

2. Naming Game Based Models

The naming game is considered especially important in modeling opinion spread through population because its simple rules that can effectively capture how small changes to individual pairwise interactions can have large effects on the opinion of the community as a whole. In its most general form, naming game simulations follow a two stage process as they advance toward consensus [24,25,26,27]. At the start, the number of words in the list of each node sees a period of growth. When the average list is sufficiently long, the speaker and listener are likely to already share an opinion. At this point the community will enter the second stage and the lists will shrink as the system moves slowly towards a consensus. The time taken to achieve consensus depends heavily on network size and structure; for example, on complete graphs the time to consensus is on the order of $N^{1/2}$ while for a two dimensional lattice it is order *N*, where *N* is the total number of nodes in the system [27].

Often of more practical importance is the naming game with only two words. The dynamics of this system can be captured in the Binary Agreement Model (BAM), which features two distinct states and one mixed state [7,8,28]. Such a model examines the spread of two opinions over time in a social network, whereas social psychology research typically examines only a single time point [29]. This system is often used when the focus of a simulation is the competition between opinions over time with dynamic network structures and multiple individuals transmitting and receiving opinions. In such a case, the consensus times are somewhat shorter as the time needed to enter the second stage is much less than in the case of many opinions. On a complete graph, time to consensus has been shown to be on the order of ln(*N*) [24,25,26,28]. This result is well known for a static network such as that presented by the complete graph, but less is known for dynamic networks. Consequently, it is of further interest to determine how to extend this BAM to an evolving network. In the case of attempting to model social processes, this becomes particularly interesting if the network evolves to favor links between like-minded individuals.

Such an evolving system can be approximated using a slight alteration to the rules of the BAM on the complete graph. In this system, a rejection parameter will be introduced depending on the weight of agreement between the chosen speaker and listener. Specifically, for interactions between individuals in the same state, the interaction is always accepted. For interactions between nodes in the mixed state and nodes in one of the pure states, the interaction is accepted with probability e^r . For interactions between nodes in different pure states, the acceptance only has a probability e^{2r} . Obviously when $r = 0$ the system reduces to that of the unchanged BAM. Mathematically, this system is very similar to one in which you have an Erdős–Rényi **(**ER) graph, a probabilistic model for generating random graphs, of some degree $\langle k \rangle$ that redraws its edges at every time step to favor same-

opinion connections. In the analogous ER graph, individuals of the same opinion are always connected while connections to other individuals would be determined by the acceptance parameter *r*. Here, the probability that an interaction is rejected is analogous to the probability that the connection between those nodes simply did not exist during that time step in the ER graph.

Fig. 1. (Left) Scaling of the average consensus time is logarithmic regardless of the value of *r*, which acts only as a pre-factor causing slight change in slope and adding a constant. This can further be seen in (Right), where by scaling the histograms of the normalized probability distributions the different values for *r* follow the same time scale as they converge to the same point (*N*=1000).

Investigation into the average time to consensus within this system shows many strong similarities to that of the standard binary agreement model. For instance, Fig. 1 (Left) shows that the time to consensus scales as ln(*N*) whether the acceptance parameter is used or not. The parameter acts only as a weak pre-factor, but has little to no effect on the general scaling with respect to the system size. This can be further seen in Fig. 1 (Right), where the

convergence of the scaled histograms shows that for a

fixed system size *N*, there is only a single (r-dependent) time scale which governs the consensus process.

In Fig. 2, the increase in consensus time can be seen with respect to *r*.

Fig. 2. (Left) For fixed N, increase in r shows a growth behavior much stronger than a linear relationship yet slower than exponential growth. It also does not scale evenly with *N,* which can be further seen on (Right). Here, when the normalized probability density functions are scaled to the average consensus time, there is still a noticeable correction to scaling $(r=1)$.

The increase is much faster than that of a linear dependence, but it is slower than exponential. One important note from examining the consensus times is that the system never gets pulled into a meta-stable state even at relatively high values of *r.* At first this seems counter-intuitive; looking back to the ER analogue graph there is the possibility that at some point the two groups of differing opinions will be connected by a single pair of nodes that would struggle to create greater overall change on the system. This pitfall is escaped, however, by the ever changing nature of the graph. Note that while the communication frequency across clusters with different opinions can be significantly reduced, the connectivity between these clusters is still dense. Consequently the system does not get stuck in such a state because only a single time step before all of the edges are redrawn and the two distinct communities get a new set of nodes connecting them. Further, the scaling of the average consensus time with the network size *N* remains logarithmic, with an *r*-dependent pre-factor, reflecting the reluctance of communications between neighbors with differing opinions.

Much more can be done with the idea of evolving (or pseudo-evolving) networks in order to further the similarities between simulated systems and real world social interactions. The first and most obvious step would be to design a truly evolving random system and compare the results to those from the complete graph with the acceptance parameter. More than that, though, evolving systems can be studied through the lens of previous important works such as those on committed agents. The committed agents research shows that the introduction of nodes that will not change their opinion regardless of interaction with others creates large scale changes in the dynamics of the system [7,8]. A committed population introduces a sharp tipping point where an initial population above the tipping point will nearly always create a rapid consensus on the network while below it the system as a whole will be largely unaffected by the committed population's presence. Such a system could easily be adapted to an evolving network in order to investigate how the tipping points will shift. Also of interest is using the acceptance parameter to change how the committed agents in such a population seek out other nodes to communicate with, mimicking the existence of small closed off groups or, at the other extreme, missionaries and others groups that seek to share their views with as many others as possible.

3. Analysis of Empirical Data Set

A key implication of the dynamic naming game model is that the opinion evolution process is qualitatively different when modelled from a static network perspective than when opinions and social ties are considered as coupled dynamic systems. Yet, we still have very limited empirical evidence as to whether social ties are subject to differential decay depending on whether persons share a given set of opinions [21]. After all if the pre-existing sharing of opinions does not impact the dynamic stability of social ties, researchers may be correct in sticking with more tractable models that conceive of the network structure as static (and thus exogenous). In addition, if people select friends based on the fact that they share an opinion, rather than changing their opinions after they select their friends, then the conditions under which an entire network may be captured by a single opinion (under the static model) change dramatically.

Here we present results of both high levels of self-selection based on preexisting opinions and high levels of variability in the dynamic stability of social ties based on whether people already share opinions (mechanisms 2 and 3 above). Data are obtained from the ongoing *NetSense* study at the University of Notre Dame [30,31]. The study equipped a cohort of roughly 200 incoming first-year students with smartphones and recorded, among other things, the calls and texts made and received (but not the content of their communications) over the last three years using a monitoring app employed on each phone. This app logged and then transmitted to a secure database a Call Data Record (CDR) for each communication event.

Each CDR contains the phone numbers of the sender and receiver along with a timestamp indicating when the event occurred. While we have thousands of CDRs for texts and calls involving a *NetSense* subject and people outside the study, we use only the CDRs in which both parties are in the *NetSense* study. This is because for both

parties involved in the communication we have data derived from surveys on their socio-demographic characteristics, tastes, and opinions as well as other variables of interest derived from a survey administered to all *NetSense* participants prior to their arrival on campus. From these CDRs we identify 505 ties formed in the first two semesters (40 weeks) among 175 students who remained in the study throughout the first year and for which we have complete data on gender, race, and parental education as an indicator of socioeconomic class.

We selected these three socio-demographic characteristics because sociological research shows that opinions tend to cluster within same race, gender, and class groupings [20,21]. Therefore, people of the same race, gender, or class are more likely to share the same opinions, while cross-gender, cross-racial, and cross-class dyads are more likely to have different opinion. Differential decay of dyad type based on the race, gender, and class mix thus indicates the network structure is not static but that it coevolves with the opinion structure.

The results are shown in Fig. 3. We find positive evidence of network/opinion co-evolution. As shown in Fig. 3 (Left), cross-race dyads decay at a faster rate than same-race dyads, with Latino-Latino pairings exhibiting remarkable dynamic durability. Within the cross-race group, some pairings are more durable than others. In particular Asian-White pairings and Latino-Black pairings decay at a slower rate than other pairs. This is consistent with sociological research that reveals higher levels of agreement among members of these racial categories [32]. In addition, as shown by Fig. 3 (Right), social-class dyads, specifically those that feature a lower-class person paired with an upper-class person are less durable than other types of dyads, particularly dyads that pair two persons of lower class origin. Sociological research shows that there is more opinion agreement within classes (especially the "working class") than there is between the upper and lower classes [33]. This result is thus consistent with homophily in the opinion/network co-evolution hypothesis: Ties between groups with less opinion consensus are less durable than ties within groups with high opinion consensus. Finally, Fig. 3 (Bottom) shows that cross-gender pairings are dynamically less stable than same-gender pairings. Given the large amount of evidence for systematic opinion differences between men and women [34], this result is consistent with a model in which persons end social connections featuring opinion disagreement at a faster rate than social network connections featuring opinion agreement [21].

Fig. 3. Survival curves for four sets of dyads classified according to ethnoracial pairing (Left), social-class pairing (Right), and gender pairing (Bottom). Estimates of hazard rates (the probability of dissolution at a given time for each dyad type) and survivor probabilities (the probability that a tie will still be active at a given time) are computed using the Kaplan–Meier estimator: at each time point *t* a risk set is specified as those objects that have survived up until *t* and an event set is those objects that experience an event in the interval from *t* to Δ*t.* Vertical hash marks indicate right-censored cases: ties active at the end of the observation period. (Left) Ethnoracial categories are White (W), Asian (A), Black (B), and Latino (L). The figure indicates that Latino–Latino (L–L) pairs have the slowest decay rate, followed by White–White (W–W), Black–Black (B–B), Asian–Asian (A–A), Asian–White (A–W), and Black–Latino (B–L) pairs. Black–White (B–W), Latino–White (L–W), Asian–Black (A– B), and Asian–Latino (A–L) have the fastest decay rates. (Right) Social-class categories are "Upper Class" (both parents have a college degree: U), "Middle Class" (only one parent has a college degree: M) and "Lower Class" (neither parent has a college degree: L). Results show dyads that match upper- and lower-class persons (L–U) decay faster than other types of dyads. (Bottom) Gender categories are Male (M) and Female (F). The results show that same gender dyads (F–F and M–M) decay at a slower rate than cross-gender dyads (M–F or F–M).

Fig. 4 summarizes results reported in [35]. The figure is a study of the coevolution of liberal-conservative opinions among *NetSense* study participants. We set out to test the hypothesis of whether the correlation over time between personal opinions and network ties can be accounted for by self-selection of persons into dyads with likeminded others (opinion homophily) or opinion change via influence after people with different opinions became tied. We used stochastic-actor based models as this simulation-based technique allows us to separate the effect of selfselection from opinion change via influence [36]. We found that for the most part persons remained fairly stable in their opinions and that they selected friends based on opinion similarity rather than changing their political opinion based on the influence of people with different opinions. This provides further evidence of another mechanism (selfselection) that may help to account for the co-evolution of the opinion distribution and the social network configuration.

Fig. 4. Evolution of the political-opinion network among *NetSense* study participants (Left). Red nodes indicate "conservative" respondents and Blue nodes indicate "liberal" respondents based on their self-placement on a seven-point scale. Two participants are connected if one sends an SMS text message or makes a voice call to another who reciprocates during that time period. Participants were surveyed four times in each time period. We used stochastic actor-based models [36] to model the effect of opinion similarity and network connectivity on the probability of keeping a tie. For the most part, self-selection of like-minded participants and exogenous opinion accounted for the majority of overtime correlation (77.4%) across different snapshots, with opinion change via influence accounting for only a small portion of the correlation (3.9%).

4. Conclusions

This paper attempts to bridge the gap between Social Impact Theory and theoretical models of the spread of opinions. Our initial results of measuring impact of connectivity dynamics in models of opinion spread are promising and will be continued to include cases with different networks structures, presence of committed agents, and clustering in the form of communities. At the same time empirical results on different probabilities of tie dissolution for nodes similar and dissimilar in certain traits are important in guiding which type of tie dynamics should be considered in theoretical models. We will also address the differences between BAM which is applied to the entire network, and the Social Impact Theory that conceptualizes opinion spread to an individual, as an impact, using three variables: (i) strength of opinion sources, (ii) proximity of opinion sources over space and time, and (iii) number of opinion sources [37]. Future work will extend this theory to modeling opinion dynamics for networks rather than individuals: Specifically, we will compare models that represent: communication frequency, measures of connectivity, and number of opinion sources.

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